

~1

1)  $f(x) = 3x$   $F(x) = 3 \cdot \frac{x^{1+1}}{1+1} = \frac{3}{2}x^2 + C$

3)  $f(x) = \frac{x^3}{3}$   $F(x) = \frac{1}{3} \cdot \frac{x^{3+1}}{3+1} = \frac{x^4}{12} + C$

2)  $f(x) = 4x^2 + x - 2$   $F(x) = \frac{4}{3}x^3 + \frac{x^2}{2} - 2x$

4)  $f(x) = \frac{1}{x^2}$   $F(x) = \frac{x^{-2+1}}{-2+1} = -\frac{1}{x} + C$

~2

1)  $f(x) = 2\sin x$   $F(x) = -2\cos x + C$

3)  $f(x) = 3\cos x - 4\sin x$   $F(x) = 3\sin x + 4\cos x$

2)  $f(x) = 5\cos x$   $F(x) = 5\sin x + C$

4)  $f(x) = 5\sin x + 2\cos x$   $F(x) = 2\sin x - 5\cos x$

~3

1)  $f(x) = x^2 + \frac{3}{\sqrt{x}}$   
 $F(x) = \frac{x^3}{3} + 3 \cdot 2\sqrt{x} + C = \frac{x^3}{3} + 6\sqrt{x} + C$

2)  $f(x) = x^3 - \frac{4}{\sqrt{x}}$   
 $F(x) = \frac{x^4}{4} - 4 \cdot 2\sqrt{x} + C = \frac{x^4}{4} - 8\sqrt{x} + C$

3)  $f(x) = \sin\left(3x + \frac{\pi}{3}\right)$   
 $F(x) = -\frac{1}{3}\cos\left(3x + \frac{\pi}{3}\right) + C$

4)  $f(x) = \cos\left(2x + \frac{\pi}{6}\right)$   
 $F(x) = \frac{1}{2}\sin\left(2x + \frac{\pi}{6}\right) + C$

~4

1)  $f(x) = (x+1)(x+3) = x^2 + 4x + 3$   
 $F(x) = \frac{x^3}{3} + 2x^2 + 3x + C$

2)  $f(x) = (1-x)(3+x) = 3 - 2x - x^2$   
 $F(x) = 3x - x^2 - \frac{x^3}{3} + C$

3)  $f(x) = \frac{x^2}{3} + \sin\left(x + \frac{\pi}{3}\right)$   
 $F(x) = \frac{x^3}{9} - \cos\left(x + \frac{\pi}{3}\right) + C$

4)  $f(x) = -\frac{x^2}{2} + \cos\left(x - \frac{\pi}{6}\right)$   
 $F(x) = -\frac{x^3}{6} + \sin\left(x - \frac{\pi}{6}\right) + C$

~5

1)  $f(x) = x + 1$ ,  $M(-2, 3)$   
 $F(-2) = \frac{x^2}{2} + x + C = \frac{(-2)^2}{2} - 2 + C = 3$   
 $C = 3$  gener  $F(x) = \frac{x^2}{2} + x + 3$   
 $x = -\frac{b}{2a} = -\frac{1}{2 \cdot \frac{1}{2}} = -1$   
 $y(-1) = \frac{1}{2} - 1 + 3 = 2,5$

2)  $f(x) = 4 + x$ ,  $M(-2, 3)$   
 $F(-2) = 4x + \frac{x^2}{2} + C = -8 + \frac{(-2)^2}{2} + C = 3$   
 $C = 9$  gener  $F(x) = \frac{x^2}{2} + 4x + 9$   
 $x = -\frac{b}{2a} = -\frac{4}{2 \cdot \frac{1}{2}} = -4$   
 $y(-4) = \frac{16}{2} - 16 + 9 = 1$

3)  $f(x) = \sin x$ ,  $M\left(\frac{\pi}{2}, 1\right)$   
 $F\left(\frac{\pi}{2}\right) = -\cos x + C = -\cos\frac{\pi}{2} + C = 1$   
 $C = 1$  gener  $F(x) = -\cos x + 1$

4)  $f(x) = \cos x$ ,  $M(\pi, -1)$   
 $F(\pi) = \sin x + C = \sin\pi + C = -1$   
 $C = -1$  gener  $F(x) = \sin x - 1$

1)  $f(x) = x^{-2}$   $M(1; -1)$   
 $F(1) = \frac{x^{-1}}{-1} + C = -\frac{1}{1} + C = -1 + C = -1$   
 $C = 0$  generic  $F(x) = -\frac{1}{x}$

2)  $f(x) = x^{-3}$   $M(-1; 0)$   
 $F(-1) = \frac{x^{-2}}{-2} + C = -\frac{1}{2 \cdot 1^2} + C = 0$   
 $C = \frac{1}{2}$  generic  $F(x) = -\frac{1}{2x^2} + \frac{1}{2}$

3)  $f(x) = 2 - \frac{1}{\cos^2 x}$ ,  $x \in [0; \frac{\pi}{2}]$   $M(\frac{\pi}{4}; \frac{\pi}{2})$   
 $F(\frac{\pi}{4}) = 2x - \operatorname{tg} x + C = 2 \cdot \frac{\pi}{4} - \operatorname{tg} \frac{\pi}{4} + C = \frac{\pi}{2}$   
 $C = 1$  generic  $F(x) = 2x - \operatorname{tg} x + 1$

4)  $f(x) = \frac{2}{\sin^2 x} + 1$   $M(\frac{\pi}{4}; \frac{\pi}{4})$   
 $F(\frac{\pi}{4}) = -2 \operatorname{ctg} x + x + C = -2 \operatorname{ctg} \frac{\pi}{4} + \frac{\pi}{4} + C = \frac{\pi}{4}$   
 $= \frac{\pi}{4} \Rightarrow C = 2$   $F(x) = -2 \operatorname{ctg} x + x + 2$

1)  $f(x) = 3x^2 + 3 \sin x$ ,  $F(x) = x^3 - 3 \cos x$

2)  $f(x) = x^4 + 4 \cos x$ ,  $F(x) = \frac{x^5}{5} + 4 \sin x$

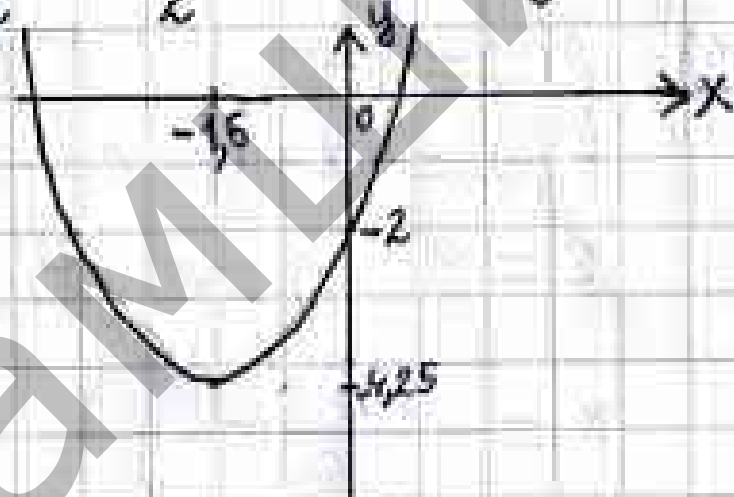
1)  $f(x) = 9x^2 + \sin 3x$   
 $F(x) = 9 \cdot \frac{x^3}{3} - \frac{1}{3} \cos 3x + C = 3x^3 - \frac{\cos 3x}{3}$

2)  $f(x) = 12x^3 - \cos 4x$   
 $F(x) = 12 \cdot \frac{x^4}{4} - \frac{1}{4} \sin 4x = 3x^4 - \frac{\sin 4x}{4}$

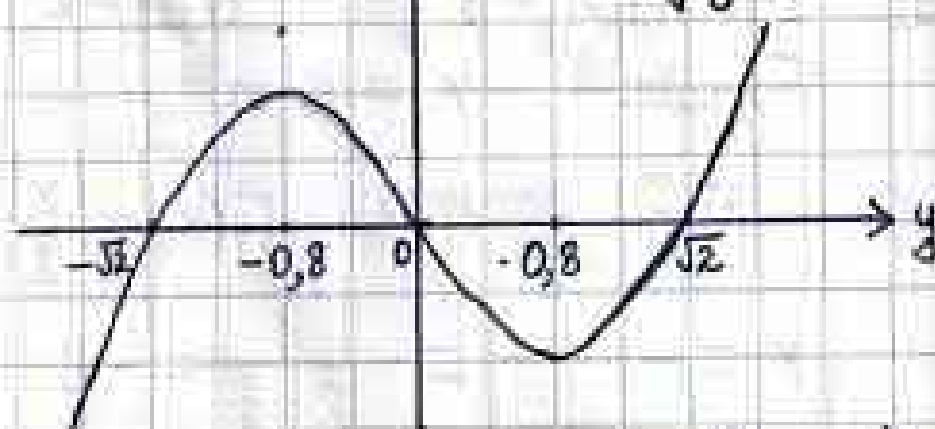
3)  $f(x) = \cos 2x - \frac{1}{\sqrt{2x-3}} + 2$   
 $F(x) = \frac{1}{2} \sin 2x - \sqrt{2x-3} + 2x + C$

4)  $f(x) = \frac{1}{\sqrt{5-2x}} + \sin 5x + 1$   
 $F(x) = -\sqrt{5-2x} - \frac{1}{5} \cos 5x + x + C$

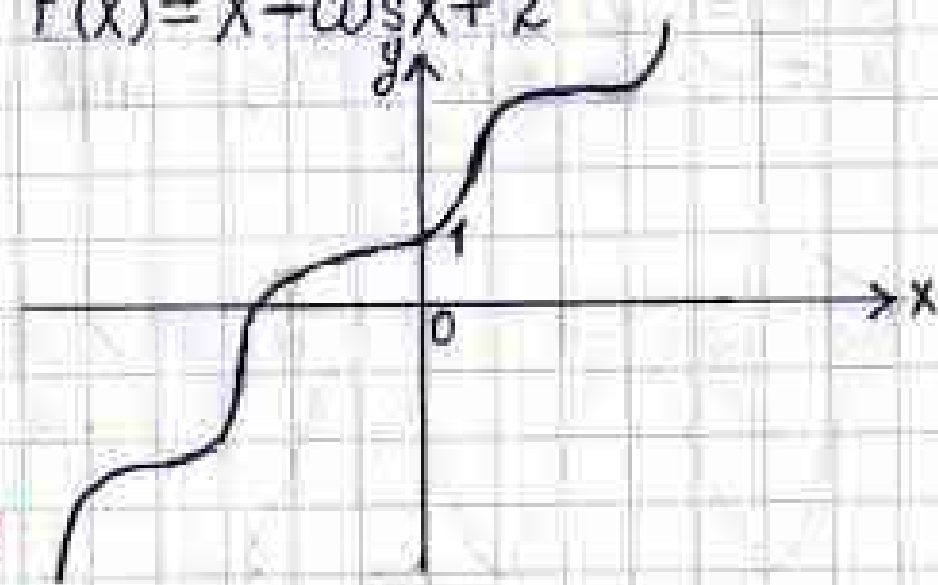
1)  $f(x) = 2x + 3$   $M(1; 2)$   
 $F(x) = x^2 + 3x + C$   
 $1 + 3 + C = 2 \Rightarrow C = -2$   
 $F(x) = x^2 + 3x - 2$   
 $x = -\frac{b}{2a} = -\frac{3}{2} = -1,5$   $y = -4,5$



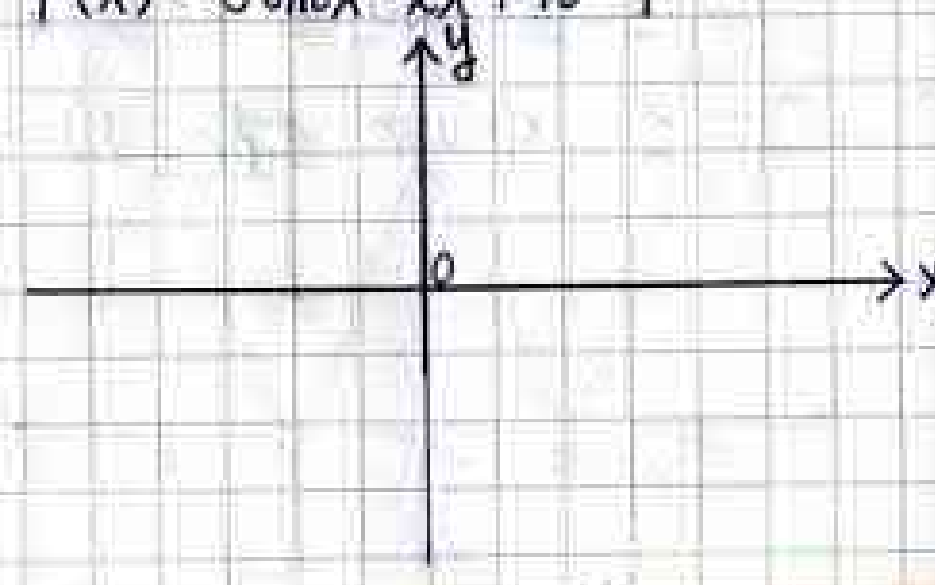
2)  $f(x) = 3x^2 - 2$   $M(2; 4)$   
 $F(x) = x^3 - 2x + C$   
 $8 - 4 + C = 4 \Rightarrow C = 0$   
 $F(x) = x^3 - 2x$   
 O<sub>y</sub> осімен (0; 0) O<sub>x</sub> осімен (-√2; 0), (√2; 0)  
 سایر نقاطی  $x = \pm \sqrt{\frac{2}{3}} \approx \pm 0,8$



3)  $f(x) = 1 + \sin x$   $M(0; 1)$   
 $F(x) = x - \cos x + C$   
 $-\cos 0 + C = 1 \Rightarrow C = 2$   
 $F(x) = x - \cos x + 2$

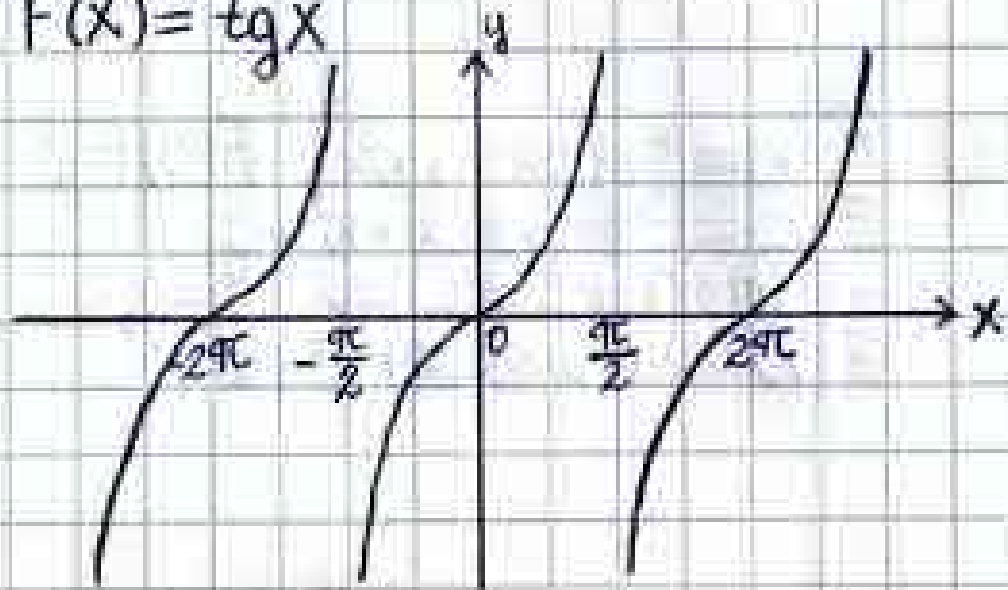


4)  $f(x) = 3 \cos x - 2$   $M(\frac{\pi}{2}; -1)$   
 $F(x) = 3 \sin x - 2x + C$   
 $3 \sin \frac{\pi}{2} - 2 \cdot \frac{\pi}{2} + C = -1$   $C = \pi - 4$   
 $F(x) = 3 \sin x - 2x + \pi - 4$

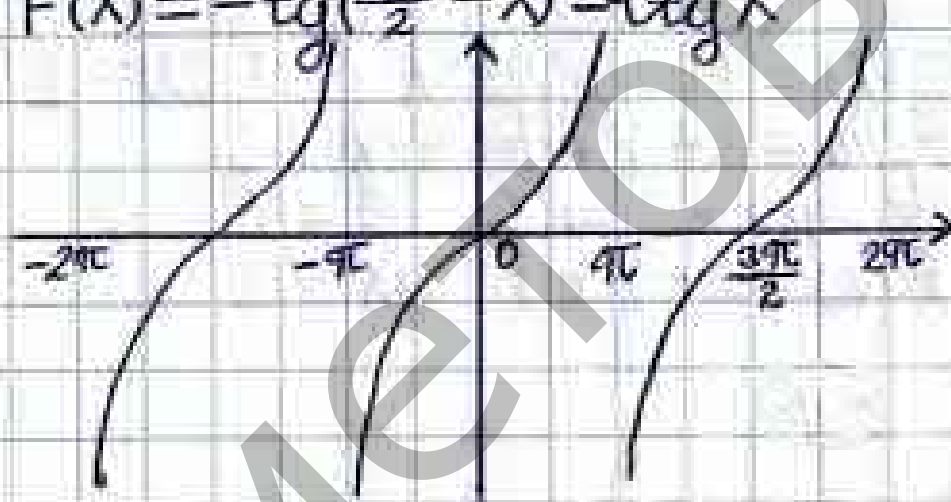


~10

1)  $f(x) = \frac{1}{\sin^2(\frac{\pi}{2} + x)}$   $M(-\frac{\pi}{4}; -1)$   
 $F(x) = -\text{ctg}(\frac{\pi}{2} + x) + C = \text{tg}x + C$   
 $\text{tg}(-\frac{\pi}{4}) + C = -1 \Rightarrow C = 0$   
 $F(x) = \text{tg}x$



2)  $f(x) = \frac{1}{\cos^2(\frac{3\pi}{2} - x)}$   $M(\frac{5\pi}{6}; \sqrt{3})$   
 $F(x) = -\text{tg}(\frac{3\pi}{2} - x) + C$   
 $-\text{tg}(\frac{3\pi}{2} - \frac{5\pi}{6}) + C = \sqrt{3} \Rightarrow C = 0$   
 $F(x) = -\text{tg}(\frac{3\pi}{2} - x) = \text{ctg}x$



~11

1)  $f(x) = (x-1)^3$   
 $F(x) = \frac{(x-1)^4}{4} + C$

2)  $f(x) = (1-2x)^2$   
 $F(x) = \frac{(1-2x)^3}{3} \cdot (-\frac{1}{2}) + C = -\frac{(2x-1)^3}{6} + C$

3)  $f(x) = \frac{1}{2\sqrt{x}} + 11x^{10}$   
 $F(x) = \sqrt{x} + x^{11} + C$

4)  $f(x) = \frac{1}{x^2} + 12x^8$   
 $F(x) = \frac{x^{-2+1}}{-2+1} + 12 \cdot \frac{x^9}{9} + C = -\frac{1}{x} + \frac{4}{3}x^9 + C$

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1)  $f(x) = x - \cos^2 x$   $x \in [0; \frac{\pi}{2}]$   
 $F(x) = \frac{x^2}{2} - \text{tg}x + C$   $M(\frac{\pi}{4}; \frac{9\pi^2}{32})$   
 $\frac{\pi^2}{16 \cdot 2} + \text{tg} \frac{\pi}{4} + C = \frac{9\pi^2}{32}$   $C = 1$   
 $F(x) = \frac{x^2}{2} - \text{tg}x + 1$

2)  $f(x) = 2\sin^2 x - x$   $x \in (0; \frac{\pi}{2}]$   
 $F(x) = -2\text{ctg}x - \frac{x^2}{2} + C$   $M(\frac{\pi}{4}; -\frac{\pi^2}{32})$   
 $-2\text{ctg} \frac{\pi}{4} - \frac{\pi^2}{16 \cdot 2} + C = -\frac{9\pi^2}{32}$   $C = 2$   
 $F(x) = -2\text{ctg}x - \frac{x^2}{2} + 2$

3)  $f(x) = x^{-3} + \cos x$   $x \in (0; +\infty)$   
 $F(x) = -\frac{1}{2x^2} + \sin x + C$   $M(0,5\pi; -\frac{1}{2\pi^2})$   
 $-\frac{1}{2 \cdot 0,25\pi^2} + \sin 0,5\pi + C = -\frac{1}{2\pi^2}$   
 $C = \frac{1}{\pi^2} - 1$   $F(x) = -\frac{1}{2x^2} + \sin x + \frac{1}{\pi^2} - 1$

4)  $f(x) = x^3 - \sin x$   $x \in (0; +\infty)$   
 $F(x) = \frac{x^4}{4} + \cos x + C$   $M(\pi; \frac{\pi^4}{4})$   
 $\frac{\pi^4}{4} + \cos \pi + C = \frac{\pi^4}{4}$   $C = 1$   
 $F(x) = \frac{x^4}{4} + \cos x + 1$

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$F(x) = (x-3)\sqrt{x-5}$   $x \in (5; +\infty)$   
 $f(x) = (x-3)\sqrt{x-5} + (x-3) \cdot (\sqrt{x-5})' =$   
 $= \sqrt{x-5} + \frac{x-3}{2\sqrt{x-5}} = \frac{x-13}{2\sqrt{x-5}}$   
 ИС:  $y = F(x)$   $\varphi = a$   $y = f(x)$   
 $\varphi = a$   $\varphi = a$   $\varphi = a$   
 балмайди

2)  $F(x) = \frac{2x-5}{3+5x}$   $x \in (-\infty; -\frac{3}{5})$   
 $f(x) = \frac{(2x-5)'(3+5x) - (2x-5)(3+5x)'}{(3+5x)^2} =$   
 $= \frac{2(3+5x) - 5(2x-5)}{(3+5x)^2} = \frac{31}{(3+5x)^2}$   
 ИС:  $y = F(x)$   $\varphi = a$   $y = f(x)$   
 $\varphi = a$   $\varphi = a$   $\varphi = a$   $\varphi = a$   
 балмайди

1)  $F'(x) = 4x^3 - 3x^2$  mit  $F(1) = 3$   
 $F(x) = x^4 - x^3 + C$   
 $1 - 1 + C = 3 \Rightarrow C = 3$   
 $F(x) = x^4 - x^3 + 3$

2)  $F'(x) = 5x^4 - 4x^3 - 2x$  mit  $F(1) = 4$   
 $F(x) = x^5 - x^3 - x^2 + C$   
 $1 - 1 - 1 + C = 4 \Rightarrow C = 5$   
 $F(x) = x^5 - x^3 - x^2 + 5$

1)  $F'(x) = 1 + x + \cos 2x$ ,  $F(0) = 1$   
 $F(x) = x + \frac{x^2}{2} + \frac{1}{2} \sin 2x + C$   
 $\frac{1}{2} \sin 0 + C = 1 \Rightarrow C = 1$   
 $F(x) = x + \frac{x^2}{2} + \frac{1}{2} \sin 2x + 1$

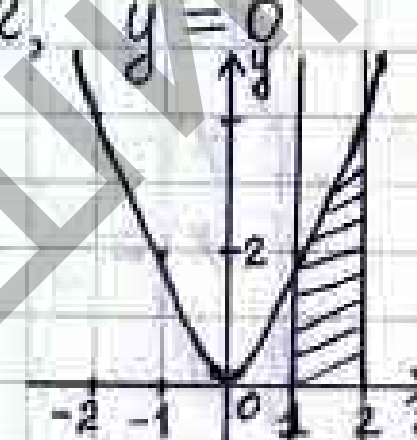
2)  $F'(x) = \sin 2x + 3x^2$ ,  $F(0) = 2$   
 $F(x) = -\frac{1}{2} \cos 2x + x^3 + C$   
 $-\frac{1}{2} \cos 0 + C = 2 \Rightarrow C = 2,5$   
 $F(x) = -0,5 \cos 2x + x^3 + 2,5$

1)  $F(x) = -\frac{1}{4} \cos 2x - \frac{1}{2} \cos x$   $f'(x) = \frac{1}{2} \sin 2x + \frac{1}{2} \sin x = \frac{1}{2} \cdot 2 \sin \frac{3x}{2} \cdot \cos \frac{x}{2}$   
 2)  $F(x) = -\frac{3}{8} \cos \frac{4x}{3} + \frac{3}{4} \cos \frac{2x}{3}$   $f'(x) = \frac{1}{2} \sin 4x - \frac{1}{2} \sin \frac{2x}{3} = \cos x \sin \frac{x}{3}$

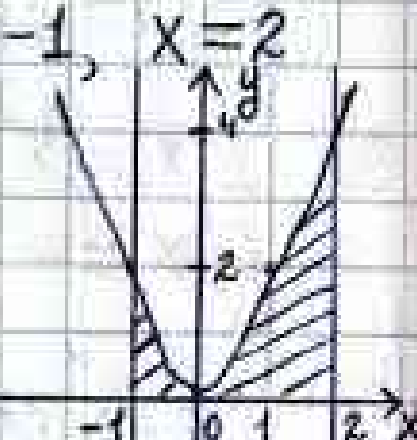
1)  $F(x) = \frac{3}{8}x - \frac{1}{4} \sin 2x + \frac{1}{32} \sin 4x$   $f'(x) = \frac{3}{8} - \frac{1}{4} \cdot 2 \cos 2x + \frac{1}{32} \cdot 4 \cos 4x =$   
 $= \frac{3}{8} - \frac{1}{2}(1 - 2 \sin^2 x) + \frac{1}{8}(1 - 2 \sin^2 2x) = \frac{3}{8} - \frac{1}{2} + \sin^2 x + \frac{1}{8} - \frac{1}{4} \sin^2 2x =$   
 $= \sin^2 x - \frac{1}{4} \cdot 4 \sin^2 x \cdot \cos^2 x = \sin^2 x - \sin^2(1 - \sin^2 x) = \sin^4 x$

2)  $F(x) = \frac{3}{8}x + \frac{1}{4} \sin 2x + \frac{1}{32} \sin 4x$   $f'(x) = \frac{3}{8} + \frac{1}{2} \cos 2x + \frac{1}{8} \cos 4x =$   
 $= \frac{3}{8} + \cos^2 x - \frac{1}{2} + \frac{1}{4} \cos^2 2x - \frac{1}{8} = \cos^2 x + \frac{1}{4} (2 \cos^2 x - 1)^2 - \frac{1}{4} =$   
 $= \cos^2 x + \frac{1}{4} (4 \cos^4 x - 4 \cos^2 x + 1) - \frac{1}{4} = \cos^4 x$

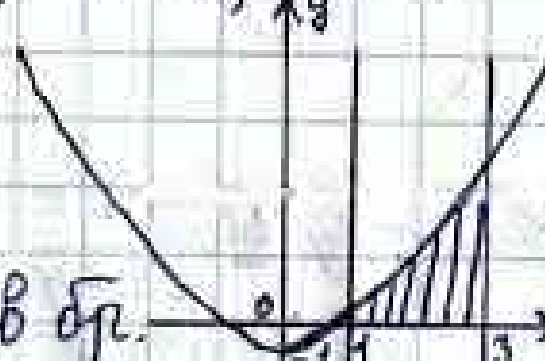
1)  $y = x^2$ ,  $x=1$ ,  $x=2$ ,  $y=0$   
 $F(x) = \frac{x^3}{3}$   
 $S = F(2) - F(1) =$   
 $\frac{8}{3} - \frac{1}{3} = 2 \frac{1}{3} \text{ kb } \overline{\text{Dr}}$




2)  $y = x^2$ ,  $y=0$ ,  $x=-1$ ,  $x=2$   
 $F(x) = \frac{x^3}{3}$   
 $S = F(2) - F(-1) =$   
 $\frac{8}{3} + \frac{1}{3} = 3 \text{ kb } \overline{\text{Dr}}$



3)  $y = 2x^2 - 1$ ,  $y=0$ ,  $x=1$ ,  $x=3$   
 $F(x) = \frac{2}{3}x^3 - x$   
 $S = F(3) - F(1) =$   
 $= 16 - \frac{2}{3} = 15 \frac{1}{3} \text{ kb } \overline{\text{Dr}}$



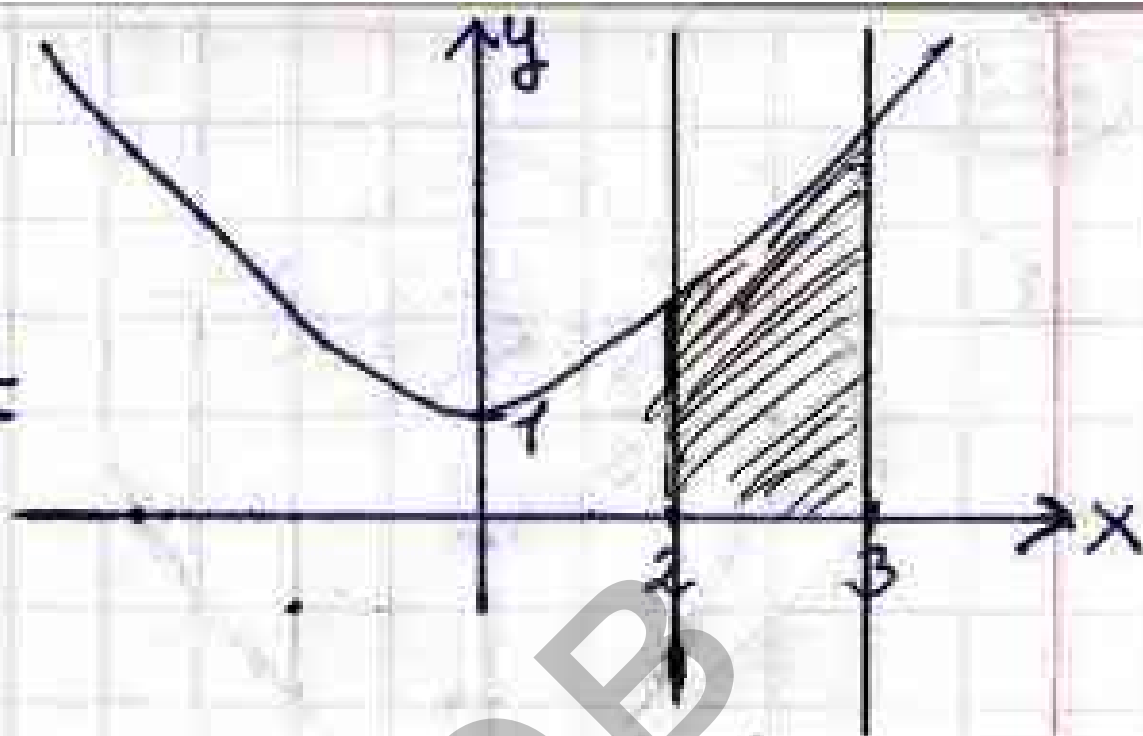
4)  $y = 2x^2 + 1$ ,  $y=0$ ,  $x=2$ ,  $x=3$   
 $F(x) = \frac{2}{3}x^3 + x$   
 $S = F(3) - F(2) =$   
 $= 21 - 7 \frac{1}{3} = 13 \frac{2}{3} \text{ kb } \overline{\text{Dr}}$



$$y = 2x^2 + 1 \quad y = 0, x = 2, x = 3$$

$$F(x) = \frac{2x^3}{3} + x$$

$$S = F(3) - F(2) = \frac{2 \cdot 27}{3} + 3 - \frac{2 \cdot 8}{3} - 2 = 13\frac{2}{3}$$



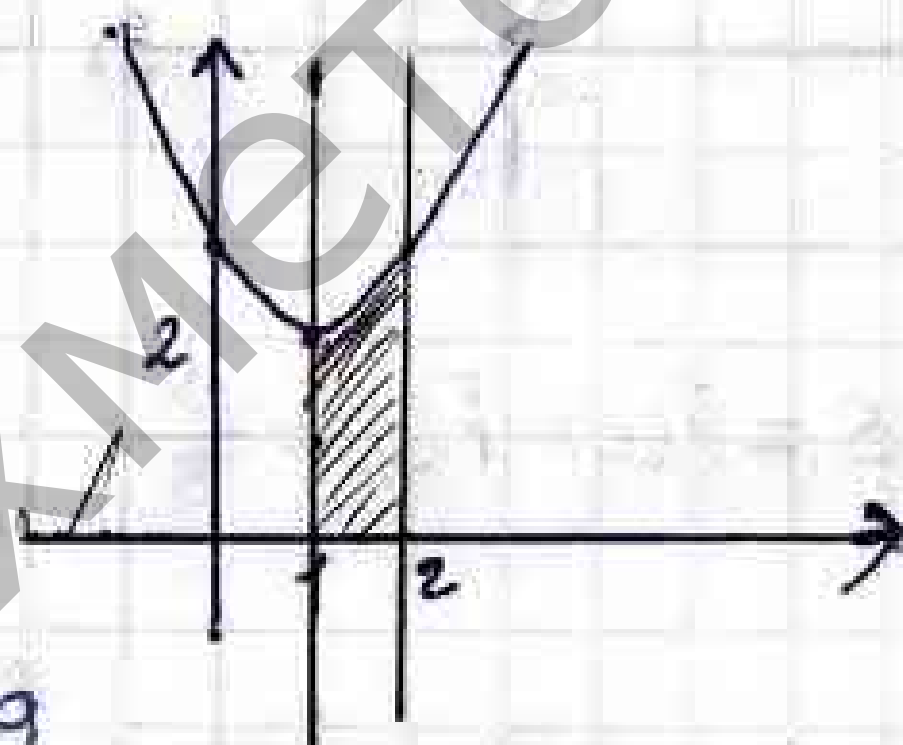
~ 19

$$1) \quad y = x^2 - 2x + 3, \quad y = 0, x = 1, x = 2$$

$$F(x) = \frac{x^3}{3} - x^2 + 3x$$

$$n = -\frac{-2}{2} = 1 \quad m = 1 - 2 + 3 = 2$$

$$S = F(2) - F(1) = \frac{8}{3} - 4 + 6 - \left(\frac{1}{3} - 1 + 3\right) = 9$$

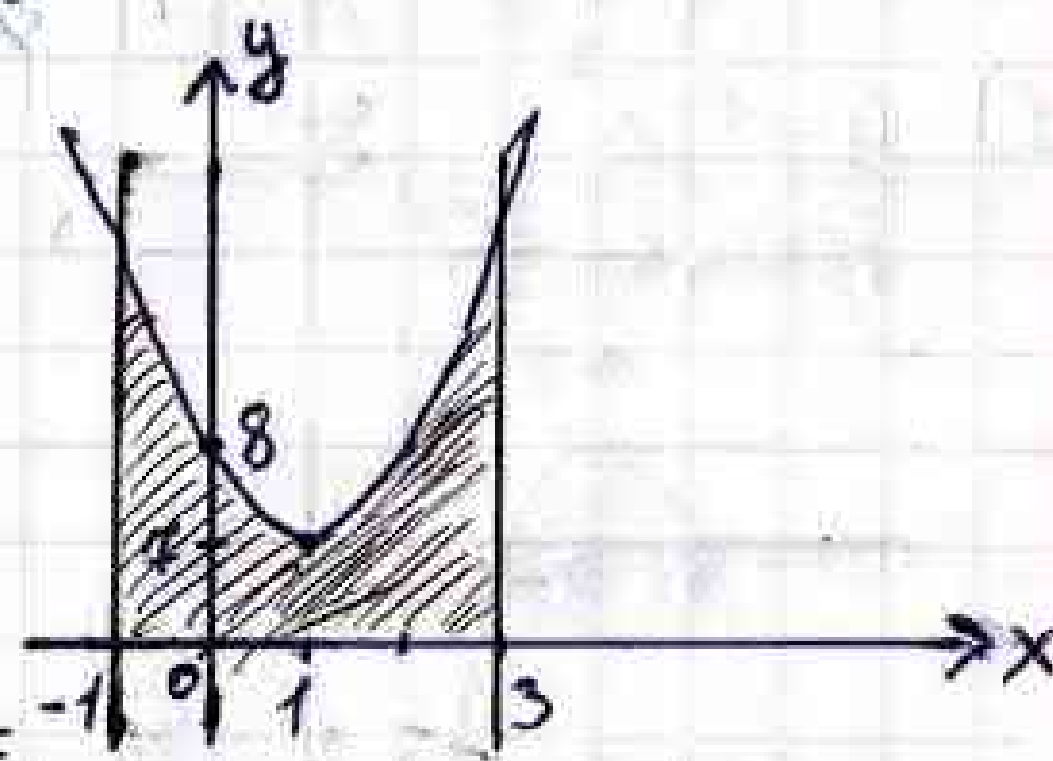


$$2) \quad y = x^2 - 2x + 8, \quad y = 0, x = -1, x = 3$$

$$F(x) = \frac{x^3}{3} - x^2 + 8x$$

$$n = -\frac{-2}{2} = 1 \quad m = 1 - 2 + 8 = 7$$

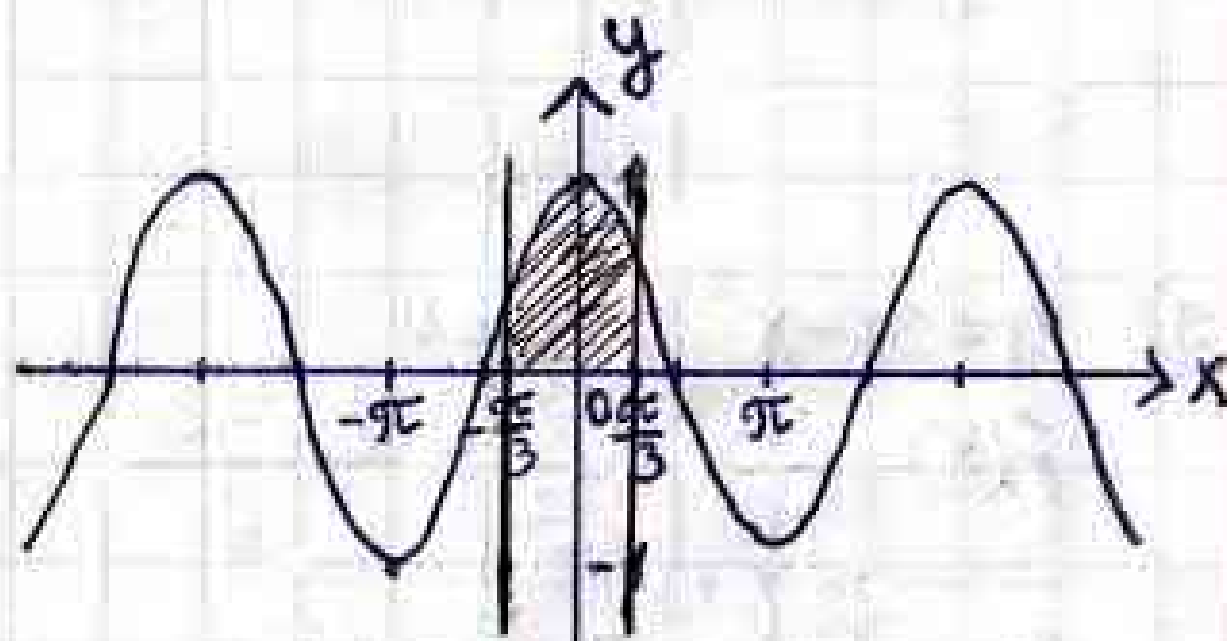
$$S = F(3) - F(-1) = \frac{27}{3} - 9 + 8 \cdot 3 - \left(-\frac{1}{3} - 1 - 8\right) = 24 - 9 - \frac{1}{3} = 14\frac{2}{3}$$



$$3) \quad y = \cos x, \quad y = 0, x = -\frac{\pi}{3}, x = \frac{\pi}{3}$$

$$F(x) = \sin x$$

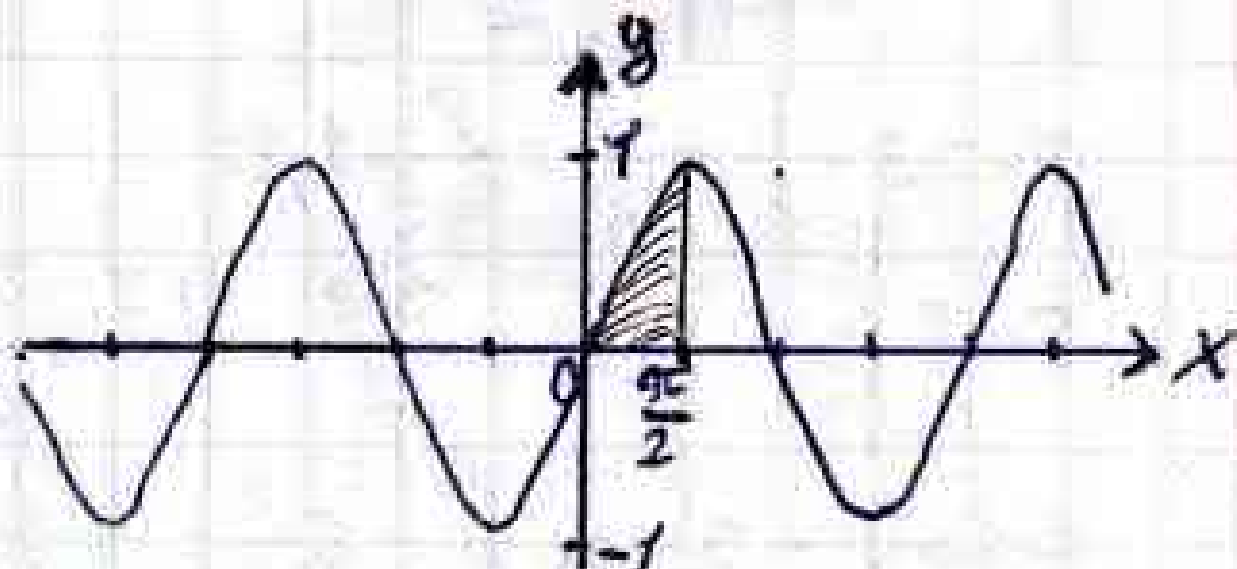
$$S = F\left(\frac{\pi}{3}\right) - F\left(-\frac{\pi}{3}\right) = \sin\frac{\pi}{3} + \sin\frac{\pi}{3} = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$$



$$4) \quad y = \sin x, \quad y = 0, x = 0, x = \frac{\pi}{2}$$

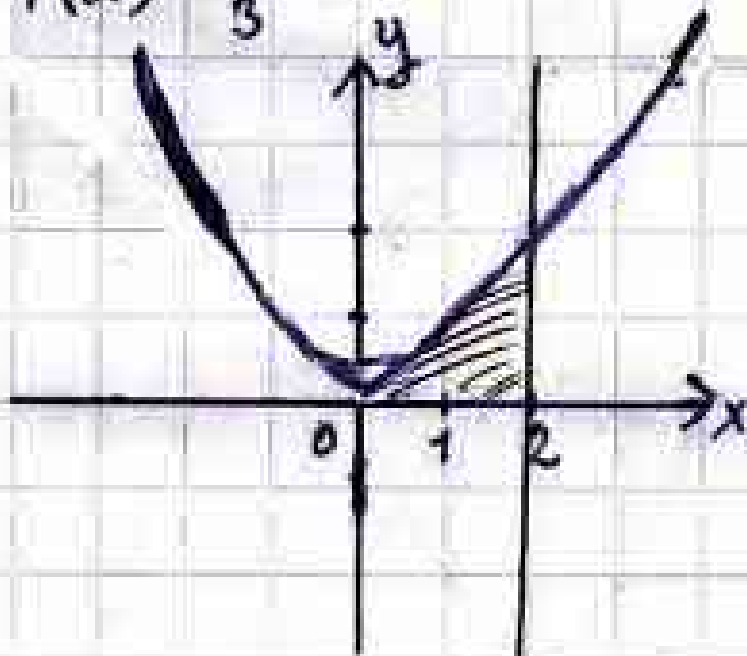
$$F(x) = -\cos x$$

$$S = F\left(\frac{\pi}{2}\right) - F(0) = -\cos\frac{\pi}{2} + \cos 0 = 1$$



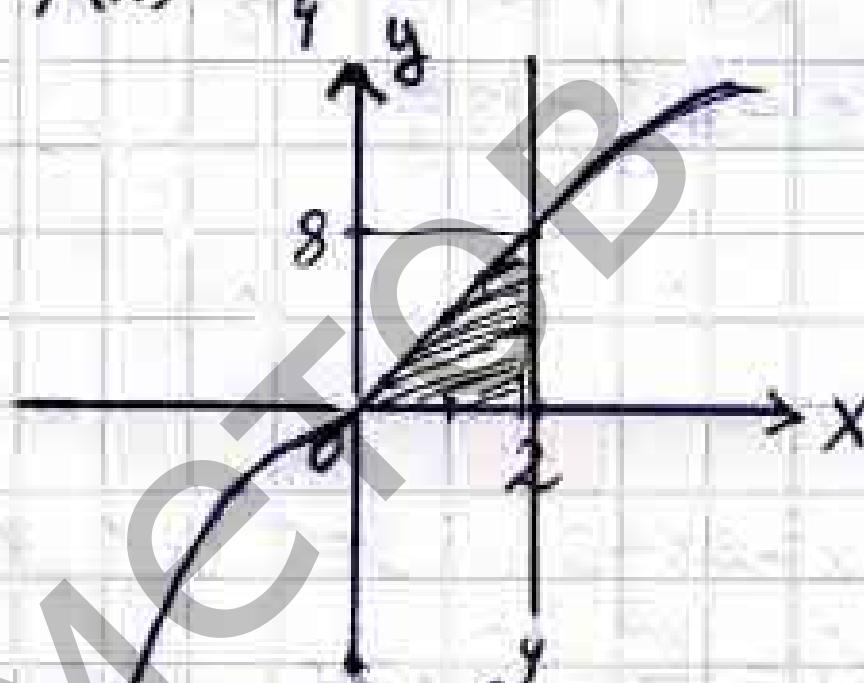
(20)

1)  $y = x^2$ ,  $y = 0$ ,  $x = 2$   
 $F(x) = \frac{x^3}{3}$



$$S = F(2) - F(0) = \frac{8}{3}$$

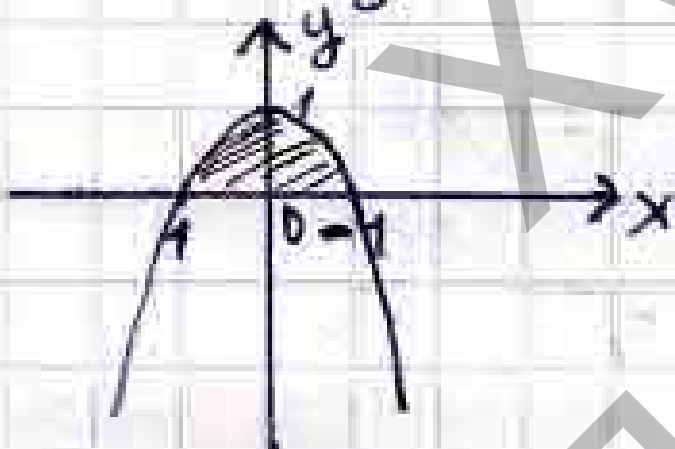
2)  $y = x^3$ ,  $y = 0$ ,  $x = 2$   
 $F(x) = \frac{x^4}{4}$



$$S = F(2) - F(0) = \frac{2^4}{4} = 4$$

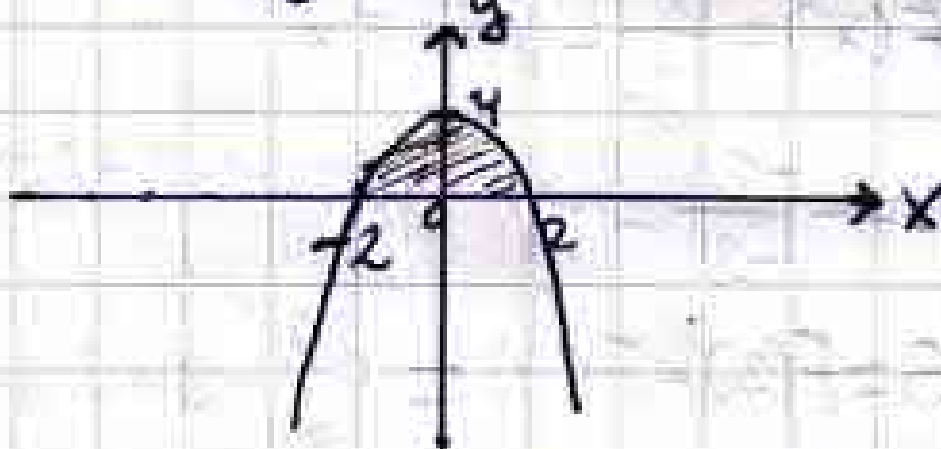
(21)

1)  $y = 1 - x^2$ ,  $y = 0$   
 $F(x) = x - \frac{x^3}{3}$



$$S = F(1) - F(-1) = 1 - \frac{1}{3} - \left(-1 + \frac{1}{3}\right) = \frac{4}{3}$$

2)  $y = -x^2 + 4$ ,  $y = 0$   
 $F(x) = -\frac{x^3}{3} + 4x$



$$S = F(2) - F(-2) = -\frac{8}{3} + 8 - \left(\frac{8}{3} - 8\right) = \frac{16}{3}$$

$$\frac{16}{3} + 16 = \frac{16}{3} + 15\frac{2}{3} = 12\frac{2}{3}$$

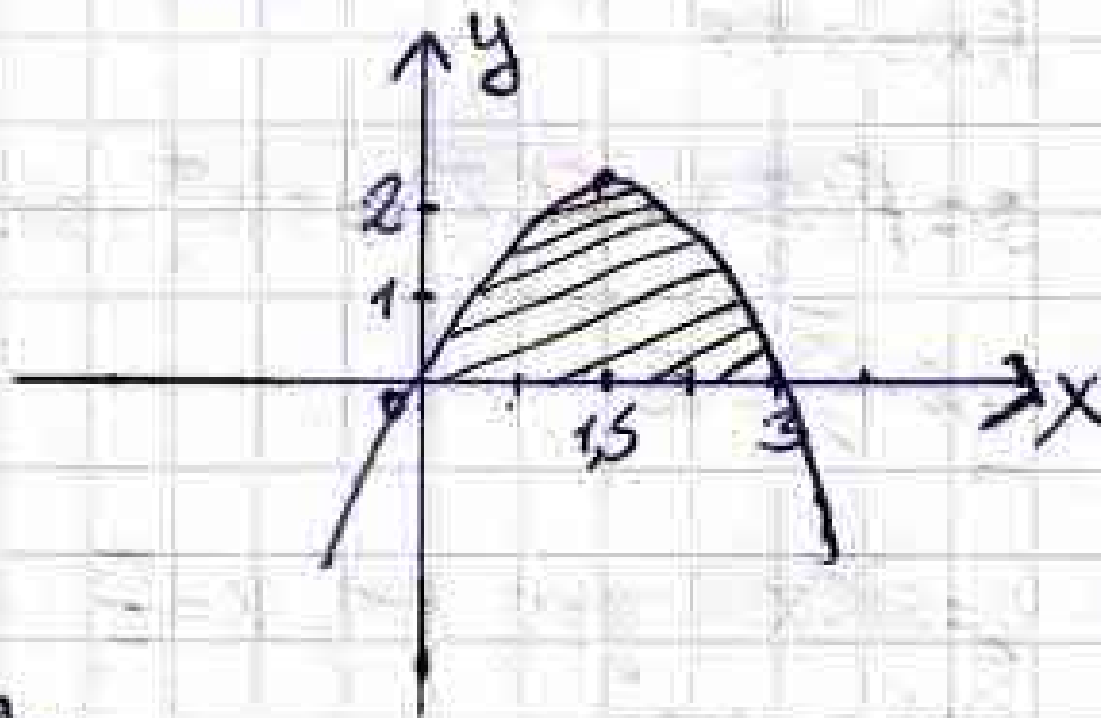
3)  $y = 3x - x^2$ ,  $y = 0$   
 $F(x) = \frac{3x^2}{2} - \frac{x^3}{3} = \frac{9x^2 - 2x^3}{6}$

$$n = -\frac{3}{2 \cdot (-1)} = \frac{3}{2} = 1,5$$

$$m = 3 \cdot \frac{3}{2} - \frac{9}{4} = \frac{9}{4} = 2,25$$

$$S = F(3) - F(0) = \frac{3 \cdot 9}{2} - \frac{27}{3} = \frac{27}{2} - 9 =$$

$$= 13\frac{1}{2} - 9 = 12\frac{3}{2} - 8\frac{2}{2} = 4\frac{1}{2}$$



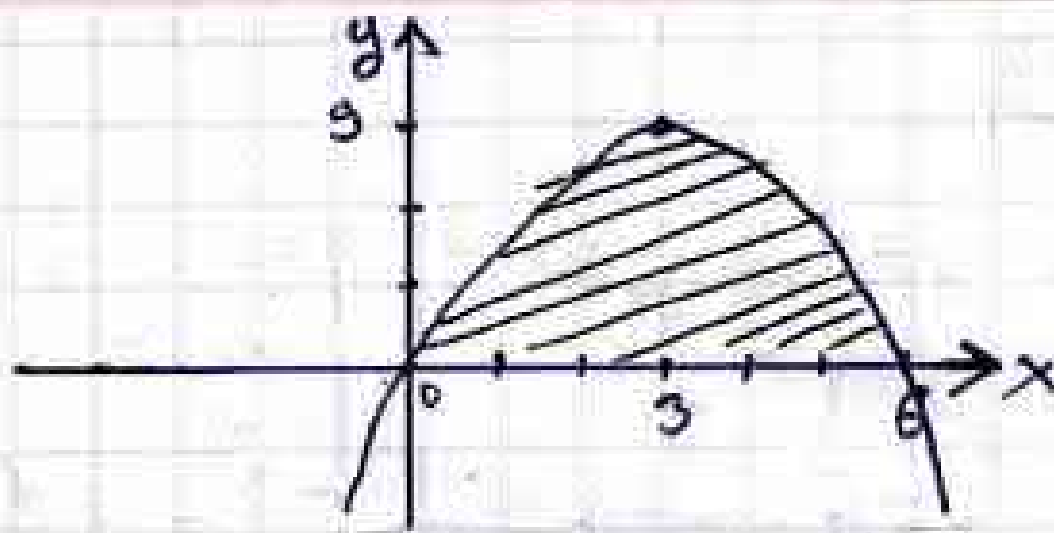
$$4) \quad y = 6x - x^2; \quad y = 0$$

$$F(x) = 3x^2 - \frac{x^3}{3}$$

$$n = -\frac{6}{2 \cdot (-1)} = 3$$

$$m = 18 - 9 = 9$$

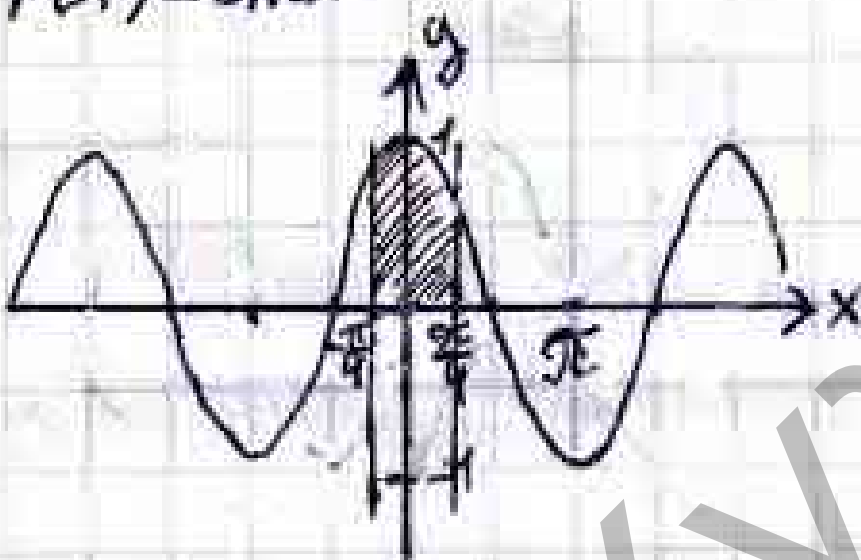
$$S = F(6) - F(0) = 3 \cdot 36 - \frac{6^3}{3} = 108 - 72 = 36$$



~22

$$1) \quad y = \cos x \quad x = -\frac{\pi}{4} \quad x = \frac{\pi}{4}$$

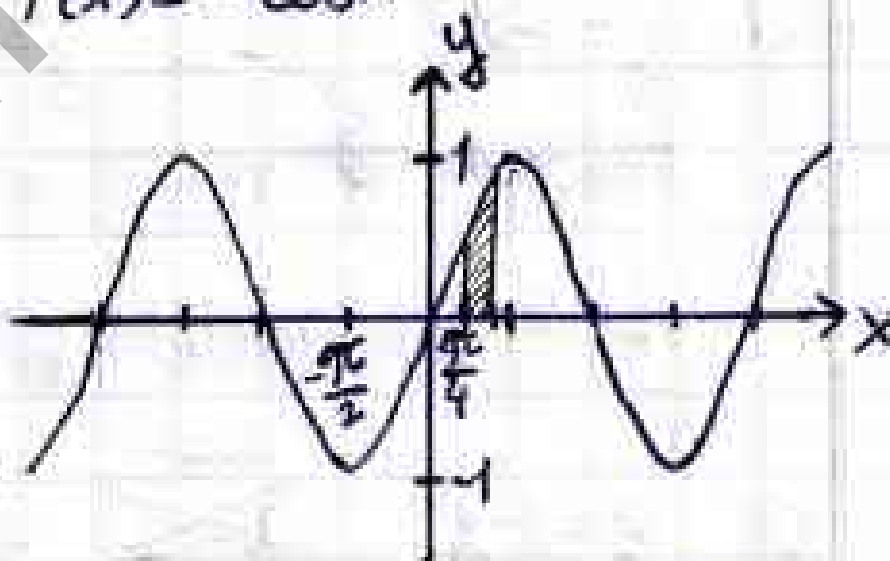
$$F(x) = \sin x$$



$$S = F\left(\frac{\pi}{4}\right) - F\left(-\frac{\pi}{4}\right) = \sin\frac{\pi}{4} + \sin\frac{\pi}{4} = \sqrt{2}$$

$$2) \quad y = \sin x \quad x = \frac{\pi}{4} \quad x = \frac{\pi}{3}$$

$$F(x) = -\cos x$$



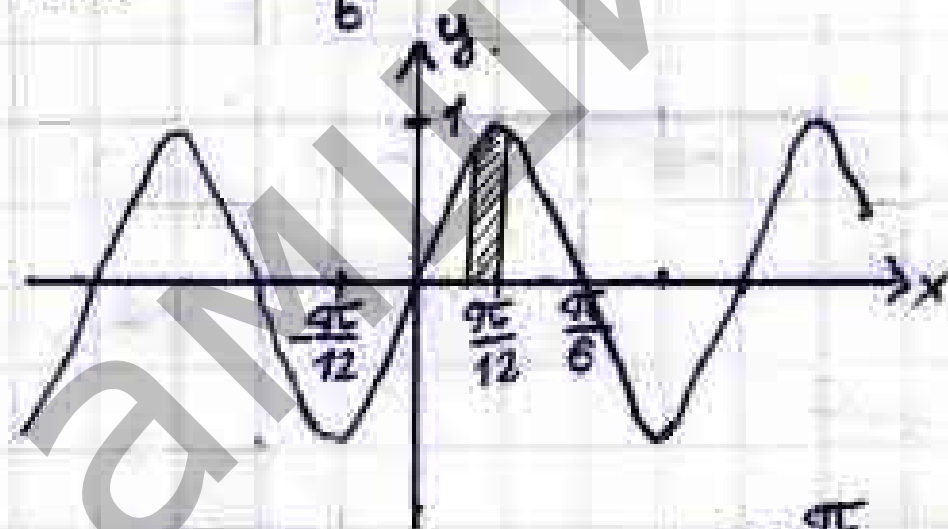
$$S = F\left(\frac{\pi}{3}\right) - F\left(\frac{\pi}{4}\right) = -\cos\frac{\pi}{3} + \cos\frac{\pi}{4} =$$

$$= -\frac{1}{2} + \frac{\sqrt{2}}{2} = \frac{\sqrt{2}-1}{2}$$

~23

$$1) \quad y = \sin 6x \quad x = \frac{\pi}{18} \quad x = \frac{\pi}{12}$$

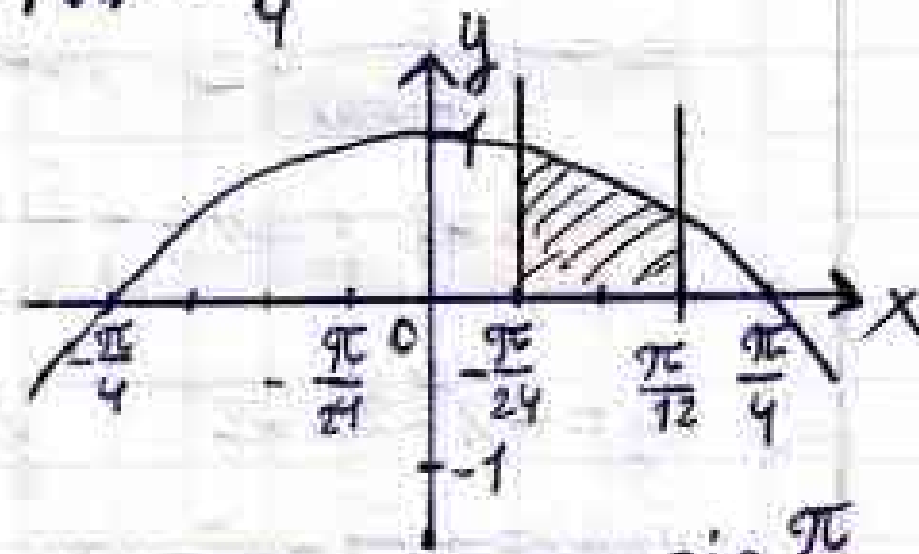
$$F(x) = -\frac{\cos 6x}{6}$$



$$S = F\left(\frac{\pi}{12}\right) - F\left(\frac{\pi}{18}\right) = -\frac{\cos\frac{\pi}{2}}{6} + \frac{\cos\frac{\pi}{3}}{6} = \frac{1}{12}$$

$$2) \quad y = \cos 4x \quad x = \frac{\pi}{24} \quad x = \frac{\pi}{12}, \quad y = 0$$

$$F(x) = \frac{\sin 4x}{4}$$



$$S = F\left(\frac{\pi}{12}\right) - F\left(\frac{\pi}{24}\right) = \frac{\sin\frac{\pi}{3}}{4} - \frac{\sin\frac{\pi}{6}}{4} =$$

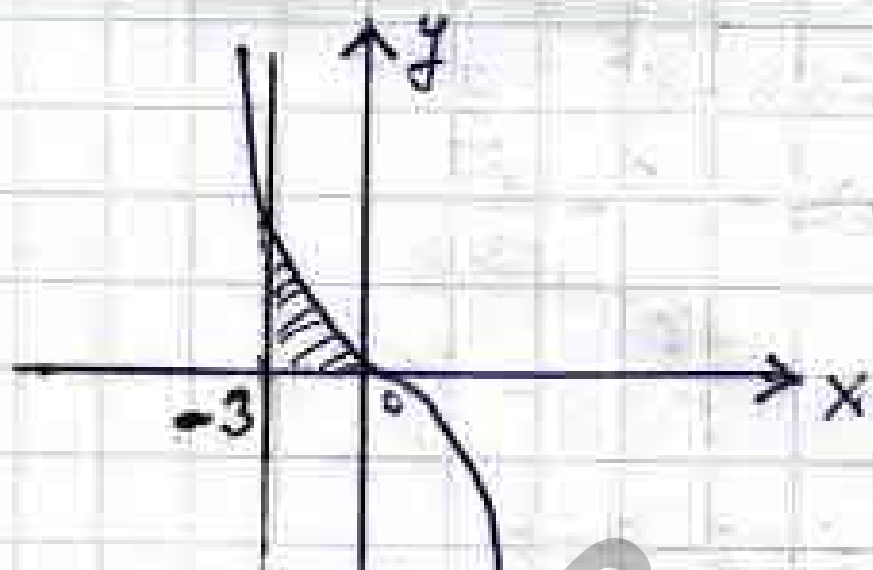
$$= \frac{\frac{\sqrt{3}}{2} - \frac{1}{2}}{4} = \frac{\sqrt{3}-1}{8}$$

~24

$$1) \quad y = -x^3 \quad x = -3, \quad y = 0$$

$$F(x) = -\frac{x^4}{4}$$

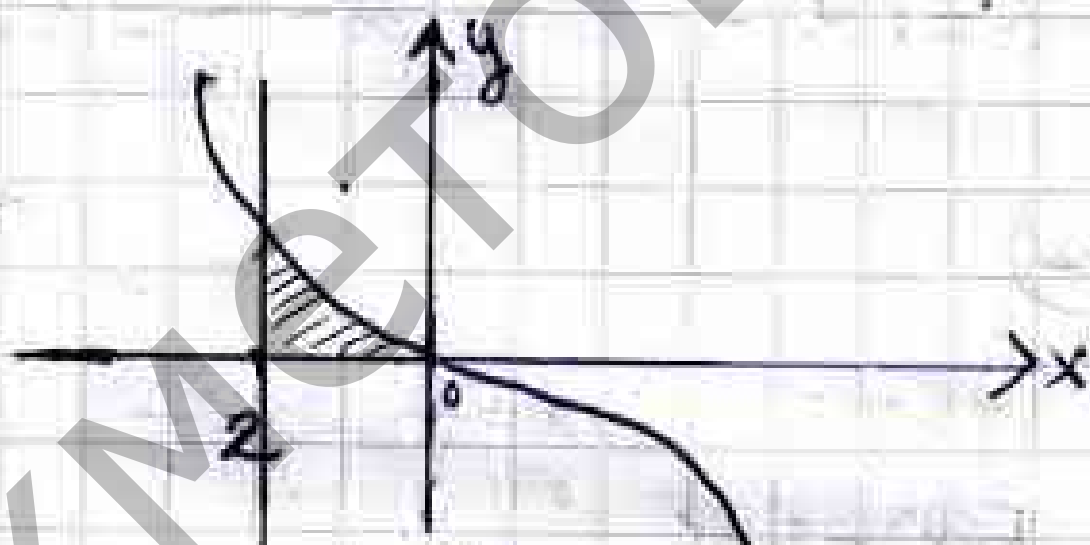
$$S = F(0) - F(-3) = -\frac{-3^4}{4} = 20\frac{1}{4}$$



$$2) \quad y = -2x^3 \quad x = -2, \quad y = 0$$

$$F(x) = -2 \cdot \frac{x^4}{4} = -\frac{x^4}{2}$$

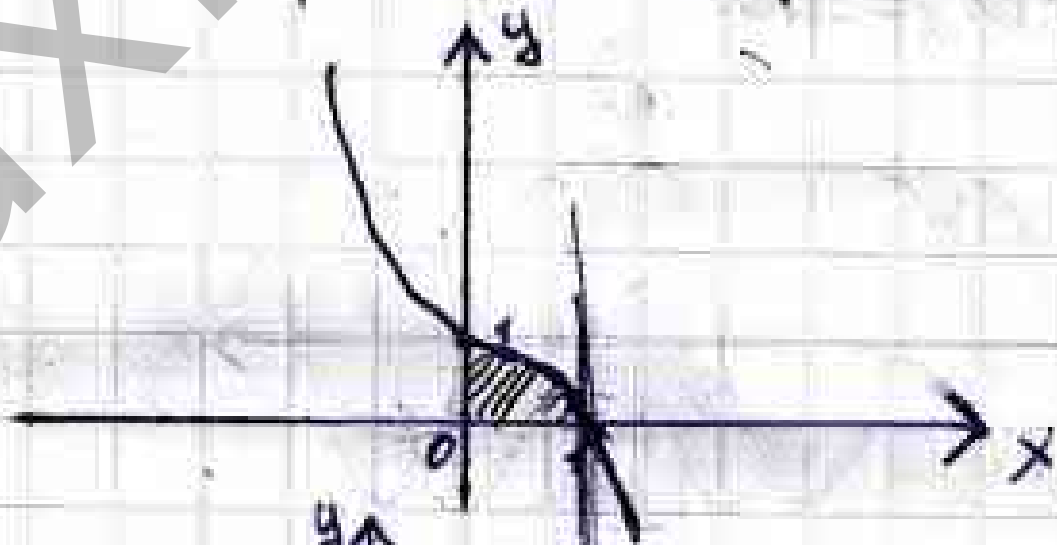
$$S = F(0) - F(-2) = 2 \cdot \frac{16}{4} = 8$$



$$3) \quad y = 1 - x^3 \quad x = 0, \quad y = 0$$

$$F(x) = x - \frac{x^4}{4}$$

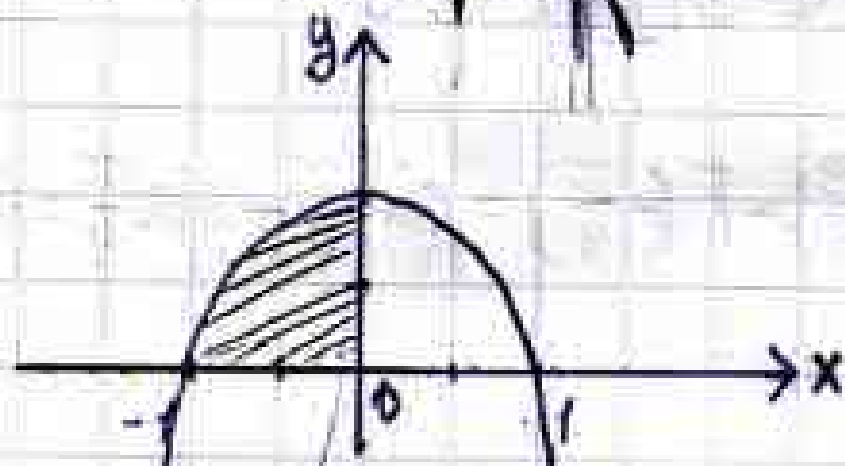
$$S = F(1) - F(0) = 1 - \frac{1}{4} = \frac{3}{4}$$



$$4) \quad y = 1 - x^2 \quad x = 0, \quad y = 0$$

$$F(x) = x - \frac{x^3}{3}$$

$$S = F(0) - F(-1) = -\left(-1 + \frac{1}{3}\right) = \frac{2}{3}$$

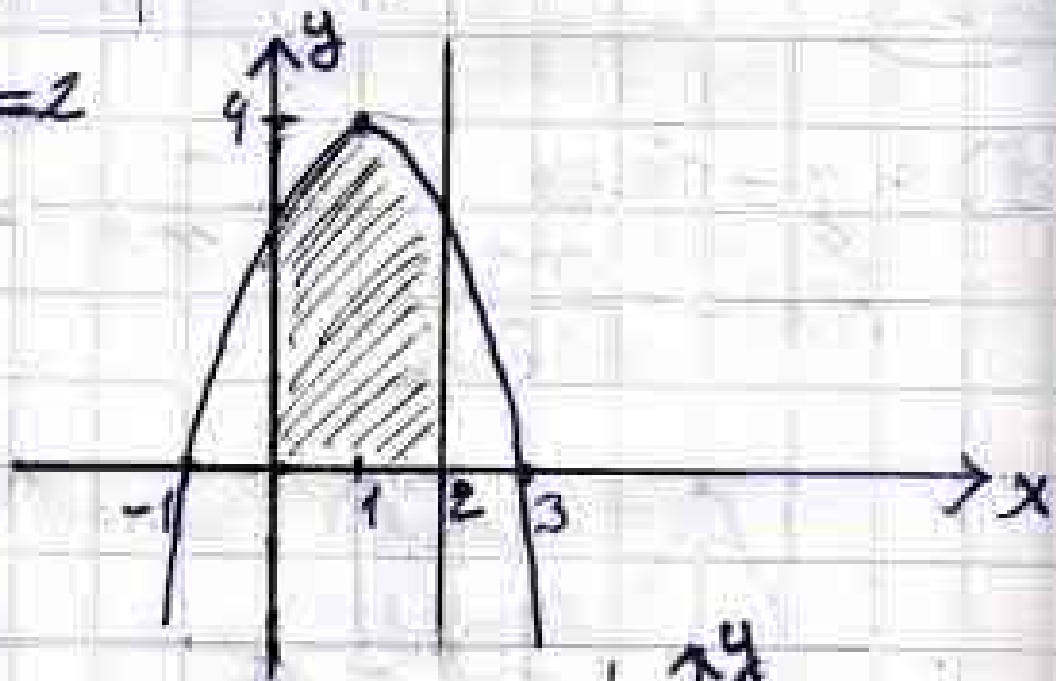


$$5) \quad y = -x^2 + 2x + 3 \quad y = 0, \quad x = 0, \quad x = 2$$

$$F(x) = -\frac{x^3}{3} + x^2 + 3x$$

$$n = -\frac{2}{2 \cdot (-1)} = 1 \quad m = -1 + 2 + 3 = 4$$

$$S = F(2) - F(0) = -\frac{8}{3} + 4 + 6 = \frac{30 - 8}{3} = \frac{22}{3}$$

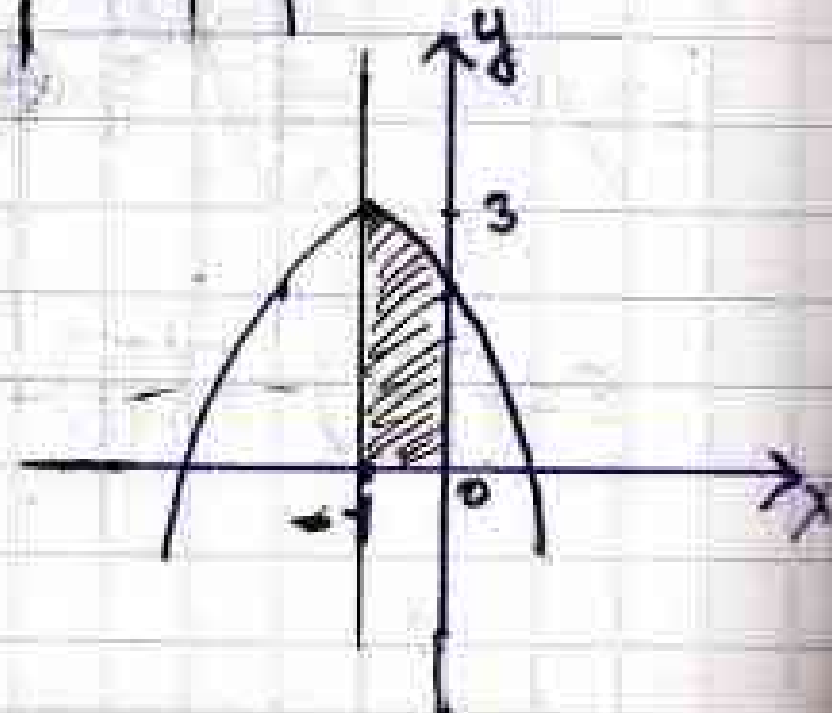


$$6) \quad y = -x^2 - 2x + 2 \quad y = 0, \quad x = -1, \quad x = 0$$

$$F(x) = -\frac{x^3}{3} - x^2 + 2x$$

$$n = -\frac{-2}{2 \cdot (-1)} = -1 \quad m = -1 + 2 + 2 = 3$$

$$S = F(0) - F(-1) = -\frac{-1}{3} - 1 - 2 = \frac{8}{3} = 2\frac{2}{3}$$

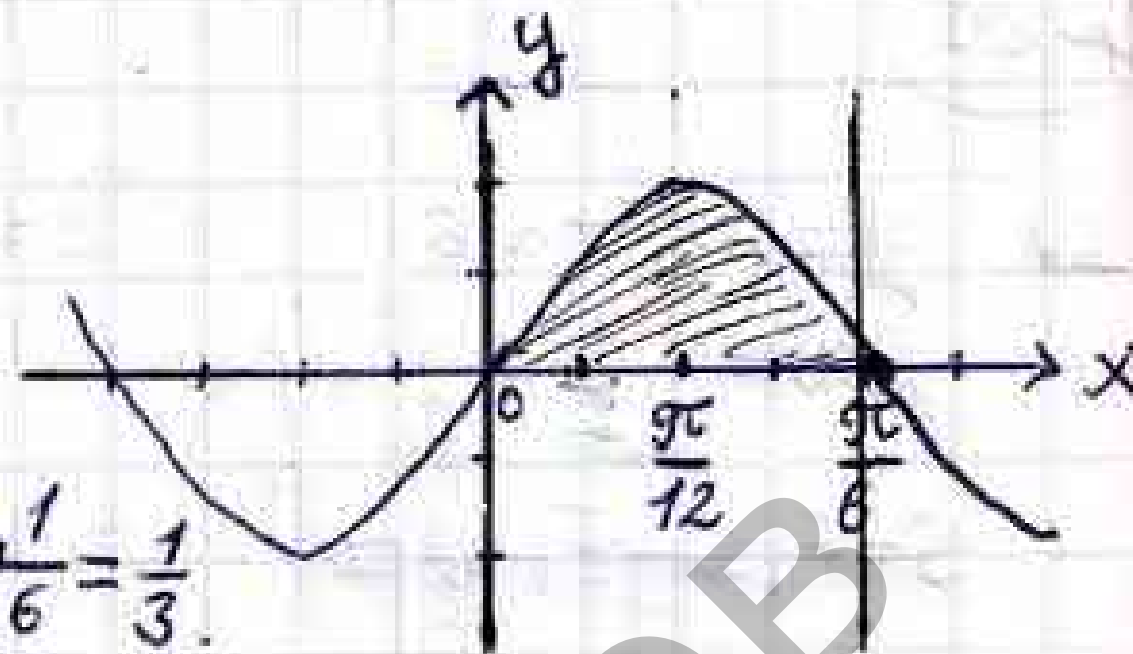




25

$$y = \sin 6x \quad y = 0 \quad 0 \leq x \leq \frac{\pi}{6}$$

$$F(x) = -\frac{\cos 6x}{6}$$



$$S = F\left(\frac{\pi}{6}\right) - F(0) = -\frac{\cos 9\pi}{6} + \frac{\cos 0}{6} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

~26

$$1) f(x) = x^2 + 2x + 3, \quad [-2; 1]$$

and

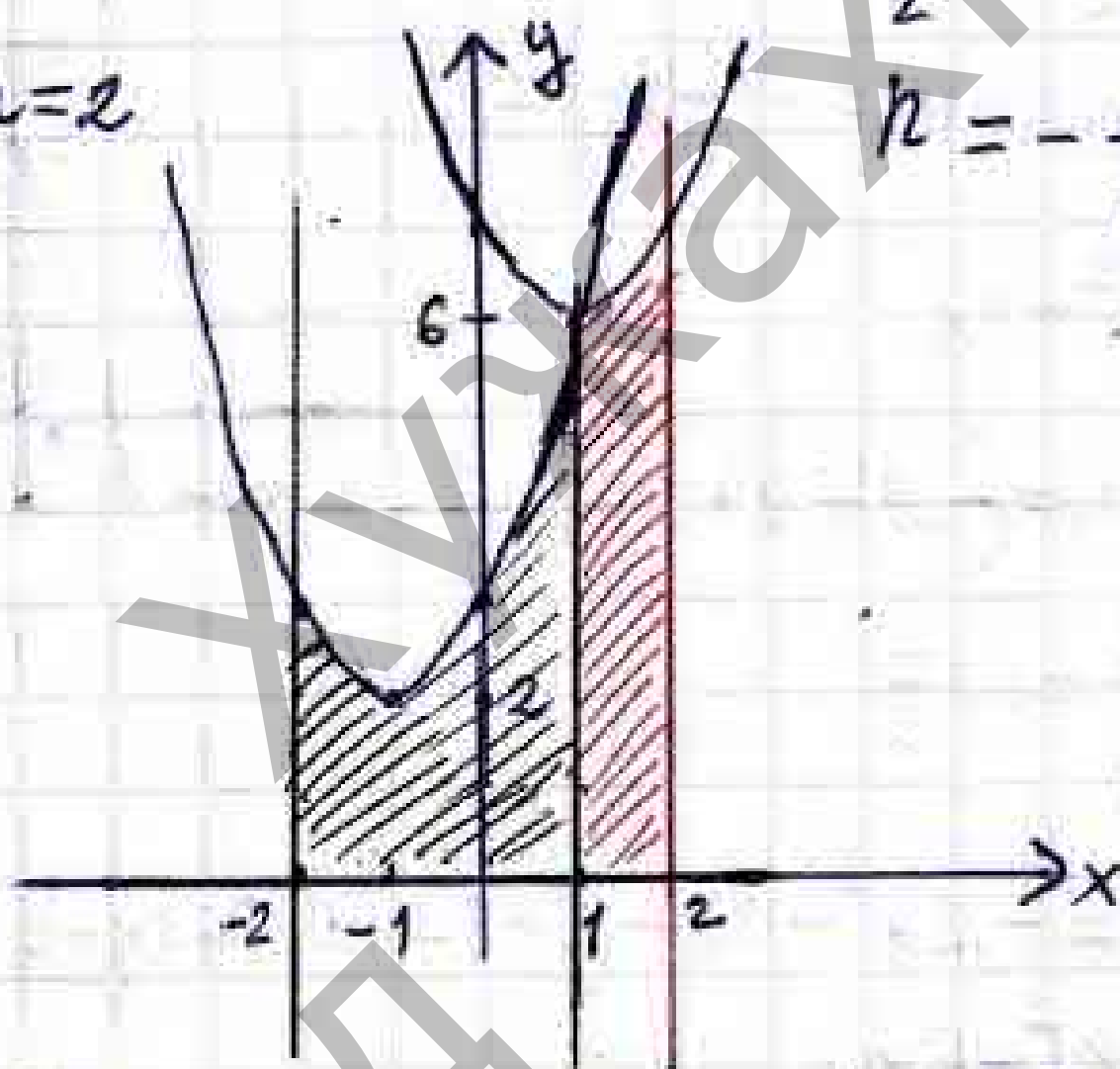
$$g(x) = x^2 - 2x + 4, \quad [1; 2]$$

$$F(x) = \frac{x^3}{3} + x^2 + 3x$$

$$F(x) = \frac{x^3}{3} - x^2 + 4x$$

$$n = -\frac{2}{2} = -1 \quad m = 2$$

$$n = -\frac{2}{2 \cdot (-1)} = 1 \quad m = 6$$



$$S_1 = F_1(1) - F_1(-2) = \left(\frac{1}{3} + 1 + 3\right) - \left(-\frac{8}{3} + 4 - 6\right) = \frac{13}{3} + \frac{14}{3} = 9$$

$$S_2 = F_2(2) - F_2(1) = \left(\frac{8}{3} - 4 + 14\right) - \left(\frac{1}{3} - 1 + 7\right) = \frac{38}{3} - \frac{19}{3} = 6$$

$$S_1 + S_2 = 9 + 6 = 15$$

24

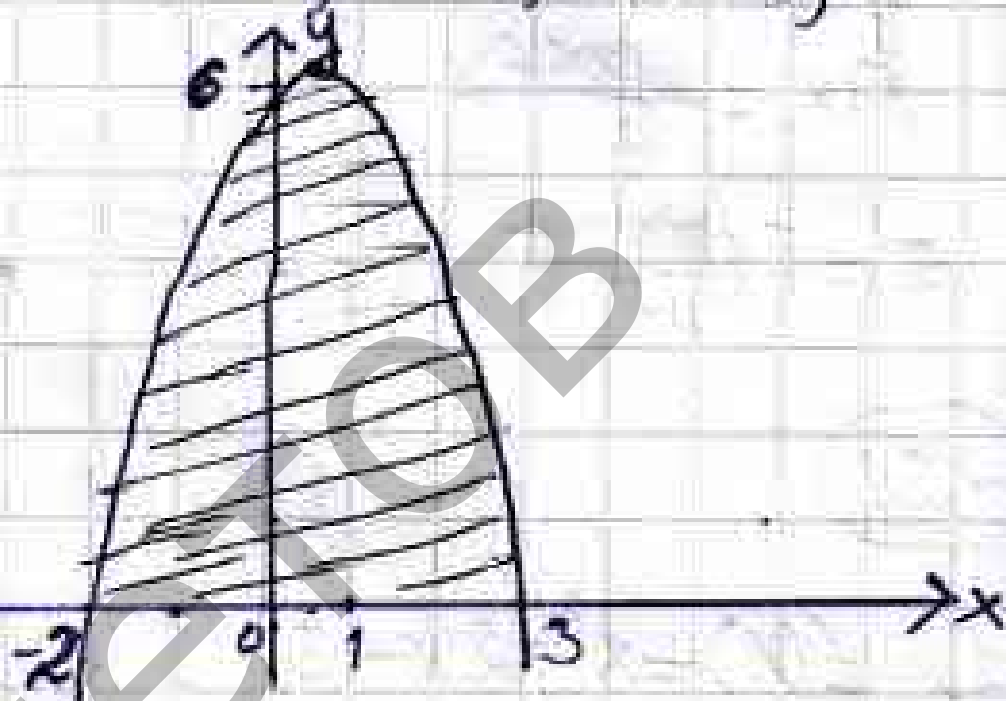
$$1) \quad y = -x^2 + x + 6 \quad n = -\frac{1}{2 \cdot (-1)} = 0,5 \quad m = -0,25 + 0,5 + 6 = 6,25$$

$$F(x) = -\frac{x^3}{3} + \frac{x^2}{2} + 6x$$

$$S = F(3) - F(-2) =$$

$$= \left(-9 + \frac{9}{2} + 18\right) - \left(-\frac{8}{3} + \frac{4}{2} - 12\right) =$$

$$= 13\frac{1}{2} + \frac{22}{3} = 13\frac{3}{6} + 7\frac{4}{3} = 20\frac{5}{6}$$



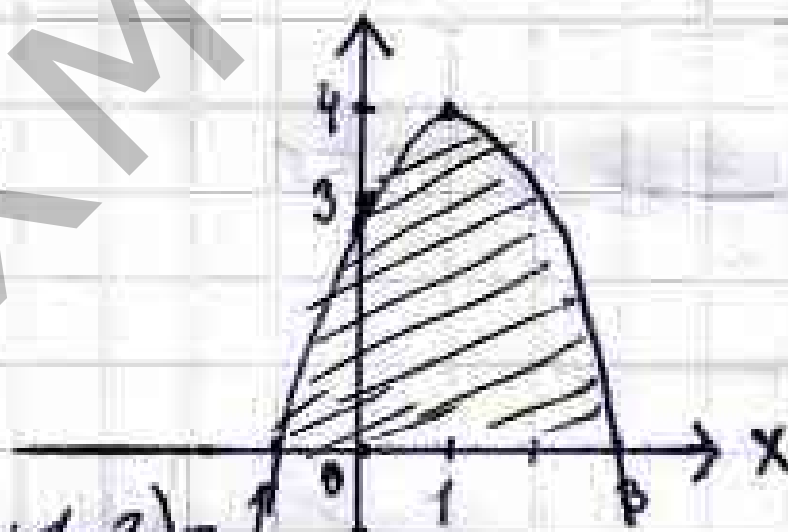
$$2) \quad y = -x^2 + 2x + 3$$

$$n = -\frac{2}{2 \cdot (-1)} = 1 \quad m = -1 + 2 + 3 = 4$$

$$F(x) = -\frac{x^3}{3} + x^2 + 3x$$

$$S = F(3) - F(-1) = \left(-9 + 9 + 9\right) - \left(-\frac{1}{3} + 1 - 3\right) =$$

$$= 9 + \frac{5}{3} = 9 + 1\frac{2}{3} = 10\frac{2}{3}$$



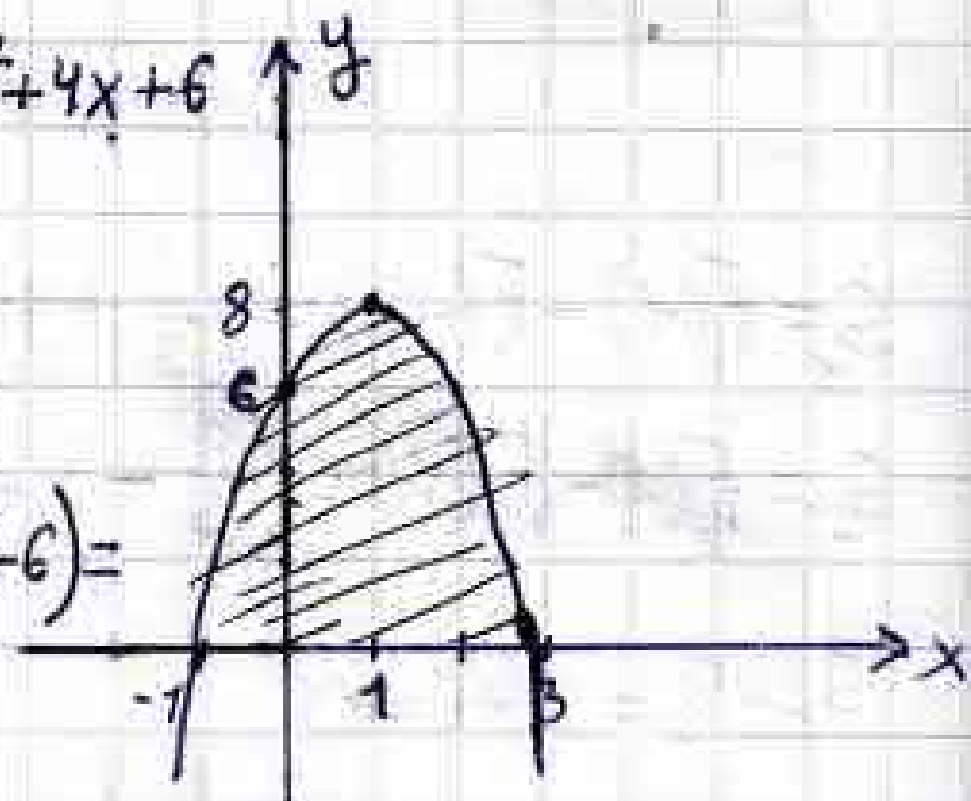
$$3) \quad y = -2(x-1)^2 + 8 = -2x^2 + 4x - 2 + 8 = -2x^2 + 4x + 6$$

$$F(x) = -\frac{2x^3}{3} + 2x^2 + 6x$$

$$n = -\frac{4}{2 \cdot (-2)} = 1 \quad m = -2 + 4 + 6 = 8$$

$$S = F(3) - F(-1) = \left(-2 \cdot 9 + 2 \cdot 9 + 18\right) - \left(-\frac{2}{3} + 2 - 6\right) =$$

$$= 18 + \frac{10}{3} = 18 + 3\frac{1}{3} = 21\frac{1}{3}$$



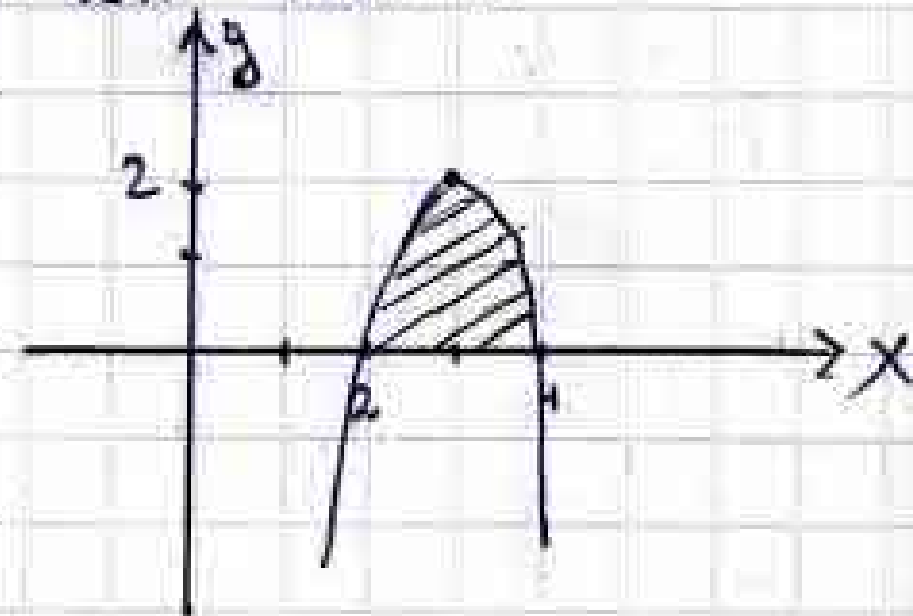
$$4) \quad y = -2(x-3)^2 + 2 = -2x^2 + 12x - 18 + 2 = -2x^2 + 12x - 16$$

$$F(x) = -\frac{2x^3}{3} + 6x^2 - 16x$$

$$n = -\frac{12}{2 \cdot (-2)} = 3 \quad m = -2 \cdot 9 + 12 \cdot 3 - 16 = 2$$

$$S = F(4) - F(2) = \left(-\frac{2 \cdot 64}{3} + 6 \cdot 16 - 16 \cdot 4\right) -$$

$$-\left(-\frac{2 \cdot 8}{3} + 6 \cdot 4 - 16 \cdot 2\right) = \frac{26}{3} = 8\frac{2}{3}$$



~28

25

$$y = \cos 5x \quad y=0 \quad x = \frac{\pi}{30} \quad x=d \quad (d < \frac{\pi}{30})$$

$$F(x) = \frac{1}{5} \sin 5x$$

$$S = F(\frac{\pi}{30}) - F(d) = 0,2$$

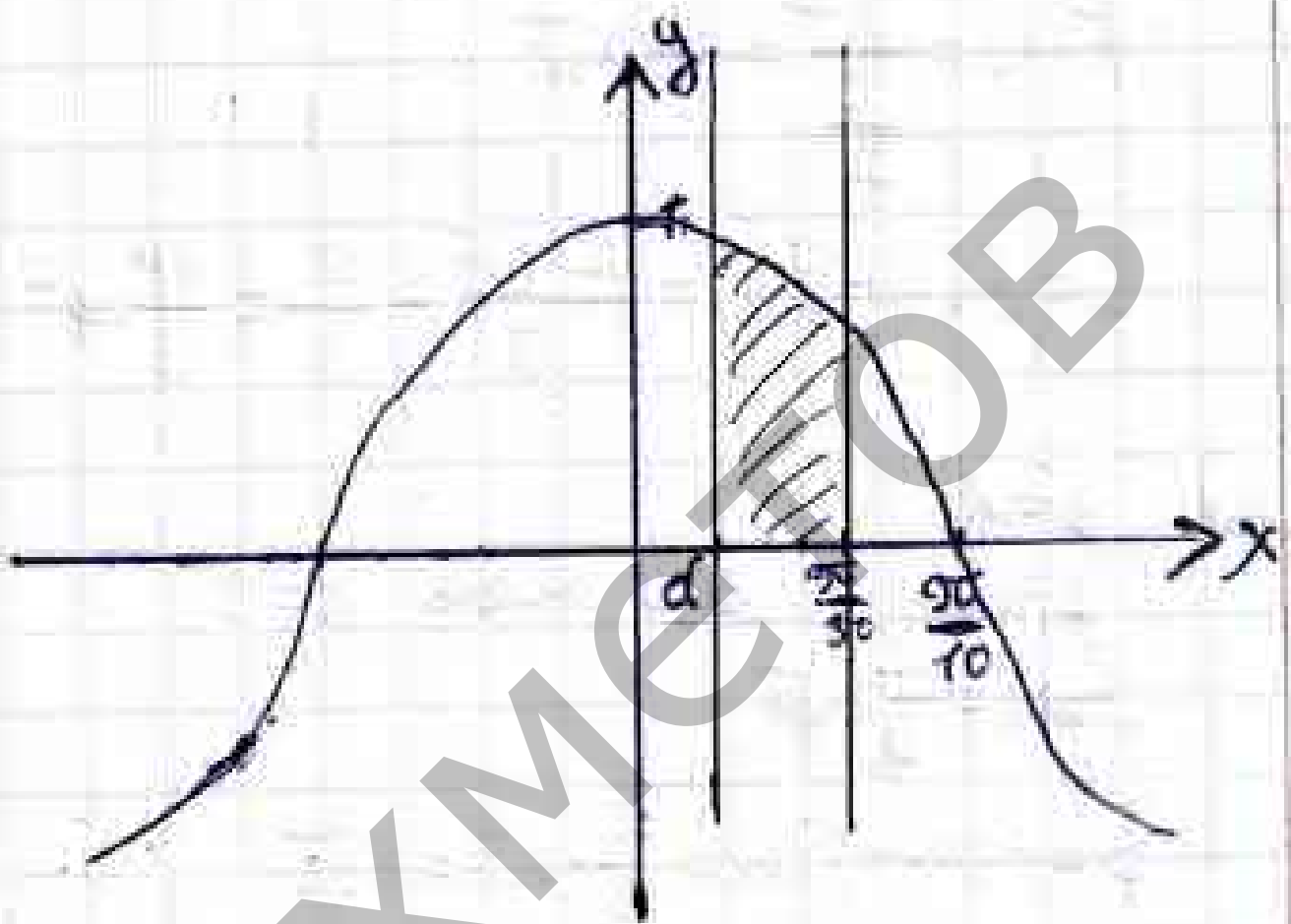
$$\frac{1}{5} \sin \frac{\pi}{6} - \frac{1}{5} \sin 5d = 0,2$$

$$-\frac{1}{5} \sin 5d = \frac{1}{5} - \frac{1}{10}$$

$$\sin 5d = -\frac{1}{2}$$

$$5d = (-1)^{n+1} \frac{\pi}{6} + \pi n$$

$$d = (-1)^{n+1} \frac{\pi}{30} + \pi n \quad n \in \mathbb{Z}$$



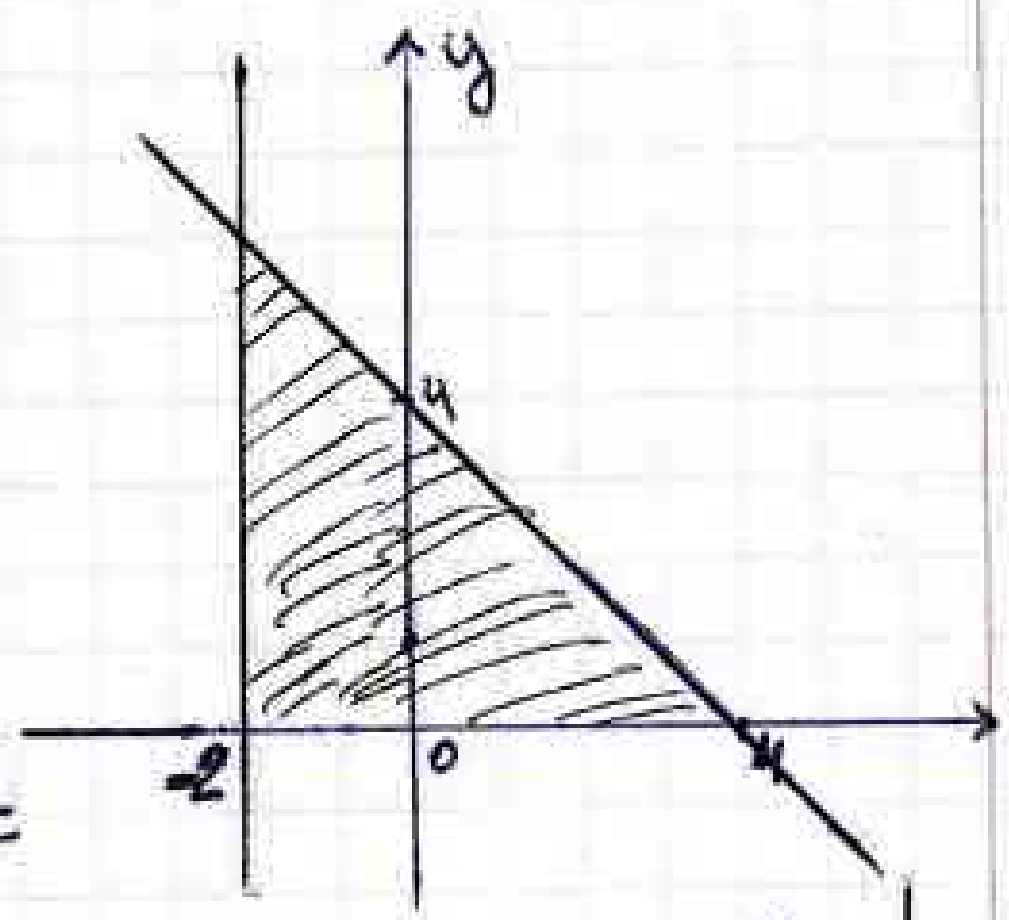
~29

1)  $f(x) = 4,5 - 0,5x^2$   $x_0 = 1$   $x = -2$   
 $f'(x) = -x$   
 $f'(x_0) = -1$   
 $f(x_0) = 4,5 - 0,5 \cdot 1^2 = 4$

$$y = 3 - 1(x-1) = 3 - x + 1 = -x + 4$$

$$F(x) = -\frac{x^2}{2} + 4x$$

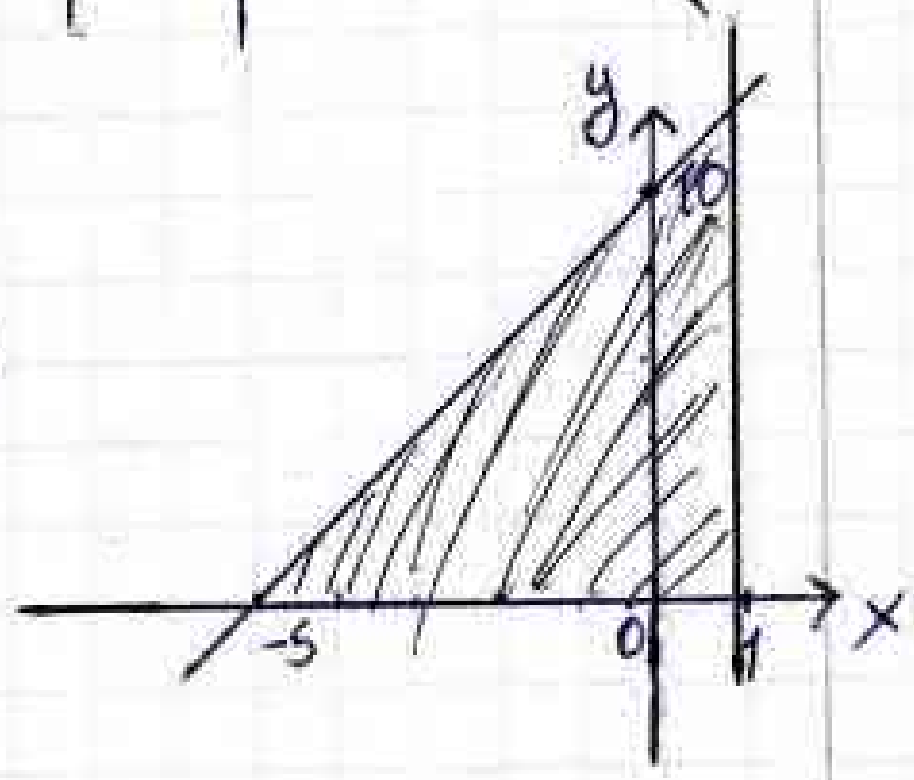
$$S = F(4) - F(-2) = \left(-\frac{16}{2} + 16\right) - \left(-\frac{4}{2} - 8\right) = 8 + 10 = 18$$



2)  $f(x) = 8 - 0,5x^2$   $x_0 = -2$   $x = 1$   
 $f'(x) = -x$   $f(x_0) = 8 - 0,5 \cdot 4 = 6$   
 $f'(x_0) = -(-2) = 2$   
 $y = 6 + 2(x+2) = 6 + 2x + 4 = 2x + 10$

$$F(x) = x^2 + 10x$$

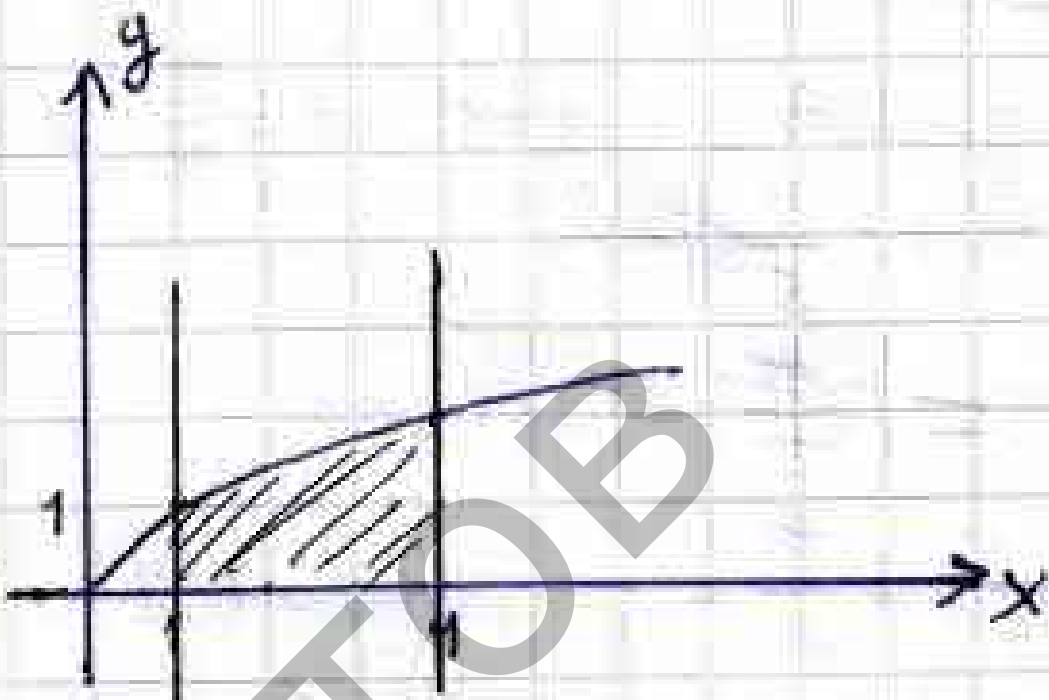
$$S = F(1) - F(-5) = (1 + 10) - (25 - 50) = 36$$



№30

1)  $y^2 = x$   
 $y = \sqrt{x}$   $x=1$   $x=4$   
 $F(x) = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3} x\sqrt{x}$

$$S = F(4) - F(1) = \frac{2 \cdot 4\sqrt{4}}{3} - \frac{2}{3} = \frac{14}{3} = 4\frac{2}{3}$$



2)  $y^2 = x$   
 $y = \sqrt{x}$   $x=0$   $x=3$   
 $F(x) = \frac{2}{3} x\sqrt{x}$

$$S = F(3) - F(0) = \frac{2}{3} \cdot 3\sqrt{3} = 2\sqrt{3}$$



ХАМШИД

ХУЖАХМЕТОВ

31

$$1) \int_{-3}^2 (2x-3) dx = x^2 - 3x \Big|_{-3}^2 = 4 - 6 - (9 + 9) = -20$$

$$3) \int_{-2}^0 (3x^2 - 10) dx = x^3 - 10x \Big|_{-2}^0 = (0 - 20) = -20$$

$$2) \int_{-2}^1 (5-4x) dx = 5x - 2x^2 \Big|_{-2}^1 = 5 - 2 - (-10 - 8) = 21$$

$$4) \int_0^2 (6x^2 - 2x + 5) dx = 2x^3 - x^2 + 5x \Big|_0^2 = 2 \cdot 8 - 4 + 5 \cdot 2 = 22$$

32

$$1) \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \cos x dx = \sin x \Big|_{\frac{\pi}{6}}^{\frac{5\pi}{6}} = \sin \frac{5\pi}{6} - \sin \frac{\pi}{6} = 1$$

$$2) \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \sin x dx = -\cos x \Big|_{\frac{\pi}{3}}^{\frac{2\pi}{3}} = -\cos \frac{2\pi}{3} + \cos \frac{\pi}{3} = 1$$

$$2) \int_{-1}^1 (5x^4 + 6x^2) dx = x^5 + 2x^3 \Big|_{-1}^1 = 1 + 2 + 1 + 2 = 6$$

$$4) \int_{-2}^1 (4x^3 + 6x) dx = x^4 + 3x^2 \Big|_{-2}^1 = 1 + 3 - 16 - 12 = -24$$

33

$$1) \int_0^{\frac{\pi}{2}} \sin^2 \frac{x}{2} dx = \int_0^{\frac{\pi}{2}} \frac{1 - \cos x}{2} dx = \frac{1}{2} x - \frac{1}{2} \sin x \Big|_0^{\frac{\pi}{2}} = \frac{1}{2} \cdot \frac{\pi}{2} - \frac{1}{2} \sin \frac{\pi}{2} = \frac{\pi}{4} - \frac{1}{2} = \frac{\pi - 2}{4}$$

$$2) \int_0^{\frac{\pi}{2}} \cos^2 \frac{x}{4} dx = \int_0^{\frac{\pi}{2}} \frac{1 + \cos x}{2} dx = \frac{1}{2} x + \frac{1}{2} \sin x \Big|_0^{\frac{\pi}{2}} = \frac{\pi}{4} + \frac{1}{2} = \frac{\pi + 2}{4}$$

$$3) \int_0^1 \frac{x^3 + x^2 + x + 1}{x+1} dx = \int_0^1 \frac{x^2(x+1) + (x+1)}{x+1} dx = \int_0^1 (x^2 + 1) dx = \frac{x^3}{3} + x \Big|_0^1 = \frac{1}{3} + 1 = 1\frac{1}{3} = \frac{4}{3}$$

$$4) \int_3^5 \frac{x^2 - 5x + 6}{x-2} dx = \int_3^5 \frac{x^2 - 2x - 3x + 6}{x-2} dx = \int_3^5 (x - 3) dx = \frac{x^2}{2} - 3x \Big|_3^5 = \frac{25}{2} - 15 - \frac{9}{2} + 9 = \frac{-5+9}{2} = 2$$

34

$$1) \int_0^{\frac{\pi}{6}} (\cos x \cos 2x - \sin x \sin 2x) dx = \int_0^{\frac{\pi}{6}} \cos(x+2x) dx = \frac{1}{3} \sin 3x \Big|_0^{\frac{\pi}{6}} = \frac{1}{3} \sin \left(3 \cdot \frac{\pi}{6}\right) - \frac{1}{3} \sin 0 = \frac{1}{6}$$

$$2) \int_0^{\frac{\pi}{6}} (\sin x \cos 3x + \cos x \sin 3x) dx = \int_0^{\frac{\pi}{6}} \sin 4x dx = -\frac{1}{4} \cos 4x \Big|_0^{\frac{\pi}{6}} = -\frac{1}{4} \cos \frac{\pi}{3} + \frac{1}{4} \cos 0 = \frac{2 - \sqrt{3}}{8}$$

$$3) \int_{\frac{1}{3}}^{\frac{1}{2}} \left(\frac{1}{2} + \frac{3}{x^2}\right) dx = \frac{1}{2}x - \frac{3}{x} \Big|_{\frac{1}{3}}^{\frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{2} - \frac{3}{2} - \left(\frac{1}{2} \cdot \frac{1}{3} - \frac{3}{\frac{1}{3}}\right) = \frac{1}{4} - \frac{3}{2} - \left(\frac{1}{6} - 9\right) = \frac{1}{4} - \frac{3}{2} - \frac{1}{6} + 9 = 8\frac{7}{10}$$

$$4) \int_{-2}^{-1} \left(x - \frac{4}{x^2}\right) dx = \frac{x^2}{2} + \frac{4}{x} \Big|_{-2}^{-1} = \frac{1}{2} - 4 - \frac{4}{-2} + \frac{4}{-2} = -3\frac{1}{2}$$

35

$$1) \int_0^{\frac{\pi}{12}} \frac{dx}{\cos^2 3x} = \frac{1}{3} \operatorname{tg} 3x \Big|_0^{\frac{\pi}{12}} = \frac{1}{3} \operatorname{tg} \frac{\pi}{4} - \operatorname{tg} 0 = \frac{1}{3}$$

$$2) \int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \frac{dx}{\sin^2 2x} = \frac{1}{2} (-\operatorname{ctg} 2x) \Big|_{\frac{\pi}{12}}^{\frac{\pi}{6}} = \frac{1}{2} (-\operatorname{ctg} (2 \cdot \frac{\pi}{6})) - \frac{1}{2} (-\operatorname{ctg} (2 \cdot \frac{\pi}{12})) = \frac{\sqrt{3}}{3}$$

$$3) \int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \sin 3x dx = -\frac{1}{3} \cos 3x \Big|_{\frac{\pi}{12}}^{\frac{\pi}{6}} = -\frac{1}{3} \cos \frac{\pi}{4} + \frac{1}{3} \cos \frac{\pi}{6} = -\frac{\sqrt{2}}{6} + \frac{\sqrt{3}}{6} = \frac{\sqrt{3} - \sqrt{2}}{6}$$

$$4) \int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \cos 4x dx = \frac{1}{4} \sin 4x \Big|_{\frac{\pi}{12}}^{\frac{\pi}{6}} = \frac{1}{4} \sin \pi - \frac{1}{4} \sin \frac{\pi}{2} = -\frac{1}{4}$$

36

$$1) \int_1^{1.5} \frac{(1-2x)^3}{(1-2 \cdot 1.5)^4} dx = \frac{1}{2} \cdot \frac{(1-2x)^4}{-8} \Big|_1^{1.5} = \frac{(1-2 \cdot 1.5)^4 - (1-2)^4}{-8} = \frac{15}{8}$$

$$2) \int_0^{\frac{1}{3}} \frac{(3x+1)^3}{(1+1)^4 (0+1)^4} dx = \frac{1}{3} \cdot \frac{(3x+1)^4}{4} \Big|_0^{\frac{1}{3}} = \frac{(1+1)^4 - (0+1)^4}{12} = \frac{15}{12}$$

$$3) \int_{-1}^4 \frac{(2-x)^3}{8} dx = \frac{1}{8} \cdot (-1) \cdot \frac{(2-x)^4}{4} \Big|_{-1}^4 = -\frac{(2-4)^4}{32} + \frac{(2+1)^4}{32} = \frac{65}{32} = 2\frac{1}{32}$$

$$4) \int_{-1}^1 \frac{(1-x)^4}{4} dx = \frac{1}{4} \cdot (-1) \cdot \frac{(1-x)^5}{5} \Big|_{-1}^1 = -\frac{(1-1)^5}{35} + \frac{(1+1)^5}{35} = \frac{32}{35}$$

37

$$1) \int_1^4 \frac{5\sqrt{x}}{x\sqrt{x}} dx = 10\sqrt{x} \Big|_1^4 = 10\sqrt{4} - 10 = 10$$

$$2) \int_{-3}^3 \frac{1}{\sqrt{1-x}} dx = -2\sqrt{1-x} \Big|_{-3}^3 = -2\sqrt{1+3} + 2\sqrt{1+8} = -4 + 6 = 2$$

$$\int_4^{11} \frac{1}{\sqrt{x+5}} dx = 2\sqrt{x+5} \Big|_4^{11} = 2\sqrt{11+5} - 2\sqrt{4+5} = 8 - 2 \cdot 3 = 2$$

$$4) \int_{-4}^{47} \frac{4}{\sqrt{x+2}} dx = 8\sqrt{x+2} \Big|_{-4}^{47} = 8 \cdot 4 - 8 \cdot 4 = 24$$

N 38

$$1) \int_0^{\frac{\pi}{2}} (1 - 2\cos^2 x) dx = \int_0^{\frac{\pi}{2}} (-\cos 2x) dx = -\frac{1}{2} \sin 2x \Big|_0^{\frac{\pi}{2}} = -\frac{1}{2} \sin \frac{\pi}{2} + \frac{1}{2} \sin 0 = -\frac{1}{2}$$

$$2) \int_0^{\frac{\pi}{3}} (2\sin 2x - 1) dx = -\cos 2x - x \Big|_0^{\frac{\pi}{3}} = -\cos \frac{2\pi}{3} - \frac{\pi}{3} + \cos 0 = -\frac{1}{2} - \frac{\pi}{3} + 1 = \frac{1 - 2\pi}{6}$$

$$3) \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x + \tan x \cot x) dx = -\cos x + x \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = -\cos \frac{\pi}{4} + \frac{\pi}{4} + \cos \frac{\pi}{4} - \frac{\pi}{4} = \frac{6\sqrt{3} - 6\sqrt{2} + \pi}{12}$$

$$4) \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (\tan 5x \cot 5x - \cos x) dx = x - \sin x \Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}} = \frac{\pi}{2} - \sin \frac{\pi}{2} - \frac{\pi}{3} + \sin \frac{\pi}{3} = \frac{\pi}{6} - \frac{1}{2} + \frac{\sqrt{3}}{2}$$

N 39

$$1) \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} 12 \sin\left(\frac{\pi}{8} - x\right) \cos\left(\frac{\pi}{8} - x\right) dx = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} 6 \sin\left(\frac{\pi}{4} - 2x\right) dx = 6 \cdot \left(-\frac{1}{2}\right) \cdot \left(-\cos\left(\frac{\pi}{4} - 2x\right)\right) \Big|_{\frac{\pi}{8}}^{\frac{3\pi}{8}} = 3 \cdot \cos \frac{\pi}{2} - 3 \cos 0 = -3$$

$$2) \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left(\cos^2\left(x + \frac{\pi}{3}\right) - \sin^2\left(x + \frac{\pi}{3}\right)\right) dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos\left(2x + \frac{2\pi}{3}\right) dx = \frac{1}{2} \sin\left(2x + \frac{2\pi}{3}\right) \Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \frac{1}{2} \left(\sin \frac{4\pi}{3} - \sin \frac{2\pi}{3}\right) = \frac{\sqrt{3}}{4}$$

$$3) \int_{\frac{\pi}{2}}^{\pi} \sin x \sin 2x dx = \int_{\frac{\pi}{2}}^{\pi} \frac{1}{2} (\cos x - \cos 3x) dx = \frac{1}{2} (\sin x - \frac{1}{3} \sin 3x) \Big|_{\frac{\pi}{2}}^{\pi} = \frac{1}{2} \left(\sin \pi - \frac{1}{3} \sin 3\pi\right) - \frac{1}{2} \left(\sin \frac{\pi}{2} - \frac{1}{3} \sin \frac{3\pi}{2}\right) = \frac{1}{2} \left(-1 - \frac{1}{3}\right) - \frac{1}{2} \left(1 - \frac{1}{3}\right) = -\frac{1}{2} \cdot \frac{4}{3} = -\frac{2}{3}$$

$$4) \int_{\frac{\pi}{2}}^{\pi} \cos x \cos 2x dx = \int_{\frac{\pi}{2}}^{\pi} \frac{1}{2} (\cos 3x + \cos x) dx = \frac{1}{6} \sin 3x + \frac{1}{2} \sin x \Big|_{\frac{\pi}{2}}^{\pi} = \frac{1}{6} \sin 3\pi + \frac{1}{2} \sin \pi - \frac{1}{6} \sin \frac{3\pi}{2} - \frac{1}{2} \sin \frac{\pi}{2} = -\frac{1}{3}$$

N 40

$$1) \int_1^x (3-2t) dt = 3t - t^2 \Big|_1^x = 3x - x^2 - 2 = 4 - 2x$$

$$x^2 - 5x + 8 = 0$$

$$\Delta = 25 - 24 = 1 \quad x_{1,2} = \frac{5 \pm 1}{2} = 2; x_2 = \frac{5 + 1}{2} = 3$$

$$2) \int_1^x (1-4t) dt = t - 2t^2 \Big|_1^x = x - 2x^2 + 1 = 12 - 9x$$

$$2x^2 - 10x + 11 = 0$$

$$\Delta = 100 - 2 \cdot 4 \cdot 11 = 12 \quad x_{1,2} = \frac{10 \pm 2\sqrt{3}}{4} = \frac{5 \pm \sqrt{3}}{2}$$

$$3) \int_x^{-1} (3t-2) dt = \frac{3t^2}{2} - 2t \Big|_x^{-1} = \frac{3}{2} + 2 - \frac{3x^2}{2} + 2x = 5 - x$$

$$\frac{3}{2}x^2 - 3x + 5 - \frac{4}{2} = 0$$

$$x^2 - 2x + 1 = 0 \quad x_{1,2} = \frac{2}{2} = 1$$

$$4) \int_x^{-2} (5t+1) dt = \frac{5t^2}{2} + t \Big|_x^{-2}$$

N 41

$$1) \int_0^x 5t dt = 5t \Big|_0^x = 5x > 1$$

$$x > \frac{1}{5} \quad \mathcal{M}: \left(\frac{1}{5}; +\infty\right)$$

$$2) \int_x^1 5t dt = 5t \Big|_x^1 = 5x^2 - 5x < 0$$

$$5x(x-1) < 0$$

$$x_1 = 0 \quad x_2 = 1 \quad \mathcal{M}: (0; 1)$$

$$3) \int_x^1 3t dt = 3t \Big|_x^1 = 3 - 3x > 9$$

$$-3x > 6$$

$$x < -2 \quad \mathcal{M}: (-\infty; -2)$$

$$4) \int_x^2 (2t-3) dt = t^2 - 3t \Big|_x^2 = 4 - 6 - x^2 + 3x > 0$$

$$x^2 - 3x + 2 < 0$$

$$\Delta = 9 - 8 = 1$$

$$x_1 = \frac{3-1}{2} = 1, \quad x_2 = \frac{3+1}{2} = 2$$

$$\mathcal{M}: (1; 2)$$

N 42

$$1) \int_0^{\pi} \sin 2x \cos 3x dx = \frac{1}{2} \int_0^{\pi} (\sin 5x - \sin x) dx = \frac{1}{2} \left(-\frac{1}{5} \cos 5x + \cos x\right) \Big|_0^{\pi} = -\frac{1}{10} \cos 5\pi + \cos \pi + \frac{1}{10} \cos 0 - \cos 0 = -\frac{1}{10}(-1) + \frac{1}{10}(-1) + \frac{1}{10} - \frac{1}{10} = \frac{1}{5} - 1 = -\frac{4}{5}$$

$$2) \int_0^{\frac{\pi}{2}} \sin 4x \sin 5x = \frac{1}{2} (\cos x - \cos 9x) = \frac{1}{2} \sin x - \frac{1}{10} \sin 9x \Big|_0^{\frac{\pi}{2}} =$$

$$= \frac{1}{2} \sin \frac{\pi}{2} - \frac{1}{10} \sin \left( \frac{\pi}{2} + \frac{8\pi}{2} \right) - \frac{1}{2} \sin 0 + \frac{1}{10} \sin 0 = \frac{1}{2} \sin \frac{\pi}{2} - \frac{1}{10} \cos 4\pi = \frac{1}{2} - \frac{1}{10} = \frac{4}{10} = \frac{2}{5}$$

$$3) \int_0^{\frac{\pi}{2}} \frac{\cos x dx}{1 - \sqrt{2} \cos \frac{x}{2}} = \int_0^{\frac{\pi}{2}} \frac{(2 \cos^2 \frac{x}{2} - 1)(\sqrt{2} \cos \frac{x}{2} + 1) dx}{-(\sqrt{2} \cos \frac{x}{2} - 1)(\sqrt{2} \cos \frac{x}{2} + 1)} = \int_0^{\frac{\pi}{2}} (1 - \sqrt{2} \cos \frac{x}{2}) dx = x - 2\sqrt{2} \sin \frac{x}{2} \Big|_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2} - 2\sqrt{2} \sin \frac{\pi}{4} + 0 + 2\sqrt{2} \sin 0 = \frac{\pi}{2} - \frac{4}{2} = \frac{\pi - 4}{2} = -\frac{4 - \pi}{2}$$

$$4) \int_0^{\frac{\pi}{2}} \frac{\cos x dx}{\sqrt{2} \sin \frac{x}{2} + 1} = \int_0^{\frac{\pi}{2}} \frac{(1 - \sqrt{2} \sin \frac{x}{2})(1 + \sqrt{2} \sin \frac{x}{2})}{(1 + \sqrt{2} \sin \frac{x}{2})} = x + 2\sqrt{2} \cos \frac{x}{2} \Big|_0^{\frac{\pi}{2}} =$$

$$= 2\sqrt{2} \cos \frac{\pi}{4} + \frac{\pi}{2} - 2\sqrt{2} \cos 0 = 2\sqrt{2} \cdot \frac{\sqrt{2}}{2} + \frac{\pi}{2} - 2\sqrt{2} = 2 + \frac{\pi}{2} - 2\sqrt{2} = \frac{4 - 4\sqrt{2} + \pi}{2}$$

N 43

$$1) \int_0^1 (2+5x)^3 dx = \frac{1}{5} \cdot \frac{(2+5x)^4}{4} \Big|_0^1 = \frac{4^4}{20} - \frac{2^4}{20} = \frac{2401 - 16}{20} = \frac{2385}{20} = 119,25$$

$$2) \int_0^1 (2x+3)^3 dx = \frac{1}{2} \cdot \frac{(2x+3)^4}{4} \Big|_0^1 = \frac{(2+3)^4}{8} - \frac{3^4}{8} = \frac{625 - 81}{8} = \frac{544}{8} = 68$$

$$3) \int_2^3 \frac{2x^3 + x^2 + 2x + 1}{1+x^2} dx = \int_2^3 \frac{2x(x^2+1) + (x^2+1)}{(x^2+1)} = x^2 + x \Big|_2^3 = 9 + 3 - (4 + 2) = 12 - 6 = 6$$

$$4) \int_{-3}^{-2} \frac{x^3 - x^2 - x + 1}{x^2 - 1} dx = \int_{-3}^{-2} \left( \frac{x(x^2-1)}{(x^2-1)} - \frac{(x^2-1)}{(x^2-1)} \right) dx = \frac{x^2}{2} - x \Big|_{-3}^{-2} = \frac{4}{2} + 2 - \left( \frac{9}{2} + 3 \right) = -3,5$$

N 44

$$1) \int_2^{12} \frac{dx}{\sqrt{3x-1}} = \frac{2}{3} \sqrt{3x-1} \Big|_2^{12} = \frac{2}{3} \sqrt{36-1} - \frac{2}{3} \sqrt{6-1} = \frac{2}{3} (\sqrt{35} - \sqrt{5})$$

$$2) \int_4^{12} \frac{dx}{\sqrt{2x+1}} = \sqrt{2x+1} \Big|_4^{12} = \sqrt{2 \cdot 12 + 1} - \sqrt{2 \cdot 4 + 1} = 5 - 3 = 2$$

$$3) \int_{-1}^0 \frac{dx}{(6x-1)^4} = \frac{1}{6} \cdot \frac{(6x-1)^{-3}}{-4+1} \Big|_{-1}^0 = -\frac{1}{18(6x-1)^3} \Big|_{-1}^0 = -\frac{1}{18 \cdot (-1)} + \frac{1}{18(-6-1)^3} = \frac{1}{18} - \frac{1}{18 \cdot 343} = \frac{342}{6174} = \frac{19}{343}$$

$$4) \int_{-1}^0 \frac{dx}{(1-2x)^5} = -\frac{1}{2} \cdot \frac{-1}{4(1-2x)^4} \Big|_{-1}^0 = \frac{1}{8(1-2x)^4} \Big|_{-1}^0 = \frac{81}{8} - \frac{1}{8 \cdot 81} = \frac{80}{8 \cdot 81} = \frac{10}{81}$$

N 45

$$1) \int_x^3 (t+1) dt = \frac{t^2}{2} + t \Big|_x^3 = \frac{9}{2} + 3 - \frac{x^2}{2} - x < 0$$

$$x^2 + 2x - 15 > 0$$

$$D = 4 + 60 = 64$$

$$x_1 = \frac{-2-8}{2} = -5 \quad x_2 = \frac{-2+8}{2} = 3$$

$$\begin{array}{c} + \quad - \quad + \\ \text{-----} \\ -5 \quad 3 \end{array} \quad \text{MC: } (-\infty; -5) \cup (3; +\infty)$$

$$2) \int_x^2 (1-t) dt = t - \frac{t^2}{2} \Big|_x^2 = 2 - \frac{4}{2} - x + \frac{x^2}{2} > 0$$

$$x^2 - x > 0$$

$$x = 0 \quad x = 1$$

$$\begin{array}{c} + \quad - \quad + \\ \text{-----} \\ 0 \quad 1 \end{array}$$

$$\text{MC: } (-\infty; 0) \cup (1; +\infty)$$

$$3) \int_{-2}^x (2-3t) dt = 2t - \frac{3t^2}{2} \Big|_{-2}^x = 2x - \frac{3x^2}{2} - 4 + \frac{3 \cdot 4}{2} > 0$$

$$\frac{3}{2} x^2 - 2x - 2 < 0$$

$$D = 4 + 4 \cdot \frac{3}{2} \cdot 2 = 16$$

$$x_1 = \frac{2-4}{3} = -\frac{2}{3} \quad x_2 = \frac{2+4}{3} = 2$$

$$\begin{array}{c} + \quad - \quad + \\ \text{-----} \\ -\frac{2}{3} \quad 2 \end{array} \quad \text{MC: } (-\frac{2}{3}; 2)$$

$$4) \int_{-3}^x (4t-1) dt = 2t^2 - t \Big|_{-3}^x = 2x^2 - x - 18 - 3 < 0$$

$$2x^2 - x - 21 < 0$$

$$D = 1 + 4 \cdot 2 \cdot 21 = 169$$

$$x_1 = \frac{1-13}{4} = -3 \quad x_2 = \frac{1+13}{4} = 3,5$$

$$\begin{array}{c} + \quad - \quad + \\ \text{-----} \\ -3 \quad 3,5 \end{array} \quad \text{MC: } (-3; 3,5)$$



~4

1)

$$\int_x^{2x} \sin 2t dt = -\frac{1}{2} \cos 2t \Big|_x^{2x} = \frac{1}{2} (-\cos 4x + \cos 2x) = \frac{1}{2}$$

$$\cos 2x - 2\cos^2 x + 1 = 1$$

$$\cos 2x(1 - 2\cos 2x) = 0$$

$$\cos 2x = 0$$

$$1 - 2\cos 2x = 0$$

$$2x = \frac{\pi}{2} + \pi n$$

$$2x = \pm \frac{\pi}{3} + 2\pi n$$

$$x = \frac{\pi}{4} + \frac{\pi n}{2}$$

$$x = \pm \frac{\pi}{6} + \pi n$$

$$n=0 \quad x = \frac{\pi}{4}$$

$$n=0 \quad x = \frac{\pi}{6} \quad x = -\frac{\pi}{6}$$

$$\text{MC: } \frac{\pi}{6}$$

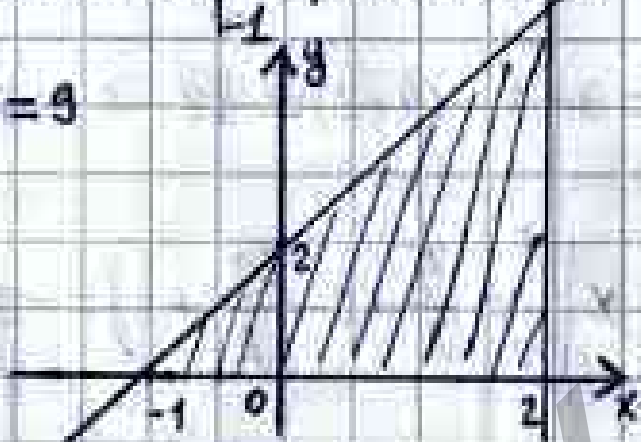
3)

~44

1)  $y = 2x + 2, y = 0, x = 2$

$$S = \int_{-1}^2 (2x + 2) dx = x^2 + 2x \Big|_{-1}^2 = 4 + 4 - 1 + 2 = 9$$

$$S = \frac{1}{2} ab = \frac{1}{2} \cdot 3 \cdot 6 = 9$$



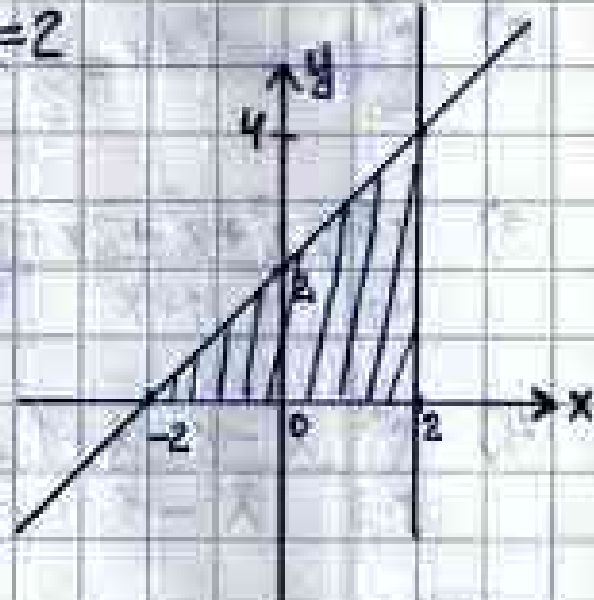
2)  $y = x + 2, y = 0, x = 2$

$$S = \int_{-2}^2 (x + 2) dx =$$

$$= \frac{x^2}{2} + 2x \Big|_{-2}^2 =$$

$$= \frac{4}{2} + 4 - \frac{4}{2} - 4 = 8$$

$$S = \frac{1}{2} ab = \frac{1}{2} \cdot 4 \cdot 4 = 8$$



~48

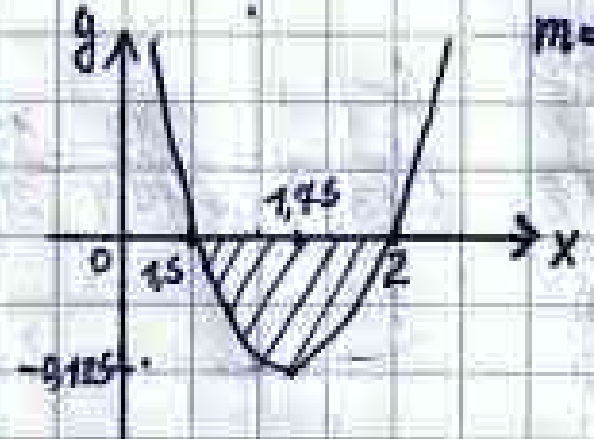
1)  $y = (x-2)(2x-3), y = 0$

$$y = 2x^2 - 7x + 6$$

$$n = \frac{-7}{2 \cdot 2} = 1,75$$

$$x_1 = 2 \quad x_2 = 1,5$$

$$m = 2 \cdot (1,75)^2 - 7 \cdot 1,75 + 6 = 0,125$$



$$S = \int_{1,5}^2 (2x^2 - 7x + 6) dx =$$

$$= \frac{2x^3}{3} - \frac{7x^2}{2} + 6x \Big|_{1,5}^2 =$$

$$= \frac{4x^3 - 21x^2 + 36x}{6} \Big|_{1,5}^2 =$$

$$= \frac{20}{6} - \frac{20,25}{6} = -\frac{0,25}{6} = -\frac{1}{24} \text{ кв. единиц}$$

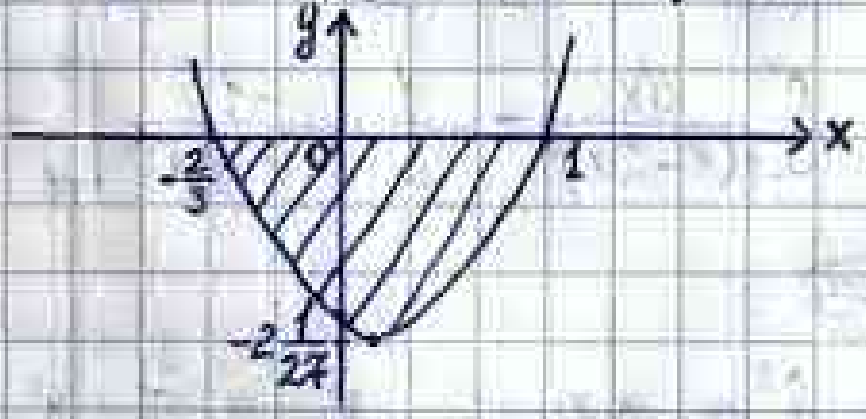
2)  $y = (3x+2)(x-1) = 3x^2 - x - 2, y = 0$

$$3x^2 - x - 2 = 0 \Rightarrow x_1 = -\frac{2}{3}, x_2 = 1$$

$$n = \frac{1}{6} \quad m = -2\frac{1}{2}$$

$$S = \int_{-\frac{2}{3}}^1 (3x^2 - x - 2) dx = x^3 - \frac{x^2}{2} - 2x \Big|_{-\frac{2}{3}}^1 =$$

$$= 1 - \frac{1}{2} - 2 - \left( \frac{8}{27} - \frac{4}{18} + \frac{4}{3} \right) = \frac{97}{27}$$



~50

1)  $f(x) = -x^2 + 4x - 4$

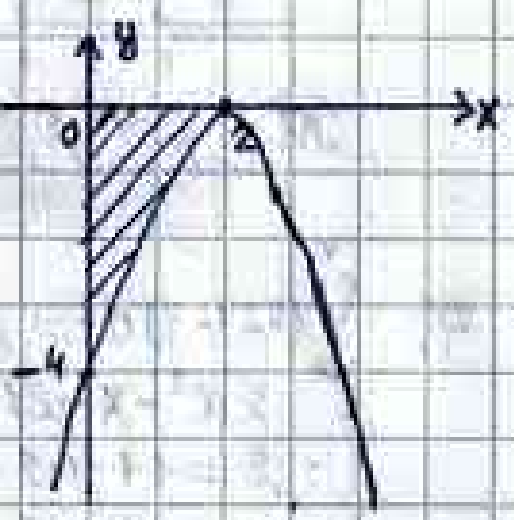
$$-x^2 + 4x - 4 = 0$$

$$x^2 - 4x + 4 = 0$$

$$D = 16 - 4 \cdot 4 = 0$$

$$x = \frac{4}{2} = 2$$

$$n = -\frac{4}{-2} = 2 \quad m = 0$$



$$S = \int_0^2 (-x^2 + 4x - 4) dx = -\frac{x^3}{3} + 2x^2 - 4x \Big|_0^2 =$$

$$= -\frac{8}{3} + 8 - 8 = -\frac{8}{3} \text{ кв. единиц}$$

2)  $f(x) = -x^2 + 6x - 9$

$$x^2 - 6x + 9 = 0$$

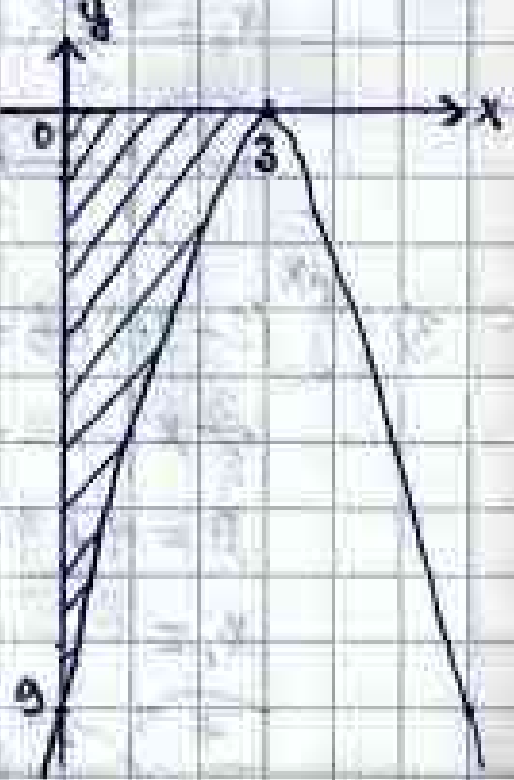
$$D = 36 - 4 \cdot 9 = 0$$

$$x = \frac{6}{2} = 3$$

$$S = \int_0^3 (-x^2 + 6x - 9) dx =$$

$$= -\frac{x^3}{3} + 3x^2 - 9x \Big|_0^3 =$$

$$= -\frac{27}{3} + 3 \cdot 9 - 9 \cdot 3 = 9$$



4

~4

1)

~4

1)

~5

1)



1)  $y = x^2 - 4x + 4$ ,  $y = 0$ ,  $x = 0$   
 $x^2 - 4x + 4 = 0$   
 $D = 16 - 4 \cdot 4 = 0$   
 $x = \frac{4}{2} = 2$   
 $S = \int_0^2 (x^2 - 4x + 4) dx =$   
 $= \left. \frac{x^3}{3} - 2x^2 + 4x \right|_0^2 =$   
 $= \frac{8}{3} - 2 \cdot 4 + 4 \cdot 2 = \frac{8}{3}$  кв. единиц

2)  $y = x^2 + 6x + 9$ ,  $y = 0$ ,  $x = 0$   
 $x^2 + 6x + 9 = 0$   
 $D = 36 - 4 \cdot 9 = 0$   
 $x = \frac{-6}{2} = -3$   
 $S = \int_{-3}^0 (x^2 + 6x + 9) dx =$   
 $= \left. \frac{x^3}{3} + 3x^2 + 9x \right|_{-3}^0 =$   
 $= \left( \frac{27}{3} + 3 \cdot 9 - 9 \cdot 3 \right) = 9$  кв. единиц

3)  $y = 4x^2 + 12x + 9$ ,  $y = 0$ ,  $x = 0$   
 $4x^2 + 12x + 9 = 0$   
 $D = 144 - 4 \cdot 36 = 0$   
 $x = \frac{-12}{2 \cdot 4} = -\frac{3}{2}$   
 $S = \int_{-\frac{3}{2}}^0 (4x^2 + 12x + 9) dx =$   
 $= \left. \frac{4x^3}{3} + 6x^2 + 9x \right|_{-\frac{3}{2}}^0 =$   
 $= 0 - \left( \frac{4 \cdot 27}{3 \cdot 8} + 6 \cdot \frac{9}{4} - 9 \cdot \frac{3}{2} \right) = \frac{9}{2}$  кв. единиц

4)  $y = 9x^2 - 6x + 1$ ,  $y = 0$ ,  $x = 0$   
 $9x^2 - 6x + 1 = 0$   
 $D = 36 - 4 \cdot 9 = 0$   
 $x = \frac{6}{2 \cdot 9} = \frac{1}{3}$   
 $S = \int_0^{\frac{1}{3}} (9x^2 - 6x + 1) dx =$   
 $= \left. 3x^3 - 3x^2 + x \right|_0^{\frac{1}{3}} =$   
 $= 3 \cdot \frac{1}{27} - 3 \cdot \frac{1}{9} + \frac{1}{3} = \frac{1}{9}$  кв. единиц

~ 51

1)  $y = 2x^2$ ,  $y = 4x$   
 $2x^2 = 4x$   
 $2x^2 - 4x = 0$   
 $x(2x - 4) = 0$   
 $x_1 = 0$ ,  $2x - 4 = 0$   
 $x_2 = 2$   
 $S = \int_0^2 4x dx - \int_0^2 2x^2 dx =$   
 $= 2x^2 \Big|_0^2 - \frac{2x^3}{3} \Big|_0^2 = 8 - \frac{2 \cdot 8}{3} = \frac{8}{3}$  кв. единиц

2)  $y = x^2$ ,  $y = -2x$   
 $x^2 = -2x$   
 $x^2 + 2x = 0$   
 $x(x + 2) = 0$   
 $x_1 = 0$ ,  $x + 2 = 0$   
 $x_2 = -2$   
 $S = \int_{-2}^0 (-2x - x^2) dx =$   
 $= -x^2 - \frac{x^3}{3} \Big|_{-2}^0 = 0 - \left( -4 + \frac{8}{3} \right) = \frac{4}{3}$  кв. единиц

~ 52

1)  $y = \sin x$ ,  $y = x + 1$ ,  $x = 0$ ,  $x = 2$   
 $S = \int_0^2 (x + 1 - \sin x) dx =$   
 $= \left. \frac{x^2}{2} + x + \cos x \right|_0^2 =$   
 $= 2 + 2 + \cos 2 - \cos 0 =$   
 $= 4 - 0,41 - 1 = 2,59$

2)  $y = \cos x$ ,  $y = 3 - x$ ,  $x = 0$ ,  $x = -1$   
 $S = \int_{-1}^0 (3 - x - \cos x) dx =$   
 $= 3x - \frac{x^2}{2} - \sin x \Big|_{-1}^0 =$   
 $= 0 - \left( -3 - \frac{1}{2} + \sin 1 \right) =$   
 $= 3,5 - 0,84 = 2,66$

~ 53

1)  $y = x^2$ ,  $x = 0$ ,  $x = 2$   
 $V = \pi \int_0^2 x^4 dx = \pi \cdot \frac{x^5}{5} \Big|_0^2 =$   
 $= \frac{32\pi}{5}$  куб. единиц

2)  $y = x^2$ ,  $x = -2$ ,  $x = 2$   
 $V = \pi \int_{-2}^2 x^4 dx = \pi \cdot \frac{x^5}{5} \Big|_{-2}^2 =$   
 $\frac{32\pi}{5} - \left( -\frac{32\pi}{5} \right) = \frac{64\pi}{5} =$   
 $= 12 \frac{4}{5} \pi$  куб. единиц

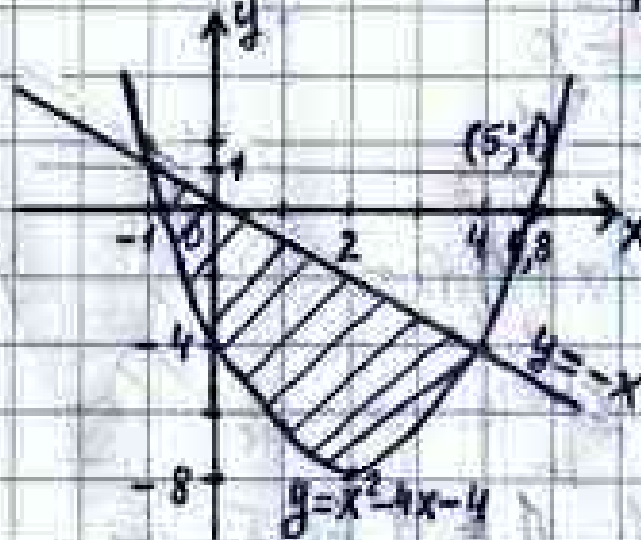
$$v = 9,8t + 0,01t^2, \quad t_0 = 0 \quad t = 4c$$

$$S(t) = \int_0^4 (9,8t + 0,01t^2) dx = 4,9t^2 + \frac{0,01 \cdot t^3}{3} \Big|_0^4 = 4,9 \cdot 16 + \frac{0,01 \cdot 64}{3} = \frac{235,2 + 0,64}{3} = \frac{235,84}{3} \text{ m}$$

~ 55

1)  $y = x^2 - 4x - 4$   
 $x^2 - 4x - 4 = -x$   
 $x^2 - 3x - 4 = 0$   
 $D = 9 + 16 = 25$   
 $x_1 = \frac{3-5}{2} = -1$   
 $x_2 = \frac{3+5}{2} = 4$

$y = -x$   
 $x^2 - 4x - 4 = 0$   
 $D = 16 + 4 \cdot 4 = 32$   
 $x_1 = \frac{4-4\sqrt{2}}{2} = 2-2\sqrt{2}$   
 $x_2 = \frac{4+4\sqrt{2}}{2} = 2+2\sqrt{2}$   
 $n = 2 \quad m = -8$

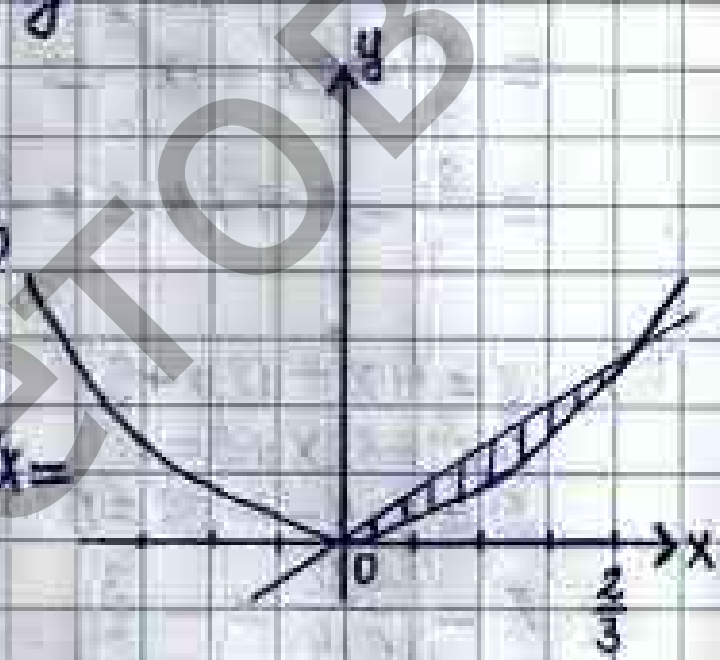


$$S = \int_{-1}^4 (x^2 - 4x - 4 - (-x)) dx = \int_{-1}^4 (x^2 - 3x - 4) dx = \left[ \frac{x^3}{3} - \frac{3x^2}{2} - 4x \right]_{-1}^4 = \frac{64}{3} - 24 - 16 - \left( -\frac{1}{3} - \frac{3}{2} + 4 \right) = \frac{100}{3} \text{ kb dipuire}$$

$$S_2(x) = \int_{-1}^4 x dx = \left[ \frac{x^2}{2} - x \right]_{-1}^4 = \frac{16}{2} - 4 - \left( \frac{1}{2} - 1 \right) = \frac{25}{2}$$

$$S_1 - S_2 = \frac{100}{3} - \frac{25}{2} = \frac{200 - 75}{6} = \frac{125}{6} \text{ kb}$$

2)  $y = 3x^2$   
 $\frac{1}{3}x^2 = 2x$   
 $3x^2 - 2x = 0$   
 $x(3x - 2) = 0$   
 $x = 0 \quad 3x - 2 = 0$   
 $x = \frac{2}{3}$



$$S = \int_0^{\frac{2}{3}} (2x - 3x^2) dx = \left[ x^2 - x^3 \right]_0^{\frac{2}{3}} = \frac{4}{9} - \frac{8}{27} = \frac{4}{27} \text{ kb dipuire}$$

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1)  $y = x^2 + 4x$   
 $x_1 = 0, y_1 = 0$   
 $x_2 = -4, y_2 = -4$   
 $A(0, 0) \quad B(-4, -4)$   
 $x - x_1 = \frac{y - y_1}{y_2 - y_1} \Rightarrow x = \frac{y + 4}{-1} = -y - 4$   
 $y = -x + 4$

$$-x^2 + 4x = -x + 4$$

$$x^2 - 5x + 4 = 0$$

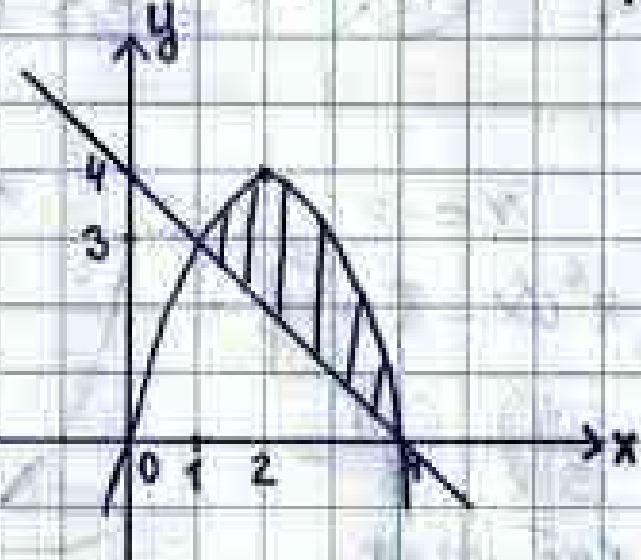
$$D = 25 - 4 \cdot 4 = 9$$

$$x_1 = \frac{5-3}{2} = 1, \quad x_2 = \frac{5+3}{2} = 4$$

$$S_1 = \int_1^4 (-x^2 + 4x) dx = \left[ -\frac{x^3}{3} + 2x^2 \right]_1^4 = -\frac{64}{3} + 32 + \frac{1}{3} - 2 = 9 \text{ kb dipuire}$$

$$S_2 = \int_1^4 (-x + 4) dx = \left[ -\frac{x^2}{2} + 4x \right]_1^4 = -8 + 16 + \frac{1}{2} - 4 = 4,5 \text{ kb dipuire}$$

$$S_1 - S_2 = 9 - 4,5 = 4,5 \text{ kb dipuire}$$

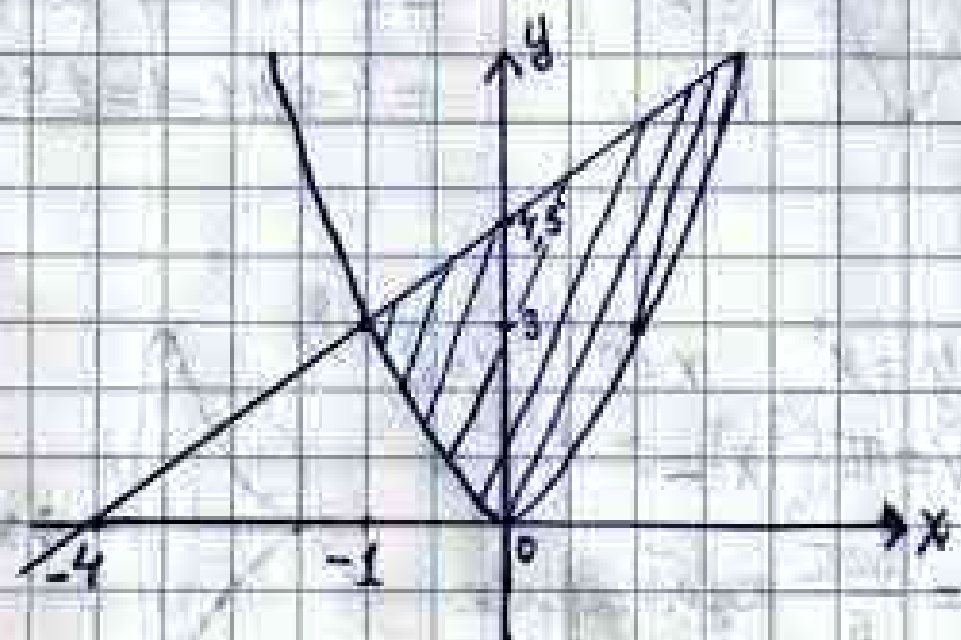


2)  $y = 3x^2$   
 $A(-3, 0) \quad B(0, 4,5)$   
 $x - x_1 = \frac{y - y_1}{y_2 - y_1} \Rightarrow x + 3 = \frac{y}{4,5} \Rightarrow y = 1,5x + 4,5$   
 $3x^2 = 1,5x + 4,5$   
 $6x^2 - 3x - 9 = 0$   
 $D = 9 + 4 \cdot 54 = 225$   
 $x_1 = \frac{3-15}{12} = -1, \quad x_2 = \frac{3+15}{12} = 1,5$

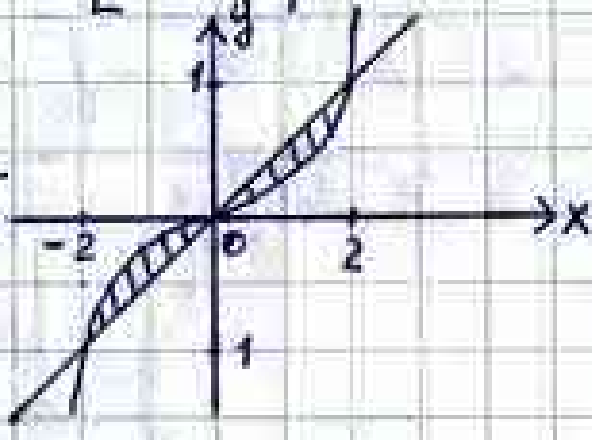
$$S_1 = \int_{-1}^{1,5} (1,5x + 4,5) dx = \left[ \frac{3}{2}x^2 + \frac{9}{2}x \right]_{-1}^{1,5} = \frac{3}{4} \cdot \frac{9}{4} + \frac{9}{2} \cdot \frac{3}{2} - \left( \frac{3}{4} - \frac{9}{2} \right) = \frac{1}{8} \text{ kb dipuire}$$

$$S_2 = \int_{-1}^{1,5} 3x^2 dx = \left[ x^3 \right]_{-1}^{1,5} = \frac{27}{8} + 1 = \frac{35}{8} \text{ kb dipuire}$$

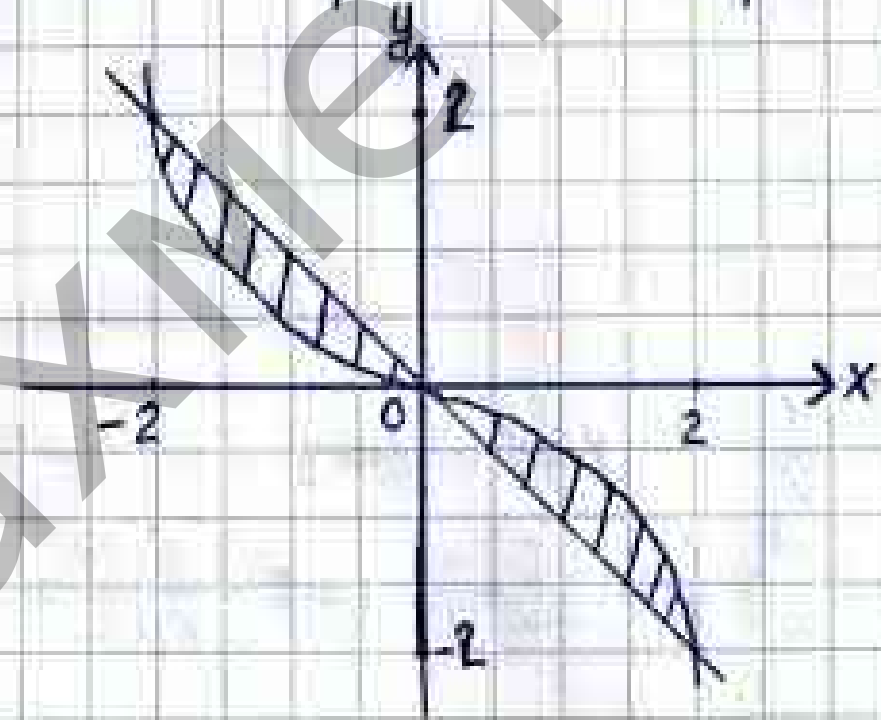
$$S_2 - S_1 = \frac{35}{8} - \frac{1}{8} = \frac{34}{8} = \frac{17}{4} = 4,25 \text{ kb dipuire}$$



1)  $y = \frac{1}{8}x^3, y = 0,5x = \frac{1}{2}x$   
 $\frac{1}{8}x^3 = \frac{1}{2}x \Rightarrow \frac{1}{2}x(\frac{1}{4}x^2 - 1) =$   
 $\frac{1}{2}x = 0 \quad \frac{1}{4}x^2 - 1 = 0$   
 $x_1 = 0 \quad x^2 = 4 \Rightarrow x = \pm 2$   
 $S_1 = \int_{-2}^0 (\frac{1}{2}x - \frac{1}{8}x^3) dx = \frac{1}{4}x^2 - \frac{1}{32}x^4 \Big|_{-2}^0 =$   
 $= 0 - (\frac{1}{4} - \frac{16}{32}) = \frac{1}{2} \text{ кв. единиц}$   
 $S_2 = \int_0^2 (\frac{1}{2}x - \frac{1}{8}x^3) dx = \frac{1}{4}x^2 - \frac{1}{32}x^4 \Big|_0^2 =$   
 $= \frac{1}{4} - \frac{16}{32} = 0 = \frac{1}{2} \text{ кв. единиц}$   
 $S_1 + S_2 =$   
 $= \frac{1}{2} + \frac{1}{2} = 1 \text{ кв.д}$

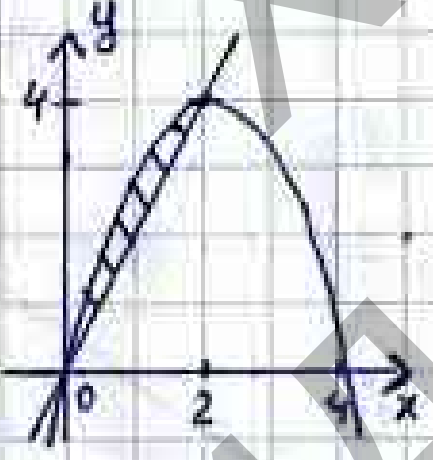


2)  $y = -\frac{1}{4}x^3, y = -x$   
 $-\frac{1}{4}x^3 = -x \Rightarrow -\frac{1}{4}x^3 + x = 0 \Rightarrow x(-\frac{1}{4}x^2 + 1) = 0$   
 $x_1 = 0 \quad -\frac{1}{4}x^2 + 1 = 0 \Rightarrow x_{2,3} = \pm 2$   
 $S_1 = \int_{-2}^0 (-x + \frac{1}{4}x^3) dx = -\frac{x^2}{2} + \frac{x^4}{16} \Big|_{-2}^0 =$   
 $0 - (-\frac{4}{2} + \frac{16}{16}) = 1$   
 $S = 2 \cdot 1 \text{ кв. единиц} = 2 \text{ кв. единиц}$

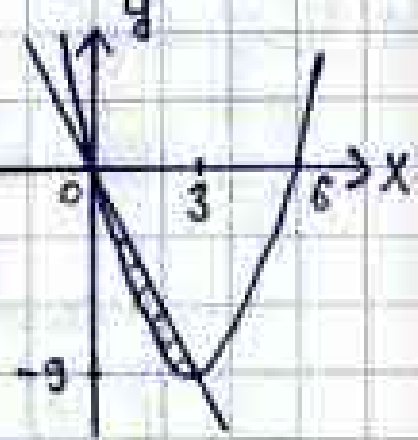


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1)  $y = 4x - x^2 = -x^2 + 4x$   
 $n = -\frac{4}{2(-1)} = 2 \quad m = -4 + 8 = 4$   
 $4x - x^2 = 0$   
 $x_1 = 0 \quad 4 - x = 0 \Rightarrow x_2 = 4$   
 $A(2, 4) \quad B(0, 0)$   
 $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} \Rightarrow \frac{x-2}{0-2} = \frac{y-4}{0-4}$   
 $\frac{x-2}{-2} = \frac{y-4}{-4} \Rightarrow y-4 = 2(x-2) \Rightarrow y = 2x$   
 $S = \int_0^2 (4x - x^2 - 2x) dx = x^2 - \frac{x^3}{3} \Big|_0^2 =$   
 $= \frac{4}{1} - \frac{8}{3} - 0 = \frac{12-8}{3} = \frac{4}{3} \text{ кв. единиц}$

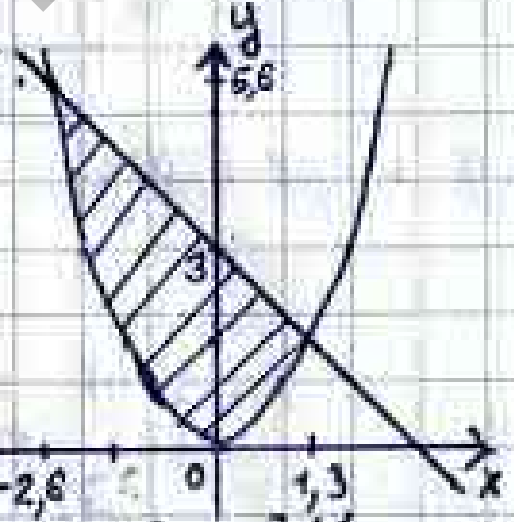


2)  $y = x^2 - 6x$   
 $n = -\frac{-6}{2} = 3 \quad m = 9 - 18 = -9$   
 $x^2 - 6x = 0$   
 $x_1 = 0 \quad x - 6 = 0 \Rightarrow x_2 = 6$   
 $A(3, -9) \quad B(0, 0)$   
 $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} \Rightarrow \frac{x-3}{0-3} = \frac{y+9}{0+9}$   
 $\frac{x-3}{-3} = \frac{y+9}{9} \Rightarrow y+9 = -3(x-3) \Rightarrow y = -3x$   
 $S = \int_0^3 (x^2 - 6x + 3x) dx = \frac{x^3}{3} - \frac{3x^2}{2} \Big|_0^3 =$   
 $= \frac{27}{3} - \frac{27}{2} = \frac{18-27}{2} = -4,5 \text{ кв. единиц}$

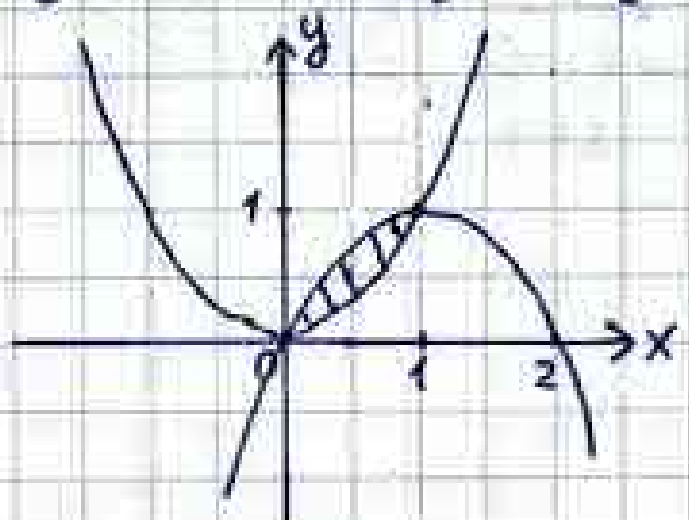


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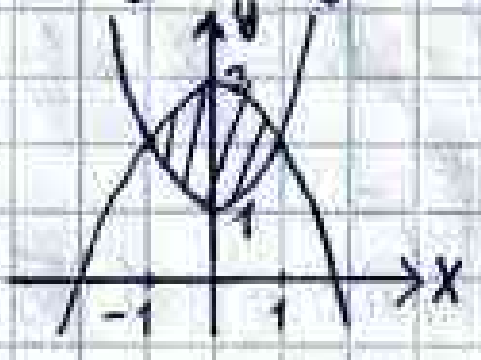
1)  $y = x^2, y = 3 - x$   
 $x^2 = 3 - x$   
 $x^2 + x - 3 = 0$   
 $D = 1 + 3 \cdot 4 = 13$   
 $x_1 = \frac{-1 - \sqrt{13}}{2} \approx -2$   
 $x_2 = \frac{-1 + \sqrt{13}}{2} \approx 1$   
 $S = \int_{-2}^1 (3 - x - x^2) dx = 3x - \frac{x^2}{2} - \frac{x^3}{3} \Big|_{-2}^1 =$   
 $= \frac{3}{1} - \frac{1}{2} - \frac{1}{3} + 6 - 2 - \frac{8}{3} = 3,5 \text{ кв. единиц}$



2)  $y = x^2, y = 2x - x^2$   
 $x^2 = 2x - x^2$   
 $2x^2 - 2x = 0$   
 $x = 0 \quad x_2 = 1$   
 $x_1 = 0 \quad x_2 = 2$   
 $S = \int_0^1 (x^2 - 2x + x^2) dx = \frac{2x^3}{3} - x^2 \Big|_0^1 =$   
 $= \frac{2}{3} - \frac{1}{1} - 0 = \frac{2-3}{3} = -\frac{1}{3} \text{ кв. единиц}$



3)  $y = x^2 + 1$ ,  $y = -x^2 + 3$   
 $x^2 + 1 = -x^2 + 3$   $-x^2 - 3 = 0$   $n=0, m=3$   
 $2x^2 - 2 = 0$   $x_1 = \sqrt{3}$   
 $x_1 = 1$   $x_2 = -1$   $x_2 = -\sqrt{3}$   
 $S = \int_{-1}^1 (-x^2 + 3 - x^2 - 1) dx = \left. -\frac{2x^3}{3} + 2x \right|_{-1}^1 =$   
 $= -\frac{2}{3} + 2 - \frac{2}{3} + 2 = \frac{1}{3} + \frac{3}{3} = \frac{16}{3} = 5\frac{1}{3}$  кв. бірлік.



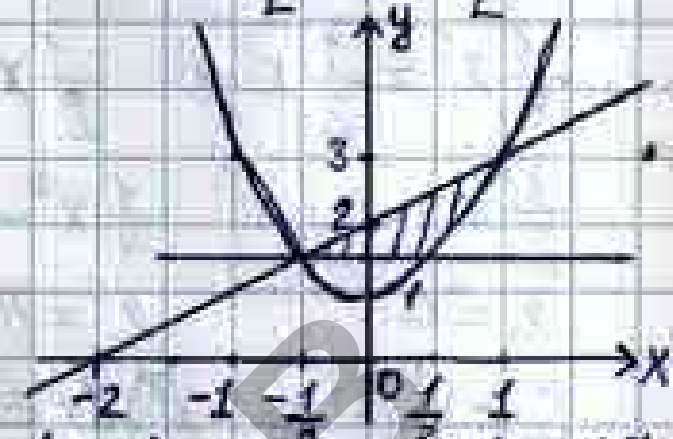
4)  $y = 2x^2 + 1$ ,  $y = x + 2$ ,  $y = 1.5$   
 $2x^2 + 1 = x + 2$   $2x^2 - x - 1 = 0$   $2x^2 + 1 = 1.5$   
 $D = 1 + 8 = 9$   $x_1 = -\frac{1}{2}$ ,  $x_2 = \frac{1}{2}$   
 $x_1 = \frac{1-3}{4} = -\frac{1}{2}$   
 $x_2 = \frac{1+3}{4} = 1$

$S_1 = \int_{-\frac{1}{2}}^1 (2x^2 + 1) dx = \left. \frac{2x^3}{3} + x \right|_{-\frac{1}{2}}^1 = \left(\frac{2}{3} + 1\right) - \left(-\frac{1}{3} + \frac{1}{2}\right) = \frac{27}{12}$

$S_2 = \int_{-\frac{1}{2}}^1 1.5 dx = \frac{3}{2}x \Big|_{-\frac{1}{2}}^1 = 0 + \frac{3}{4} = \frac{3}{4}$

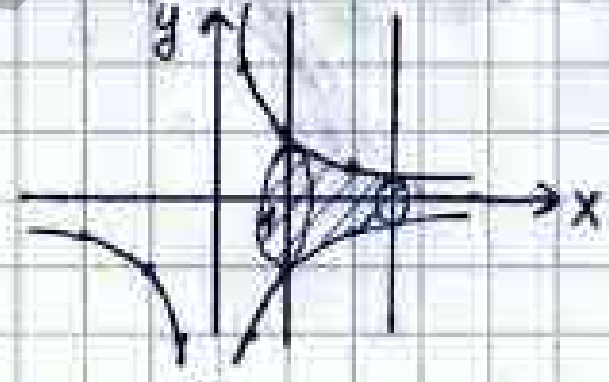
$S_3 = \int_{\frac{1}{2}}^1 (2x^2 + 1) dx = \left. \frac{2x^3}{3} + x \right|_{\frac{1}{2}}^1 = \left(\frac{2}{3} + 1\right) - \left(\frac{1}{3 \cdot 4} + \frac{1}{2}\right) = \frac{13}{12}$

$S_1 - S_2 - S_3 = \frac{27}{12} - \frac{3}{4} - \frac{13}{12} = \frac{5}{12}$  кв. бірлік.



~60)  $y = \frac{1}{x}$   $x=1$   $x=3$

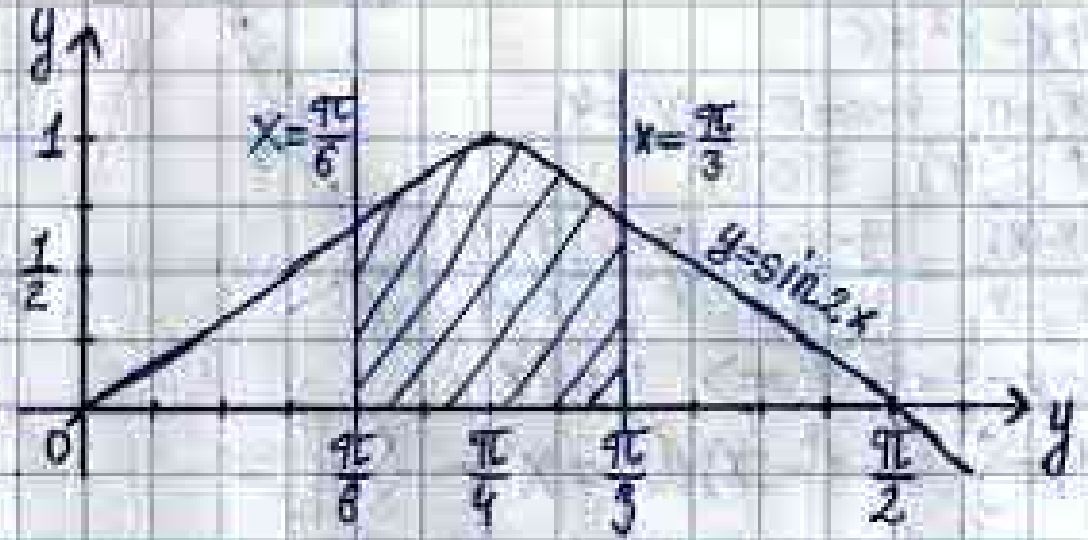
$V = \pi \int_1^3 y^2 dx = \pi \int_1^3 \frac{1}{x^2} dx = \pi \left( -\frac{1}{x} \right) \Big|_1^3 = \pi \left( -\frac{1}{3} + 1 \right) = \frac{2\pi}{3}$



~61)  $v = Rt + at^2$   $S(t) = \int_0^t (Rt + at^2) dt = \left. \left( \frac{Rt^2}{2} + \frac{at^3}{3} \right) \right|_0^t = 8R + \frac{16}{3}a$

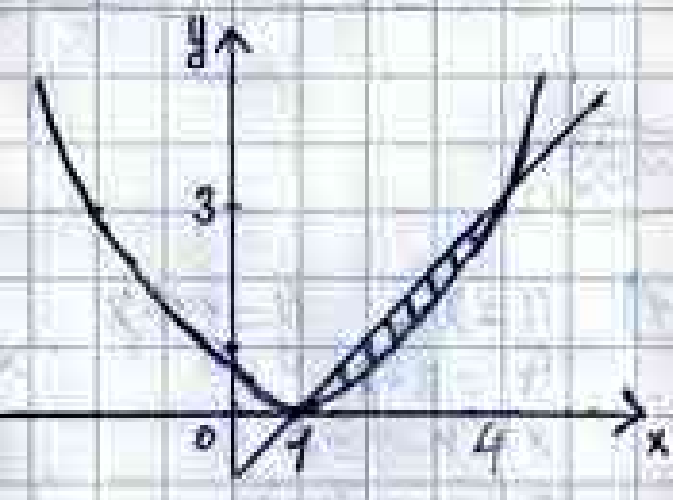
~62)  $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin 2x dx = -\frac{1}{2} \cos 2x \Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}} =$

$= -\frac{1}{2} \cos \frac{2\pi}{3} + \frac{1}{2} \cos \frac{\pi}{3} =$   
 $= -\frac{1}{2} \cdot \left(-\frac{1}{2}\right) + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$  кв. бірлік



~63)  $y = x^2 - 2x + 1$   
 $n = -\frac{-2}{2} = 1$   $m = 0$

$y' = 2x - 2$   
 $2x - 2 = 0$   
 $x = 1$  Ох осімен (1; 0) қимасын  
 $x^2 - 2x + 1 = 2x - 2$   
 $x^2 - 4x + 3 = 0$   
 $x_1 = 1$   $x_2 = 3$  екі р-а қимасын



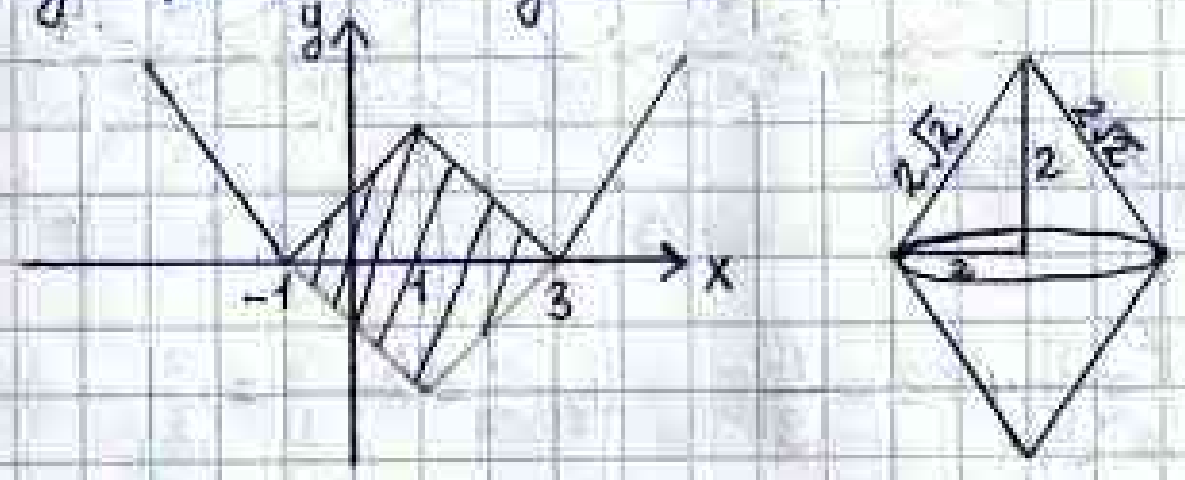
~64)  $f(x) = 2x - 4$   
 $F(x) = x^2 - 4x + C$   $A(0; 4)$   
 $0^2 - 4 \cdot 0 + C = 4 \Rightarrow C = 4$   
 $F(x) = x^2 - 4x + 4$

$x^2 - 4x + 4 = 2x - 4$   
 $x^2 - 6x + 8 = 0$   
 $D = 36 - 32 = 4$   
 $x_1 = \frac{6-2}{2} = 2$ ,  $x_2 = \frac{6+2}{2} = 4$

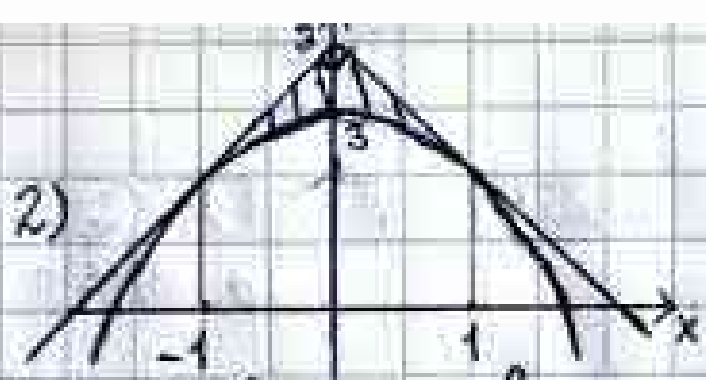
$S_1 = \int_2^4 (2x - 4) dx = x^2 - 4x \Big|_2^4 = 16 - 16 - 4 + 8 = 4$   
 $S_2 = \int_2^4 (x^2 - 4x + 4) dx = \left. \frac{x^3}{3} - 2x^2 + 4x \right|_2^4 = \frac{8}{3}$   
 $S_1 - S_2 = 4 - \frac{8}{3} = \frac{4}{3}$  кв. бірлік

~66

1)  $y = ||x-1|-2|$   $y=0$   $x=0$



$V_k = 2 \cdot V_k = 2 \cdot \frac{1}{3} \cdot S_{\text{осн}} \cdot H = \frac{2}{3} \pi R^2 H =$   
 $= \frac{2}{3} \pi \cdot 4 \cdot 2 = \frac{16\pi}{3}$   $\text{кв. дм}$



2)  $-0,5x^2 + 3 = x + 8$   
 $0,5x^2 + x - 3 + 8 = 0$   
 $D = 1 + 4 \cdot 0,5(3-8) = 0 \Rightarrow b = 3,5$   
 $-0,5x^2 + 3 = x + 3,5$   
 $x = -1$   $x = 1$   
 $S = \int_{-1}^1 ((x+3,5) - (-\frac{1}{2}x^2 + 3)) dx +$   
 $+ \int_0^1 ((-x+3,5) - (-\frac{1}{2}x^2 + 3)) dx = \frac{1}{3} \text{ кв. дм}$

$k_1 = \text{tg } 45^\circ = 1$   
 $k_2 = \text{tg } 135^\circ = -1$   
 $y = kx + b$   
 $y_1 = -x + 8$   
 $y_2 = x + 8$

~68

$y = -\frac{1}{4}x^2 + 1$   $n=0; m=1$

$-\frac{1}{4}x^2 + 1 = 0 \Rightarrow x_1 = -2, x_2 = 2$   $Ox$  осімен кутелері  $(-2; 0), (2; 0)$

тананма тендеуі:  
 $y = -\frac{1}{4}x^2 + 1$   $x_0 = 2;$

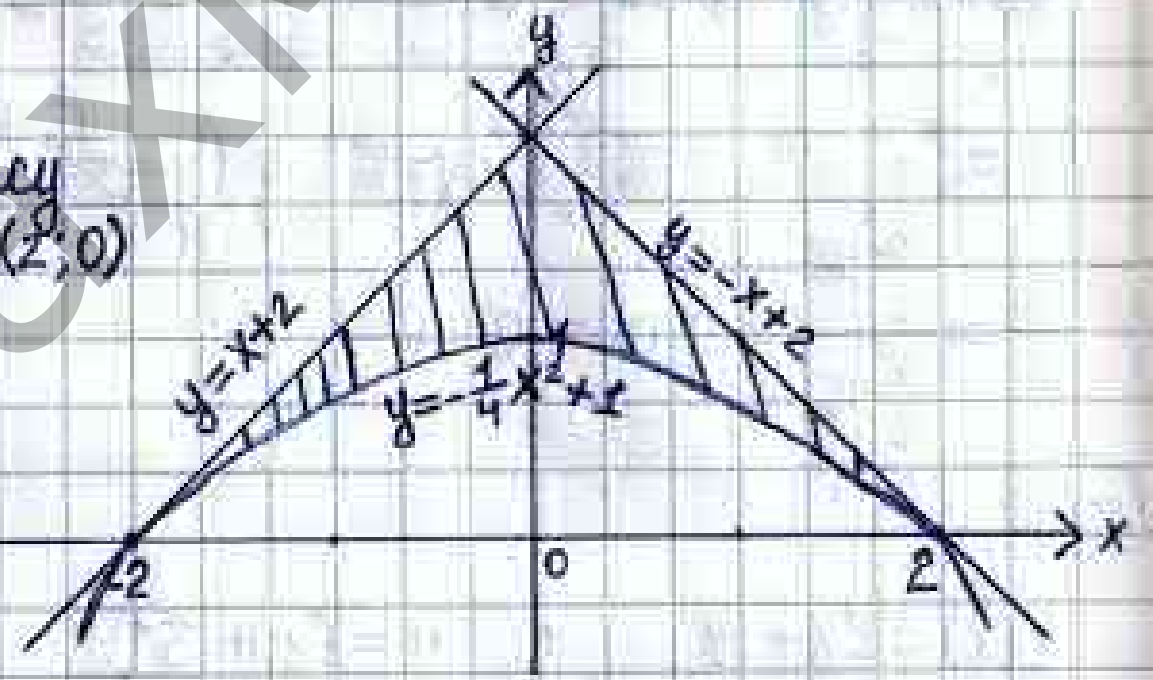
$y'(2) = -\frac{1}{2}x = -\frac{1}{2} \cdot 2 = -1; y(2) = -\frac{1}{4} \cdot 4 + 1 = 0.$

$y = f(x_0) + f'(x_0)(x-x_0) \Rightarrow y = 0 - 1(x-2) = x+2$

деген  $y_1 = -x+2$   $y_2 = -f(x)+2 = x+2$

$S_1 = \int_0^2 (-x+2) dx = -\frac{x^2}{2} + 2x \Big|_0^2 = -\frac{4}{2} + 4 = 2;$   $S_2 = \int_0^2 (-\frac{1}{4}x^2 + 1) dx = -\frac{x^3}{12} + x \Big|_0^2 = -\frac{8}{12} + 2 = -\frac{2}{3} + 2 = \frac{4}{3}$

$S_1 - S_2 = 2 - \frac{4}{3} = \frac{6-4}{3} = \frac{2}{3}$   $S = 2 \cdot \frac{2}{3} = \frac{4}{3} \text{ кв. дм}$



~69

$y = -x^2 + 4x$   $n=2; m=4$

$x^2 + 4x = 0 \Rightarrow x_1 = 0, x_2 = 4$   $Ox$  осімен кутелері

тананма тендеуі:  
 $y = -x^2 + 4x$   $x_0 = 0$

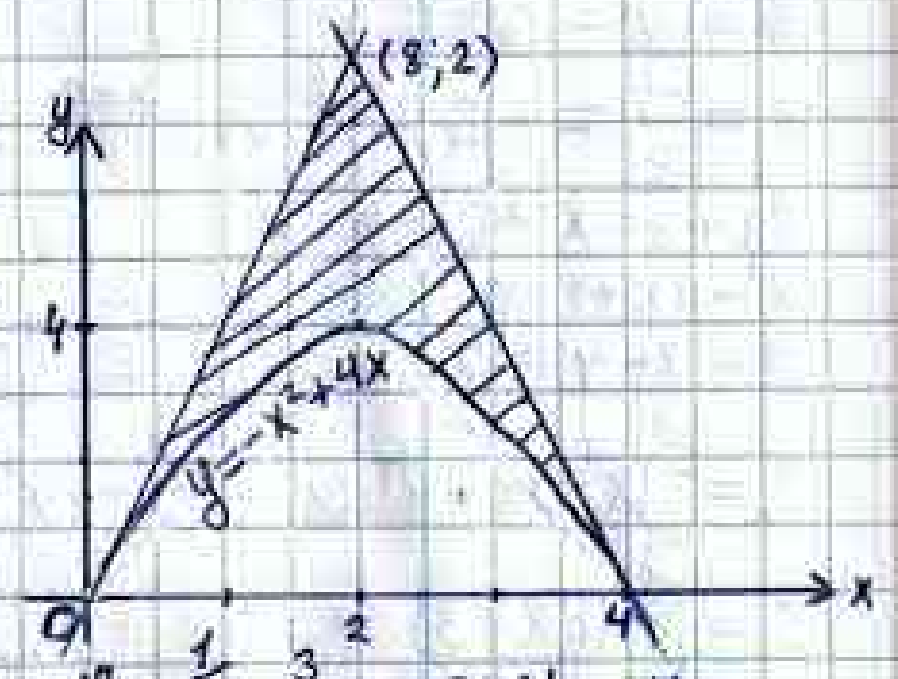
$y'(0) = -2x + 4 = -2 \cdot 0 + 4 = 4; y(0) = -0 + 4 \cdot 0 = 0$

$y = f(x_0) + f'(x_0)(x-x_0) \Rightarrow y = 0 + 4(x-0) = 4x$

деген  $y_1 = 4x$   $y_2 = -4x$

$S_1 = \int_0^2 4x dx = 2x^2 \Big|_0^2 = 2 \cdot 4 = 8;$   $S_2 = \int_0^2 (-x^2 + 4x) dx = -\frac{x^3}{3} + 2x^2 \Big|_0^2 = -\frac{8}{3} + 8 = \frac{-8+24}{3} = \frac{16}{3}$

$S_1 - S_2 = 8 - \frac{16}{3} = \frac{24-16}{3} = \frac{8}{3}$   $S = 2 \cdot \frac{8}{3} = \frac{16}{3} = 5 \frac{1}{3} \text{ кв. дм}$



~40

$y = \int_0^{x+1} 3t^2 dt = t^3 \Big|_0^{x+1} = (x+1)^3 - x^3 = x^3 + 3x^2 + 3x + 1 - x^3 = 3x^2 + 3x + 1, y=1$

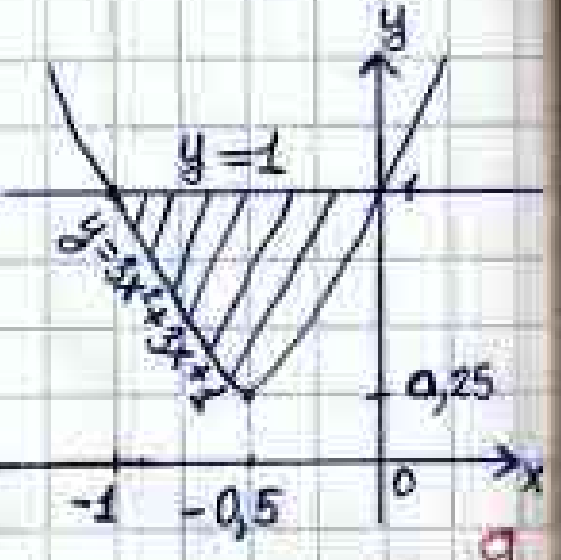
$y=0$   $3x^2 + 3x + 1 = 0$   $x=0$   $y=1$   
 $D = 9 - 12 = -3 < 0$   $Ox$  осімен кутелері;  $Oy$  осімен  $(0; 1)$ .

$n = -\frac{b}{2a} = -\frac{3}{2 \cdot 3} = -0,5$   $m = 0,25$

$3x^2 + 3x + 1 = 1 \Rightarrow x_1 = 0, x_2 = -1$

$S_1 = \int_{-1}^0 dx = x \Big|_{-1}^0 = 1, S_2 = \int_{-1}^0 (3x^2 + 3x + 1) dx = x^3 + \frac{3x^2}{2} + x \Big|_{-1}^0 =$

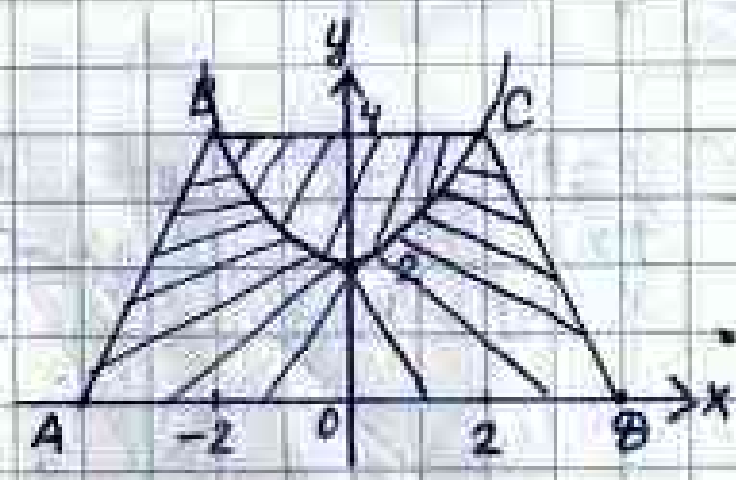
$= 0 - (-1 + \frac{3}{2} - 1) = 2 - 1,5 = 0,5$   $S_1 - S_2 = 1 - 0,5 = 0,5 \text{ кв. дм}$



41

$A(-4;0) \quad B(-2;4) \quad C(2;4) \quad D(4;0) \quad y = \frac{1}{2}x^2 + 2$

$\frac{1}{2}x^2 + 2 = 0 \quad n=0 \quad m=2$   
 $x^2 = -4$  Ох осімен қиылыспайды  $x=0 \quad y=2$  Оу осімен  $(0;2)$



$S_{ABCD} = \frac{a+b}{2} \cdot h = \frac{4+8}{2} \cdot 4 = 24$

$S_1 = \int_0^2 4 dx = 4x \Big|_0^2 = 8 \quad S_2 = \int_0^2 (\frac{1}{2}x^2 + 2) dx = \frac{x^3}{6} + 2x \Big|_0^2 = \frac{8}{6} + 2 \cdot 2 = \frac{16}{3}; \quad S_1 - S_2 = 8 - \frac{16}{3} = \frac{8}{3}$

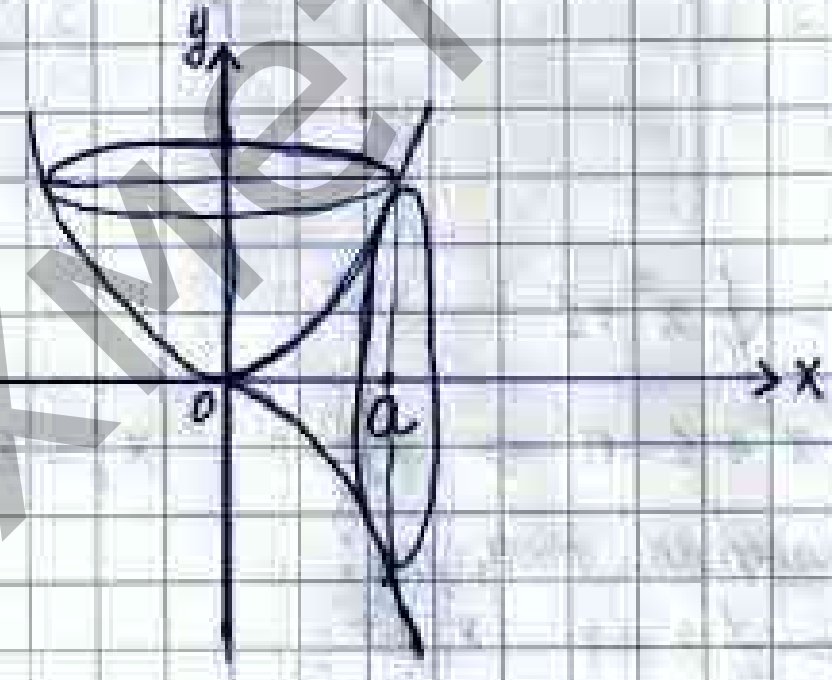
$S = 2 \cdot \frac{8}{3} = \frac{16}{3} \quad 24 - \frac{16}{3} = \frac{56}{3} \quad \frac{16}{3} : \frac{56}{3} = \frac{2}{7} = 2:7$

42

$y = x^2 \quad x=0 \quad x=a$   
 $V_1 = \pi \int_0^a y^2 dx = \pi \int_0^a x^4 dx = \pi \frac{x^5}{5} \Big|_0^a = \frac{a^5}{5} \pi$

$x = \sqrt{y} \quad V_2 = \pi \int_0^a x^2 dy = \pi \int_0^a y dy = \frac{y^2}{2} \Big|_0^a = \frac{a^2}{2} \pi$

$\frac{V_1}{V_2} = \frac{\frac{a^5}{5} \pi}{\frac{a^2}{2} \pi} = \frac{2}{5} a^3 = 0,4 a^3$



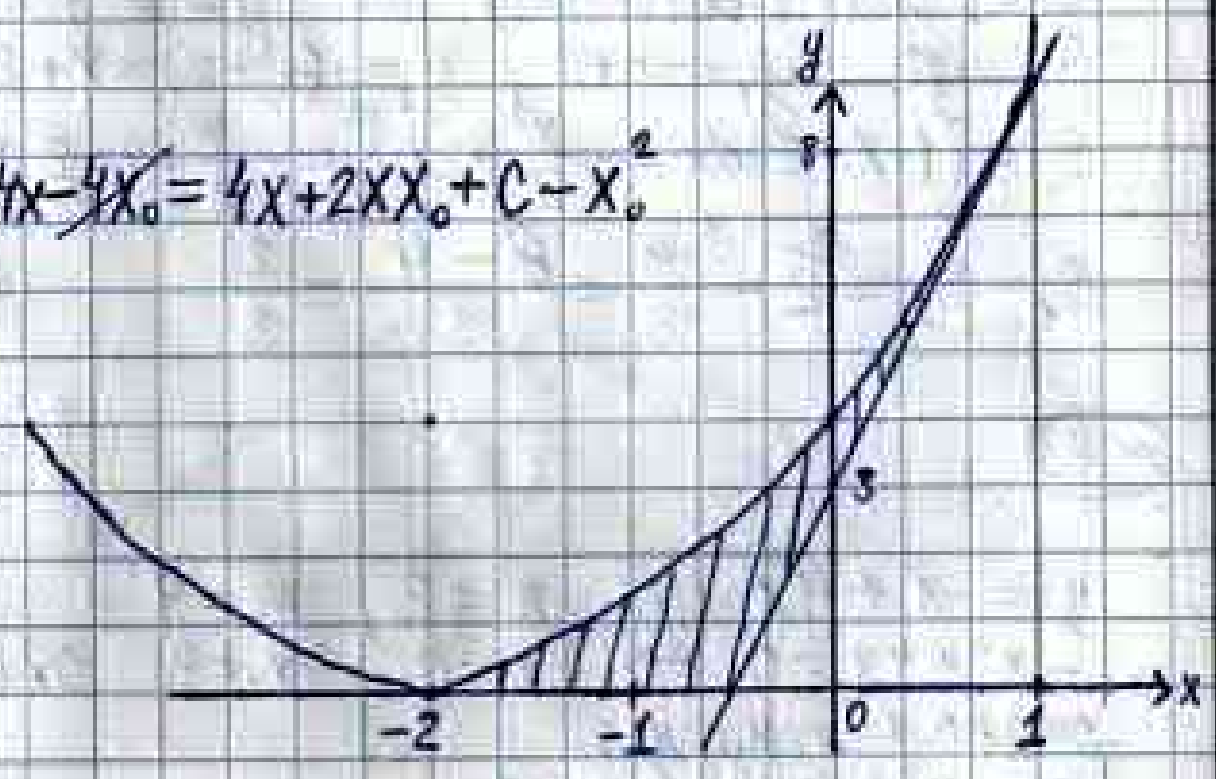
44

$f(x) = 2x + 4 \quad y = 6x + 3; \quad y = 0$   
 $F(x) = x^2 + 4x + C$

$y = F(x_0) + f(x)(x-x_0)$   
 $6x + 3 = x_0^2 + 4x_0 + C + (2x+4)(x-x_0) = x_0^2 + 4x_0 + C + 2x_0x - 2x_0^2 + 4x - 4x_0 = 4x + 2x_0x + C - x_0^2$   
 $6x = 4x + 2x_0x \Rightarrow x_0 = 1 \quad 3 = C - x_0^2$   
 $F(x) = x^2 + 4x + 4 = 0 \quad 3 = C - 1^2 \Rightarrow C = 4$

$D = 16 - 16 = 0$   
 $x = \frac{-4}{2} = -2$  егер  $x=0 \quad y=4$

$6x + 3 = x^2 + 4x + 4$   
 $x^2 - 2x + 1 = 0 \quad x = \frac{2}{2} = 1; \quad y = 9$   
 $D = 4 - 4 = 0$



$S_1 = \int_{-2}^1 (x^2 + 4x + 4) dx = \frac{x^3}{3} + 2x^2 + 4x \Big|_{-2}^1 = \frac{1}{3} + 2 + 4 - (-\frac{8}{3} + 8 - 8) = 4$

$S_2 = \int_{\frac{1}{2}}^1 (6x + 3) dx = 3x^2 + 3x \Big|_{\frac{1}{2}}^1 = 3 + 3 - (3 \cdot \frac{1}{4} + 3 \cdot \frac{1}{2}) = 6 + \frac{3}{4} = 6 \frac{3}{4} \quad S = S_1 - S_2 = 4 - 6 \frac{3}{4} = \frac{1}{4}$

Узунд мөхсөр!

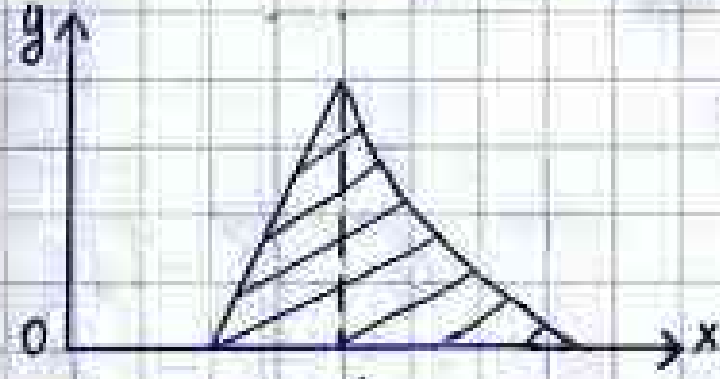
1)  $F(x) = 2 - \cos x$      $f(x) = \sin x$     (B)

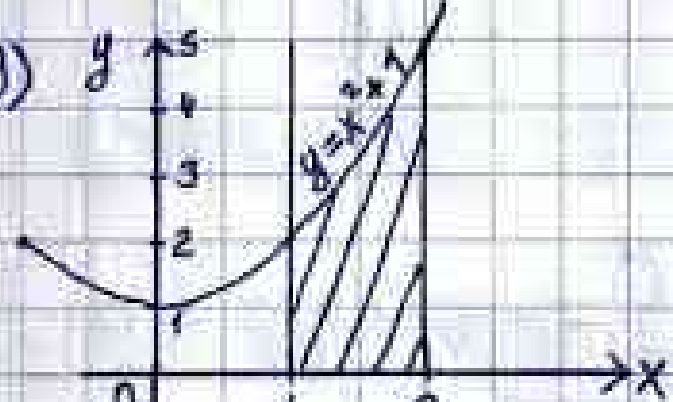
2)  $f(x) = 5x^4 - 2x$      $F(x) = x^5 - x^2$     (D)

4)  $F(x) = \frac{3}{x-2}$      $f(x) = -\frac{3}{(x-2)^2}$     (C)  $(-\infty, 2) \cup (2, +\infty)$

5)  $y = x^2 - 2x$      $A(-1, -1)$   
 $F(x) = \frac{x^3}{3} - x^2 + C$      $-\frac{1}{3} - 1 + C = -1$      $C = \frac{1}{3} \Rightarrow F(x) = \frac{x^3}{3} - x^2 + \frac{1}{3}$     (C)

7)  $\int_2^4 10x dx = 5x^2 \Big|_2^4 = 5 \cdot 16 - 5 \cdot 4 = 60$     (D)

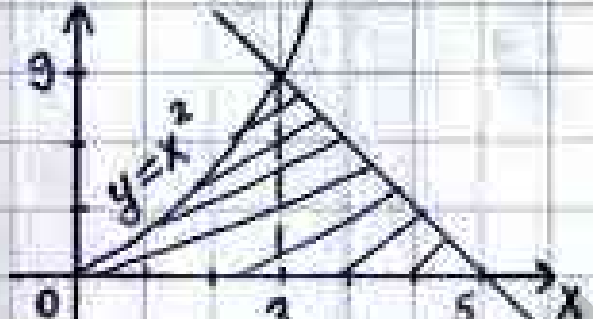
8)     (B)  $S = S_{\text{трап}} + S_{\Delta}$

9)      $S = \int_1^2 (x^2 + 1) dx = \frac{x^3}{3} + x \Big|_1^2 = (\frac{8}{3} + 2) - (\frac{1}{3} + 1) = 3\frac{1}{3}$     (D)

10)  $\sqrt{2} \int_0^{\frac{\pi}{4}} \cos x dx = \sqrt{2} \sin x \Big|_0^{\frac{\pi}{4}} = \sqrt{2} (\sin \frac{\pi}{4} - \sin 0) = \sqrt{2} \cdot \frac{\sqrt{2}}{2} = 1$     (C)

11)  $\int_1^b 8 dx = 8x \Big|_1^b = 8b - 8 = 8 \Rightarrow b = 2$     (C)

12)  $y = x^2 - 4x + 5$      $y = 5$     *ауганш*  
 $x^2 - 4x + 5 = 5$      $S = \int_0^4 (5 - x^2 + 4x - 5) dx = 2x^2 - \frac{x^3}{3} \Big|_0^4 = 2 \cdot 16 - \frac{64}{3} = \frac{32}{3} = 10\frac{2}{3}$     (A)

13)      $S = \int_0^3 x^2 dx = \frac{x^3}{3} \Big|_0^3 = 9$   
 $S_{\Delta} = 9 \cdot 2 = 18$      $S = 9 + 18 = 27$     (C)

14)  $\int_0^4 \frac{x^3 + 1}{x^2 - x + 1} dx = \int_0^4 \frac{(x+1)(x^2 - x + 1)}{(x^2 - x + 1)} dx = \frac{x^2}{2} + x \Big|_0^4 = \frac{16}{2} + 4 = 12$     (D)

15)  $\int_0^{\frac{\pi}{2}} 14 \sin x dx = -14 \cos x \Big|_0^{\frac{\pi}{2}} = -14 \cos \frac{\pi}{2} + 14 \cos 0 = 14$     (C)

16)  $\int_0^6 \frac{x^4 - 1}{x + 1} dx = \int_0^6 \frac{(x^2 + 1)(x - 1)(x + 1)}{(x + 1)} dx = \int_0^6 (x^3 + x - x^2 - 1) dx = \frac{x^4}{4} + \frac{x^2}{2} - \frac{x^3}{3} - x \Big|_0^6 = 324 + 18 - 72 - 6 = 264$     (B)

17)  $\int_0^x 12t^2 dt = 4t^3 \Big|_0^x = 4x$      $4x = 4 \Rightarrow x = 1$     (D)

18)  $\int_{-2}^x 4 dt = 4t \Big|_{-2}^x = 4x + 8$      $4x + 8 > 0 \Rightarrow x > -2$     (A)

19)  $F(x) = 7,5x^2 - 10$      $f(x) = 15x$     (B)

20)  $V = \pi \int_0^3 y^2 dx$      $y = x^2$      $x = 0, x = 3$

$V = \pi \int_0^3 x^4 dx = \frac{x^5}{5} \pi \Big|_0^3 = \frac{243\pi}{5}$     (A)

~ 75

$$1) \sqrt[3]{99 \cdot 64 \cdot 100} = 7 \cdot 8 \cdot 10 = 560$$

$$3) \sqrt{a^4 \cdot 8^2 \cdot c^6} = a^2 b c^3$$

$$2) \sqrt[3]{8 \cdot 27 \cdot 125} = 2 \cdot 3 \cdot 5 = 30$$

$$4) \sqrt[3]{m^2 \cdot K^{12} \cdot t^6} = m^2 K^4 t^2$$

~ 76

$$1) \frac{\sqrt{49}}{\sqrt{225}} = \frac{7}{15}$$

$$2) \sqrt[3]{\frac{8 \cdot 125}{343}} = \frac{2 \cdot 5}{7} = \frac{10}{7}$$

$$3) \sqrt[4]{\frac{1}{625} \cdot \frac{1}{16}} = \frac{1}{20}$$

$$4) \sqrt[5]{\frac{486}{32}} = \frac{3}{2}$$

~ 77

$$1) \sqrt[5]{32 a^{10}} = 2 a^2$$

$$2) \sqrt[6]{128 a^{12} b^{18} c^6} = 2 \sqrt[3]{2 a^2 b^3 c}$$

$$3) \sqrt[3]{64 m^6 n^9 p^3} = 4 m^2 n^3 p$$

$$4) \sqrt[4]{\frac{16 x^8 y^{12} z^4}{81 x^4 y^8 z^3}} = \frac{2 x y z}{3}$$

~ 78

$$1) \sqrt{3} = \sqrt{3}$$

$$2) \sqrt{4} = \sqrt{2^2} = 2$$

$$3) \sqrt[3]{7} = \sqrt[3]{7}$$

$$4) \sqrt[5]{11} = \sqrt[5]{11}$$

$$5) \sqrt{a \sqrt{a}} = \sqrt[4]{a^3}$$

$$6) \sqrt[4]{9 a^8} = \sqrt[4]{9} a^2 = \frac{3}{2} a^2$$

$$7) \sqrt[5]{m n} = \sqrt[5]{m n}$$

$$8) \sqrt[3]{\frac{a}{8}} = \sqrt[3]{\frac{a}{8}}$$

~ 79

$$1) \frac{\sqrt{2 \cdot 3}}{\sqrt[5]{486}} + \sqrt[3]{27 \cdot 2^6} = \sqrt[4]{\frac{1}{81}} + 3 \cdot 2^2 = 12 \frac{1}{3}$$

$$3) \sqrt[3]{27 \cdot 4^3} - \sqrt{\frac{81}{256}} = 3 \cdot 4 - \frac{9}{16} = 11 \frac{7}{16}$$

$$2) \sqrt[3]{216 \cdot 7^3} - \sqrt{\frac{32}{243}} = 6 \cdot 7 - \frac{2}{3} = 41 \frac{1}{3}$$

$$4) 5 - \left( 3 \sqrt{\frac{16}{81}} + \sqrt[3]{0,125} \right) = 5 - \left( 3 \cdot \frac{2}{3} + 0,5 \right) = 2,5$$

~ 80

$$1) 1 - \sqrt{2 \frac{7}{9}} + 0,3 \cdot \sqrt[4]{256} = 1 - \frac{5}{3} + 1,2 = \frac{8}{15}$$

$$3) \frac{11}{0,15 \cdot \sqrt[3]{64000} - 0,29 \sqrt[3]{8000}} = \frac{11}{0,15 \cdot 40 - 0,29 \cdot 20} = 55$$

$$2) 2 \sqrt{1 \frac{11}{25}} - 1 \frac{2}{5} + 0,7 \sqrt[3]{0,216} = 2 \cdot \frac{6}{5} - \frac{7}{5} + 0,7 \cdot 0,6 = 2,4 - 1,4 + 0,42 = 1,42$$

$$4) \frac{2,5 \sqrt[3]{10000} + \frac{3}{4} \sqrt{1,44} - 2,09 \sqrt[3]{1,331}}{2,5 \cdot 10 + \frac{3}{4} \cdot 1,2 - 2,09 \cdot 1,1} = \frac{25 + 0,9 - 1,9}{25 + 0,9 - 1,9} = 24$$

~ 81

$$1) \sqrt[3]{8 + \sqrt{37}} \cdot \sqrt{8 - \sqrt{37}} = \sqrt[3]{64 - 37} = \sqrt[3]{27} = 3$$

$$3) (\sqrt{3 - \sqrt{5}} + \sqrt{3 + \sqrt{5}}) \cdot 0,2^2 =$$

$$2) \sqrt[3]{7 - \sqrt{47}} \cdot \sqrt[3]{7 + \sqrt{47}} = \sqrt[3]{49 - 47} = \sqrt[3]{2} = 2$$

$$(3 - \sqrt{5} + 2 \sqrt{3 - \sqrt{5}} + 3 + \sqrt{5}) \cdot 25 = 1250$$

$$4) (\sqrt{6 - \sqrt{11}} - \sqrt{6 + \sqrt{11}})^2 \cdot \left(\frac{2}{3}\right)^2 =$$

$$(6 - \sqrt{11} - 2 \sqrt{36 - 11} + 6 + \sqrt{11}) \cdot \frac{4}{9} = 3$$

~ 82

$$1) 2 \sqrt[3]{81} + \sqrt[3]{125} + \sqrt[3]{64} = 2 \cdot 3 + 5 + 2 = 3$$

$$3) \sqrt[3]{375} - \frac{2}{3} \sqrt[3]{1029} + 0,75 \sqrt[3]{192} - 0,2 \sqrt[3]{3000} =$$

$$2) 5 \sqrt[3]{-8} + \sqrt[3]{16} - \sqrt[3]{729} = 5 \cdot (-2) + 2 - 3 = -11$$

$$5 \sqrt[3]{3} - \frac{2}{3} \cdot 7 \sqrt[3]{3} + 0,75 \cdot 4 \sqrt[3]{3} - 0,2 \cdot 10 \sqrt[3]{3} = 4 \sqrt[3]{3}$$

$$4) \frac{4}{3} \sqrt[3]{162} - 0,2 \sqrt[3]{1250} + 0,75 \sqrt[3]{512} - 4 \sqrt[3]{2} =$$

$$4 \cdot \sqrt[3]{2} - 0,2 \cdot 5 \sqrt[3]{2} + 0,75 \cdot 4 \sqrt[3]{2} - 4 \sqrt[3]{2} = -\sqrt[3]{2}$$

~ 83

$$1) (2\sqrt{145} - 3\sqrt{28} + 2\sqrt{63})^2 - 60 \sqrt[3]{1000} = (2 \cdot 5 \sqrt{4} - 3 \cdot 2 \sqrt{7} + 2 \cdot 3 \sqrt{7})^2 - 60 \cdot 10 = 100 \cdot 7 - 600 = 100$$

$$2) \frac{1}{3} (2\sqrt{150} + 3\sqrt{24} - 5\sqrt{54})^2 + 15 \sqrt[4]{625} = \frac{1}{3} (2 \cdot 5 \sqrt{6} + 3 \cdot 2 \sqrt{6} - 5 \cdot 3 \sqrt{6})^2 + 15 \cdot 5 = \frac{1}{3} \cdot 6 + 75 = 74$$

$$3) (\sqrt{5 + 2\sqrt{6}} + \sqrt{3 + \sqrt{2}}) \cdot \sqrt[3]{\sqrt{2} - \sqrt{3}} = \sqrt{(5 + 2\sqrt{6})(\sqrt{2} - \sqrt{3})^2} + \sqrt[3]{2 - 3} = \sqrt{(5 + 2\sqrt{6})(2 - 2\sqrt{6} + 3)} + (-1) = 0$$

1.2



$$4) \sqrt{20,25} + \sqrt[3]{24} - \sqrt[4]{0,1296} - \frac{2}{5} \sqrt[3]{375} + \frac{1}{3} \sqrt[5]{\frac{7}{32}} = 4,5 - 2\sqrt[3]{5} - 0,6 + \frac{1}{3} \cdot \frac{2}{2} - \frac{2}{5} \cdot 5\sqrt[3]{3} = 4,4$$

~84

$$1) \sqrt{47-4\sqrt{33}} + \sqrt{47+4\sqrt{33}} = \sqrt{(2\sqrt{11}-\sqrt{3})^2} + \sqrt{(2\sqrt{11}+\sqrt{3})^2} = 2\sqrt{11} - \sqrt{3} + 2\sqrt{11} + \sqrt{3} = 4\sqrt{11}$$

$$2) \sqrt{31-6\sqrt{26}} - \sqrt{31+6\sqrt{26}} = \sqrt{(3\sqrt{2}-\sqrt{13})^2} - \sqrt{(3\sqrt{2}+\sqrt{13})^2} = 3\sqrt{2} - \sqrt{13} - 3\sqrt{2} - \sqrt{13} = -2\sqrt{13}$$

$$3) \frac{\sqrt[3]{7} - \sqrt[3]{189} + \sqrt[3]{56}}{\sqrt[3]{7}} = \frac{\sqrt[3]{7} - 3\sqrt[3]{7} + 2\sqrt[3]{7}}{\sqrt[3]{7}} = 0$$

$$4) \sqrt[3]{5\sqrt{2}-7} \cdot \sqrt{3+2\sqrt{2}} = \sqrt[3]{(\sqrt{2}-1)^3} \cdot \sqrt{(1+\sqrt{2})^2} = (\sqrt{2}-1)(1+\sqrt{2}) = 2-1=1$$

~85

$$1) \sqrt{a^3} \sqrt[3]{a} \cdot \sqrt[3]{a} \sqrt{a} = \sqrt{a^{\frac{3}{2}}} \cdot \sqrt[3]{a^{\frac{2}{3}}} = a^{\frac{2}{2} + \frac{1}{3}} = a^{\frac{7}{6}}$$

$$2) \sqrt[4]{b^3} \sqrt[3]{b^2} \cdot \sqrt[3]{b^2} \sqrt{b} = \sqrt[4]{b^{\frac{3}{2}}} \cdot \sqrt[3]{b^{\frac{4}{3}}} = b^{\frac{5}{12} + \frac{3}{4}} = \sqrt[12]{b^9}$$

$$3) \sqrt[4]{\frac{a}{8}} \sqrt{\frac{a}{8}} \cdot \sqrt{\frac{a}{8}} \sqrt[4]{\frac{a}{8}} = \sqrt[4]{\left(\frac{a}{8}\right)^{\frac{3}{2}}} \cdot \sqrt{\left(\frac{a}{8}\right)^{\frac{3}{4}}} = \frac{a^{\frac{3}{8} + \frac{3}{8}}}{8} = \frac{a}{8}$$

$$4) \sqrt[3]{a^3} \sqrt[3]{a^3} \sqrt[3]{a} \sqrt[3]{a} = (a^{\frac{1}{3}} \cdot a^{\frac{1}{3}})^{\frac{1}{3}} \cdot a^{\frac{2}{3}} = a^{\frac{1}{9} + \frac{2}{3}} = a^{\frac{7}{9}}$$

~86

$$1) \frac{67^2 - 58^2}{\sqrt{53^2 - 28^2}} = \frac{(67-58)(67+58)}{\sqrt{(53-28)(53+28)}} = \frac{9 \cdot 125}{\sqrt{25 \cdot 81}} = 9$$

$$2) \frac{\sqrt{113^2 - 112^2}}{19^2 - 11^2} = \frac{\sqrt{(113-112)(113+112)}}{(19-11)(19+11)} = \frac{1}{8}$$

$$3) \left(3\sqrt{\frac{2}{3}} - \sqrt{24} + \sqrt{6}\right) \cdot \left(2\sqrt{\frac{2}{3}} + 3\sqrt{\frac{3}{2}}\right) = \left(3\sqrt{\frac{2}{3}} - 2\sqrt{6} + \sqrt{6}\right) \left(2\sqrt{\frac{2}{3}} + 3\sqrt{\frac{3}{2}}\right) = \left(\sqrt{\frac{2}{3}} - \sqrt{6}\right) \left(2\sqrt{\frac{2}{3}} + 3\sqrt{\frac{3}{2}}\right) = 0$$

$$4) (\sqrt[3]{16} - 2\sqrt[3]{2} + 4\sqrt[3]{54}) \cdot (5\sqrt[3]{4} - 3\sqrt[3]{\frac{1}{2}}) = (2\sqrt[3]{2} - 2\sqrt[3]{2} + 12\sqrt[3]{2}) (5 \cdot 2\sqrt[3]{\frac{1}{2}} - 3\sqrt[3]{\frac{1}{2}}) = 12\sqrt[3]{2} \cdot 7\sqrt[3]{\frac{1}{2}} = 84$$

~87

$$1) \sqrt[3]{16+8\sqrt{5}} + \sqrt[3]{16-8\sqrt{5}} = \sqrt[3]{(1+\sqrt{5})^3} + \sqrt[3]{(1-\sqrt{5})^3} = 1 + \sqrt{5} + 1 - \sqrt{5} = 2$$

$$2) \sqrt[3]{26-15\sqrt{3}} + \sqrt[3]{26+15\sqrt{3}} = \sqrt[3]{(2+\sqrt{3})^3} + \sqrt[3]{(2-\sqrt{3})^3} = 2 + \sqrt{3} + 2 - \sqrt{3} = 4$$

$$3) \sqrt[4]{125} \cdot \sqrt[4]{5} : (\sqrt{5} \sqrt{2} \cdot \sqrt{200}) = 5^{\frac{4}{4}} \cdot 5^{\frac{1}{4}} : (\sqrt{5} \cdot \sqrt{2} \cdot \sqrt{200}) = 5 : \sqrt{2} = \frac{5}{\sqrt{2}}$$

4)

~88

$$1) \frac{\sqrt[6]{a^3} \sqrt[6]{a^{-1}}}{\sqrt[3]{a^2}} = \frac{\sqrt[6]{a^{\frac{2}{3}}}}{a^{\frac{2}{3}}} = \frac{a^{\frac{2}{18}}}{a^{\frac{2}{9}}} = a^{\frac{2}{18} - \frac{4}{18}} = \frac{1}{\sqrt[3]{a}}$$

$$2) \frac{\sqrt[4]{x^3} \sqrt[3]{x}}{\sqrt[3]{x}} = \frac{\sqrt[4]{x^{\frac{3}{2}}}}{\sqrt[3]{x}} = \frac{x^{\frac{3}{8}}}{x^{\frac{1}{3}}} = x^{\frac{3}{8} - \frac{1}{3}} = \sqrt[24]{x^5}$$

$$3) \frac{\sqrt[4]{a^{-1} b^2} \sqrt[4]{a b^3}}{\sqrt[3]{a^2 b^{-2}} \sqrt[3]{a^3 b}} = \frac{\sqrt[4]{a^{-\frac{1}{4} + \frac{3}{4}} b^{\frac{5}{2} + \frac{3}{2}}}}{\sqrt[3]{a^{\frac{2}{3} + 1} b^{-\frac{2}{3} + 1}}} = \frac{\sqrt[4]{a^{\frac{1}{2}} b^4}}{\sqrt[3]{a^{\frac{5}{3}} b^{\frac{1}{3}}}} = a^{-\frac{1}{8} - \frac{11}{24}} b^{\frac{5}{8} + \frac{7}{24}} = a^{-\frac{25}{24}} b^{\frac{29}{24}} = \frac{b^{\frac{29}{24}}}{a^{\frac{25}{24}}}$$

$$4) \frac{\sqrt[5]{x^{-2} y} \sqrt[5]{x y^{-1}}}{\sqrt[5]{x y^{-1}} \sqrt[5]{x^2 y^{-2}}} = \frac{\sqrt[5]{x^{-\frac{1}{5}} y^{\frac{4}{5}}}}{\sqrt[5]{x^{\frac{3}{5}} y^{-\frac{1}{5}}}} = \frac{x^{-\frac{1}{10}} y^{\frac{4}{10}}}{x^{\frac{3}{10}} y^{-\frac{1}{10}}} = x^{-\frac{4}{10}} y^{\frac{5}{10}} = \frac{y}{x}$$

~89

$$1) \frac{\sqrt[10]{27^9} \cdot \sqrt[5]{9}}{\sqrt[3]{3} \cdot \sqrt[5]{3^4} \cdot \sqrt[2]{27}} = \frac{3^{\frac{12}{10}} \cdot 3^{\frac{2}{5}}}{3^{\frac{1}{3}} \cdot 3^{\frac{4}{5}} \cdot (3^{\frac{3}{2}})^{\frac{1}{2}}} = \frac{3^{\frac{12}{10} + \frac{2}{5}}}{3^{\frac{1}{3} + \frac{4}{5} + \frac{3}{2}}} = \frac{3^{\frac{16}{10}}}{3^{\frac{16}{10}}} = 1$$

$$2) \frac{\sqrt[5]{80}}{\sqrt[3]{20} \cdot \sqrt[4]{50}} + \frac{2}{5} \sqrt[5]{\frac{64}{125}} = \frac{\sqrt[5]{2^4 \cdot 5}}{\sqrt[3]{2^2 \cdot 5} \cdot \sqrt[4]{2 \cdot 5^2}} + \frac{2}{5} \cdot \frac{4}{5} = \frac{2^{\frac{4}{5}} \cdot 5^{\frac{1}{5}}}{2^{\frac{2}{3} + \frac{1}{2}} \cdot 5^{\frac{1}{3} + \frac{1}{2}}} + \frac{8}{25} = \frac{2^{\frac{4}{5}} \cdot 5^{\frac{1}{5}}}{2^{\frac{7}{6}} \cdot 5^{\frac{5}{6}}} + \frac{8}{25} = \frac{2^{\frac{4}{5} - \frac{7}{6}} \cdot 5^{\frac{1}{5} - \frac{5}{6}}}{1} + \frac{8}{25} = 2^{-\frac{1}{30}} \cdot 5^{-\frac{1}{6}} + \frac{8}{25} = 1 + 0,5 = 1,5$$

30)  $11^{\frac{2}{3}} = \sqrt[3]{121}$

2)  $0,7^{\frac{5}{4}} = \sqrt[4]{\frac{10^5}{7^5}} = \frac{10}{7}$

3)  $\left(\frac{3}{10}\right)^{0,75} = \sqrt[4]{\frac{27}{1000}}$

4)  $(-21)^{\frac{1}{5}} = -\sqrt[5]{21} = -21^{\frac{1}{5}}$

5)  $a^{2,5} = \frac{1}{a^2} \cdot \sqrt{a}$

6)  $(b+1)^{1,5} = (b+1) \sqrt{b+1}$

7)  $(a-2b)^{\frac{3}{2}} = \frac{(a-2b)^3}{\sqrt{a-2b}}$

8)  $(x-y)^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{x-y}}$

~ 91

1)  $8^{\frac{1}{3}} = \sqrt[3]{2^3} = 2$

2)  $16^{\frac{3}{4}} = \sqrt[4]{4096} = 8$

3)  $64^{\frac{1}{2}} = \frac{1}{8}$

4)  $0,25^{\frac{1}{2}} = \frac{1}{0,5} = 2$

5)  $0,36^{\frac{1}{2}} = \sqrt{0,6^2} = 0,6$

6)  $(-27)^{\frac{1}{3}} = 27 \cdot 3 = 81$

7)  $\left(\frac{2}{4}\right)^{\frac{1}{2}} = \frac{3}{2}$

8)  $32^{\frac{1}{5}} = \frac{1}{2}$

~ 92

1)  $\sqrt[3]{a^2} = a^{\frac{2}{3}}$

2)  $\sqrt[3]{b^3} = b^{\frac{3}{3}}$

3)  $\sqrt[3]{a^2+b^2} = (a^2+b^2)^{\frac{1}{3}}$

4)  $\sqrt[3]{x-y} = (x-y)^{\frac{1}{3}}$

5)  $\sqrt[5]{a^2 b^3} = a^{\frac{2}{5}} b^{\frac{3}{5}}$

6)  $\frac{1}{\sqrt{a}} = a^{-\frac{1}{2}}$

7)  $\frac{1}{\sqrt{a+b}} = (a+b)^{-\frac{1}{2}}$

8)  $\sqrt[3]{a-b} = 2(a-b)^{-\frac{1}{3}}$

~ 93

1)  $4^{\frac{1}{2}} \cdot 16^{\frac{3}{4}} \cdot 16^{\frac{3}{4}} \cdot 32^{\frac{1}{5}} \cdot 2^3 = 2 \cdot \frac{1}{(2^5)^4} \cdot 8 = 1$

2)  $24^{\frac{1}{3}} \cdot 81^{\frac{1}{4}} \cdot \left(\frac{27}{8}\right)^{-\frac{1}{3}} \cdot \left(\frac{2}{3}\right)^3 = \sqrt[4]{(3^4)^3} \cdot \frac{2}{3} \cdot \frac{8}{27} = 16$

3)  $64^{\frac{2}{3}} : 64^{\frac{1}{2}} = \sqrt[3]{(4^3)^2} : 8 = 4^2 : 8 = 16 : 8 = 2$

4)  $429^{\frac{1}{2}} : 429^{\frac{1}{2}} = 27 : 9 = 3$

~ 94

1)  $a^{\frac{13}{4}} : a^{\frac{2}{3}} = a^{\frac{39}{12} - \frac{8}{12}} = a^{\frac{31}{12}} = a^{\frac{31}{12}} \sqrt[12]{a}$

2)  $(x+y)^{\frac{4}{5}} : (x+y)^{\frac{2}{5}} = (x+y)^{\frac{2}{5}}$

3)  $a^{\frac{2}{3}} \cdot x^{\frac{3}{5}} \cdot a^{\frac{3}{4}} \cdot x^{\frac{2}{3}} = a^{\frac{2}{3} + \frac{3}{4}} \cdot x^{\frac{3}{5} + \frac{2}{3}} = a^{\frac{15}{12} + \frac{19}{12}} \cdot x^{\frac{19}{15}}$

4)  $b^{\frac{4}{12}} \cdot y^{\frac{5}{6}} \cdot b^{\frac{2}{3}} \cdot y^{\frac{3}{4}} = b^{\frac{4}{12} + \frac{2}{3}} \cdot y^{\frac{5}{6} + \frac{3}{4}} = b^{\frac{15}{12}} \cdot y^{\frac{19}{12}}$

~ 95

1)  $4^{1,5} - 9^{-0,5} + \left(\frac{1}{64}\right)^{\frac{2}{3}} = 4 \cdot 2 - \frac{1}{3} + 4^2 = 23 \frac{2}{3}$

2)  $8^{\frac{2}{3}} - \left(\frac{1}{16}\right)^{-0,75} + \left(\frac{1}{4}\right)^{1,5} = 4 - 8 + \frac{1}{4} \cdot \frac{1}{2} = -4 + \frac{1}{8} = -\frac{31}{8}$

3)  $(25^{-\frac{1}{3}} - 36^{\frac{1}{2}}) \cdot (16^{\frac{1}{4}} + 216^{\frac{1}{3}})^0 = \frac{1}{5} - \frac{5}{6} = -\frac{29}{30}$

4)  $\left(\frac{2}{5}\right)^{-3} \cdot \left(\frac{6}{4}\right)^{-\frac{3}{2}} = \frac{125}{8} \cdot \frac{4}{25} \cdot \frac{2}{8} = 1$

~ 96

1)  $(a^{\frac{3}{4}})^{\frac{5}{6}} = a^{\frac{3}{4} \cdot \frac{5}{6}} = a^{\frac{5}{8}} = \sqrt[8]{a^5}$

2)  $(a^{\frac{5}{8}})^{\frac{3}{10}} = a^{\frac{5}{8} \cdot \frac{3}{10}} = a^{\frac{1}{4}} = \sqrt[4]{a}$

3)  $((a+x)^{\frac{2}{5}})^{\frac{1}{4}} = (a+x)^{\frac{2}{5} \cdot \frac{1}{4}} = \sqrt[20]{a+x}$

4)  $\left(\frac{a-b}{a+b}\right)^{-\frac{2}{3}} = \left(\frac{a-b}{a+b}\right)^{\frac{2}{3} \cdot \frac{3}{2}} = \frac{a+b}{a-b}$

~ 97

1)  $(49^{\frac{2}{3}})^{\frac{3}{4}} = 49^{\frac{2}{3} \cdot \frac{3}{4}} = \sqrt{49} = 7$

2)  $(625^{\frac{3}{8}})^{\frac{2}{3}} = 625^{\frac{3}{8} \cdot \frac{2}{3}} = \frac{1}{5}$

3)  $(64^{\frac{1}{6}})^{-\frac{2}{3}} = 64^{\frac{1}{6} \cdot \frac{2}{3}} = \frac{1}{2}$

4)  $\left(\left(\frac{4}{25}\right)^{\frac{2}{3}}\right)^{\frac{3}{4}} = \left(\frac{4}{25}\right)^{\frac{2}{3} \cdot \frac{3}{4}} = \sqrt{\frac{4}{25}} = \frac{2}{5}$

5)  $\left(\left(\frac{3}{8}\right)^{-\frac{5}{2}}\right)^{\frac{2}{15}} = \left(\frac{27}{8}\right)^{-\frac{5}{2} \cdot \frac{2}{15}} = \frac{2}{3}$

6)  $\left(\left(\frac{36}{25}\right)^{-\frac{2}{3}}\right)^{\frac{3}{4}} = \left(\frac{81}{25}\right)^{-\frac{2}{3} \cdot \frac{3}{4}} = \frac{5}{9}$

~ 98

1)  $12^{\frac{3}{4}} > 12^{\frac{3}{2}}$

2)  $8^{\frac{3}{2}} > 8^{\frac{4}{3}}$

3)  $\left(\frac{1}{18}\right)^{\frac{5}{4}} > \left(\frac{1}{18}\right)^{\frac{6}{5}}$

4)  $\left(\frac{1}{5}\right)^{1,5} < \left(\frac{1}{5}\right)^{\frac{5}{3}}$

~ 99

1)  $\left(\left(\frac{1}{2}\right)^{\sqrt{2}}\right)^{-\sqrt{8}} = \left(\frac{1}{2}\right)^{\sqrt{2} \cdot (-\sqrt{8})} = \left(\frac{1}{2}\right)^{-\sqrt{16}} = \left(\frac{1}{2}\right)^{-4} = 16$

2)  $\left(\left(\sqrt[3]{6}\right)^{\sqrt{3}}\right)^{-3\sqrt{3}} = \sqrt[3]{6}^{-3 \cdot 3} = 6^{\frac{1}{3} \cdot (-9)} = \frac{1}{216}$

$$8^{\frac{1}{3}} - \left(\frac{1}{16}\right)^{-0,75} + \left(\frac{1}{9}\right)^{1,5} = 4 - 16 + \frac{1}{27} = -12 \frac{1}{27}$$

$$4) \left(64^{-\frac{1}{2}} + \frac{3}{8}\right)^0 \cdot \left(343^{\frac{1}{3}} - 81^{\frac{1}{2}}\right) = 7 - 9 = -2$$

~ 100

$$1) -0,027^{-\frac{1}{3}} + \left(\frac{1}{6}\right)^{-1} - 3^{-1} + (5,5)^0 = -0,3^{-1} + 6 - \frac{1}{3} + 1 = -\frac{10}{3} - \frac{1}{3} + \frac{7}{3} = \frac{-11+7}{3} = -\frac{4}{3}$$

$$2) \left(\left(\frac{3}{7}\right)^{0,5}\right)^{-0,5} - 7,5 - (\sqrt{4^3})^2 - 2 \cdot (-2)^4 = 1 - 7,5 - 4^{\frac{2}{3} \cdot 2} - 32 = 38,5 - 4 \cdot \sqrt{4^2} = 38,5 - 4 \cdot 2 = 46,5$$

$$3) (0,008)^{\frac{2}{3}} \cdot (0,64)^{0,5} : (0,04)^{-0,5} : (0,25)^{-1,5} = (\sqrt[3]{0,008})^2 \cdot 0,8 : 0,2^{-1} : (\sqrt{0,25})^{-3} = 0,01 \cdot 0,8 : 5 : 8 = 0,0008$$

$$4) 0,125^{-\frac{1}{3}} - \left(-\frac{1}{6}\right)^{-2} + 256^{0,75} + (1,2)^0 \cdot 0,5^{-1} - 36 + (\sqrt{256})^3 + 1 = 2 - 36 + 4^3 + 1 = 64 - 33 = 31$$

~ 101

$$1) \frac{a^{\frac{1}{2}} \sqrt{a^3}}{a^{\frac{1}{4}} a^{\frac{3}{8}}} = a^{-\frac{1}{2} + \frac{3}{2} - (-\frac{1}{4}) - \frac{3}{8}} = \sqrt{a}$$

$$2) \frac{x^{\frac{1}{3}} \sqrt[3]{x}}{x^{-\frac{1}{3}}} = x^{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = a^{\frac{10}{6}} = a^{\frac{5}{3}} \sqrt{a}$$

$$3) \frac{a - 16a^{0,5}}{5a^{0,25} + 20} = \frac{a^{0,5}(a^{0,25} - 4)(a^{0,25} + 4)}{5(a^{0,25} + 4)} = \frac{a^{0,5}(a^{0,25} - 4)}{5}$$

$$4) \frac{x^{\frac{4}{3}}y + xy^{\frac{4}{3}}}{\sqrt[3]{x} + \sqrt[3]{y}} = \frac{xy(\sqrt[3]{x} + \sqrt[3]{y})}{(\sqrt[3]{x} + \sqrt[3]{y})} = xy$$

~ 102

$$1) \left(\frac{1}{2}\right)^{12} \cdot 4^{\frac{\sqrt{3}}{2}} \cdot \left(\frac{1}{8}\right)^{\sqrt{27}} \cdot 16^3 = 2^{-12} \cdot 2^{\sqrt{3}} \cdot 2^{-3 \cdot 3\sqrt{3}} \cdot 2^{4 \cdot 3} = 2^{-12 + \sqrt{3} - 9\sqrt{3} + 12} = 2^{-8\sqrt{3}} = (4^{\sqrt{3}})^{-4}$$

$$2) \frac{12^{\sqrt{18}}}{4^{\sqrt{108}}} \cdot \frac{2^{27\sqrt{3}}}{6^{\sqrt{24}}} = \frac{12^{4\sqrt{3}} \cdot 2^{27\sqrt{3}}}{4^{6\sqrt{3}} \cdot 6^{3\sqrt{3}}} = \left(\frac{2^4 \cdot 6^4 \cdot 2^{27}}{2^{12} \cdot 6^3}\right)^{\sqrt{3}} = \left(\frac{6^4 \cdot 2^{31}}{6^3 \cdot 2^{12}}\right)^{\sqrt{3}} = (6 \cdot 2^{12})^{\sqrt{3}}$$

~ 103

$$1) \left(1 \frac{61}{64}\right)^{\frac{2}{3}} + 198^0 - \left(9^{-0,4} \cdot 5^{\frac{1}{2}} \cdot 3^{\frac{4}{5}}\right)^{-2} + (0,0081)^{\frac{1}{4}} = \sqrt[3]{\frac{125}{64}}^{-2} + 1 - \left(3^{-0,8+0,8} \cdot \sqrt{5}\right)^{-2} + \frac{3}{10} = \frac{2}{25} + \frac{5}{10} - \frac{10}{5} = 1,74$$

$$2) \left(-3 \frac{3}{8}\right)^{-\frac{2}{3}} + 27^{-\frac{2}{3}} \left(9^{0,5}\right)^5 \cdot 3^{-2} + \left(\left(\frac{4}{9}\right)^3\right)^0 - \left(-\frac{1}{2}\right)^{-2} = \sqrt[3]{\frac{27}{8}}^{-2} + 3^{-2+5 \cdot 2} + 1 - 4 = \frac{4}{9} + 3 - 3 = \frac{4}{9}$$

$$3) \left(\frac{9}{16}\right)^{-\frac{1}{10}} \cdot \left(\frac{25}{36}\right)^{-\frac{3}{2}} - \left(\left(\frac{4}{3}\right)^{-\frac{1}{2}}\right)^{-\frac{2}{5}} \cdot \left(\frac{6}{5}\right)^{-3} = \left(\frac{3}{4}\right)^{-\frac{1}{5}} \cdot \left(\frac{5}{6}\right)^{-3} - \left(\frac{4}{3}\right)^{\frac{1}{5}} \cdot \left(\frac{5}{6}\right)^3 = \left(\frac{4}{3}\right)^{\frac{1}{5}} \cdot \left(\frac{5}{6}\right)^3 - \left(\frac{4}{3}\right)^{\frac{1}{5}} \cdot \left(\frac{5}{6}\right)^3 = 0$$

$$4) \left(9^{-\frac{2}{3}}\right)^{\frac{3}{4}} - \left(25^{\frac{3}{2}}\right)^{\frac{1}{10}} + \left(\left(\frac{3}{4}\right)^{-1} \cdot \left(\frac{2}{9}\right)^{\frac{6}{4}}\right)^0 : 36^{-\frac{1}{2}} = 9^{-\frac{1}{2}} - 1 \cdot \sqrt{36} = \frac{1}{3} - \frac{3}{6} = -\frac{1}{3}$$

$$5) \left(4^{\frac{1}{4}} + \left(\frac{1}{2^{-\frac{1}{2}}}\right)^{-\frac{4}{3}}\right) \cdot \left(4^{-0,25} - (2\sqrt{2})^{-\frac{4}{3}}\right) = \left(4^{\frac{1}{4}} + 2^{-2}\right) \cdot \left(4^{-\frac{1}{4}} - 2^{-2}\right) = 4^{-\frac{1}{2}} - 2^{-4} = \frac{1}{2} - \frac{1}{16} = \frac{8-1}{16} = \frac{7}{16}$$

$$6) \left(\frac{1}{3} \left(0,027^{\frac{2}{3}} + 15 \cdot 0,0016^{\frac{3}{4}} + 0,1 \cdot 243^{\frac{3}{5}}\right)\right)^{\frac{1}{2}} = \left(\frac{1}{3} (0,09 + 15 \cdot 0,008 + 0,1 \cdot 27)\right)^{\frac{1}{2}} = \frac{\sqrt{97}}{10}$$

~ 104

$$1) \left(\frac{2}{9}\right)^{\sqrt{5}} < \left(\frac{2}{8}\right)^{\sqrt{5}}$$

$$2) \left(\frac{\sqrt{5}}{3}\right)^{\sqrt{3}} > \left(\frac{3\sqrt{5}}{4}\right)^{\sqrt{3}}$$

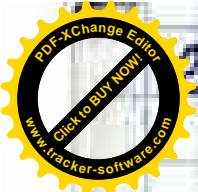
$$3) \left(\frac{9}{5}\right)^{1,2} > \left(\frac{9}{6}\right)^{1,2}$$

$$4) \left(\frac{\sqrt{2}}{2}\right)^{-2,8} < \left(\frac{\sqrt{2}}{3}\right)^{-2,8}$$

~ 105

$$1) \frac{\left(a^{\frac{1}{3}} - b\right)^2 \cdot \left(\frac{b}{a^{\frac{1}{3}}} + \frac{a^{\frac{1}{3}}}{b} + 1\right)}{\frac{b^2}{a^{\frac{1}{3}}} - \frac{b}{a^{\frac{1}{3}}} + \frac{a^{\frac{1}{3}}}{b^2} - \frac{a^{\frac{1}{3}}}{b}} = \frac{\left(a^{\frac{1}{3}} - b\right)^2 \cdot \left(\frac{b^2 + a^{\frac{1}{3}}b + a^{\frac{2}{3}}}{a^{\frac{1}{3}}b}\right)}{\frac{b^3(b - a^{\frac{1}{3}}) + a(a^{\frac{1}{3}} - b)}{a^{\frac{2}{3}}b^2}} = \frac{\left(a^{\frac{1}{3}} - b\right)(a - b)}{a^{\frac{1}{3}}b} \cdot \frac{a^{\frac{2}{3}}b^2}{\left(a^{\frac{1}{3}} - b\right)(a - b)} = a^{\frac{1}{3}}b$$

1.5



$$\begin{aligned}
 & \frac{1}{a^{\frac{1}{4}} + a^{\frac{1}{2}} + 1} + \frac{1}{a^{\frac{1}{4}} - a^{\frac{1}{2}} + 1} - \frac{2\sqrt{a} - 2}{a^{\frac{1}{2}} - a^{\frac{1}{4}} + 1} = \frac{a^{\frac{1}{2}} - 1}{(a^{\frac{1}{4}})^3 - 1} + \frac{a^{\frac{1}{2}} + 1}{(a^{\frac{1}{4}})^3 + 1} - \frac{(a^{\frac{1}{4}} + 1)2(a^{\frac{1}{4}} - 1)}{(a^{\frac{1}{4}})^3 + 1} = \\
 & = \frac{a^{\frac{1}{2}} - 1}{(a^{\frac{1}{4}} - 1)(a^{\frac{1}{4}} + 1)} + \frac{a^{\frac{1}{2}} + 1}{(a^{\frac{1}{4}} - 1)(a^{\frac{1}{4}} + 1)} - \frac{2(a^{\frac{1}{4}} - 1)}{(a^{\frac{1}{4}} - 1)(a^{\frac{1}{4}} + 1)} = \frac{2a^{\frac{1}{2}} - 2}{(a^{\frac{1}{4}} - 1)(a^{\frac{1}{4}} + 1)} = \frac{2(a^{\frac{1}{2}} - 1)}{(a^{\frac{1}{4}} - 1)(a^{\frac{1}{4}} + 1)} = \frac{4}{a + \sqrt{a} + 1}
 \end{aligned}$$

~106

$$\begin{aligned}
 1) \quad & \frac{2a^{-\frac{1}{3}}}{a^{\frac{2}{3}} - 3a^{-\frac{1}{3}}} - \frac{a^{\frac{2}{3}}}{a^{\frac{2}{3}} - a^{\frac{2}{3}}} - \frac{a+1}{a^{\frac{1}{2}} - 4a + 3} = \frac{a^{-1}}{2a^{\frac{1}{3}}} - \frac{a^{-3}}{a^{\frac{2}{3}}} - \frac{1}{a+1} = \\
 & = \frac{2a - 2 - a + 3 - a - 1}{(a-3)(a-1)} = 0
 \end{aligned}$$

$$\begin{aligned}
 2) \quad & \left( \frac{x^{\frac{4}{3}} + 8x^{\frac{1}{3}}y}{x^{\frac{2}{3}} - 2\sqrt[3]{xy} + 4y^{\frac{2}{3}}} - 2\sqrt[3]{xy} \right)^6 = \left( \frac{(x^{\frac{4}{3}} + 8x^{\frac{1}{3}}y)(x^{\frac{1}{3}} + 2y^{\frac{1}{3}})}{(x^{\frac{2}{3}} - 2\sqrt[3]{xy} + 4y^{\frac{2}{3}})(x^{\frac{1}{3}} + 2y^{\frac{1}{3}})} - 2\sqrt[3]{xy} \right)^6 = \\
 & = \left( \frac{x^{\frac{1}{3}}(x+8y)(x^{\frac{1}{3}} + 2y^{\frac{1}{3}})}{(x+8y)} - 2\sqrt[3]{xy} \right)^6 = (x^{\frac{1}{3}} + 2\sqrt[3]{xy} - 2\sqrt[3]{xy})^6 = x^{\frac{2}{3} \cdot 6} = x^4
 \end{aligned}$$

~107

$$\begin{aligned}
 1) \quad & (x^{-2} + a^{\frac{2}{3}}x^{-\frac{4}{3}})^{-\frac{1}{2}} + (a^{-2} + a^{-\frac{4}{3}}x^{-\frac{2}{3}})^{-\frac{1}{2}} = (x^{-2}(1 + a^{\frac{2}{3}}x^{\frac{2}{3}}))^{-\frac{1}{2}} + (a^{-2}(1 + a^{\frac{2}{3}}x^{-\frac{2}{3}}))^{-\frac{1}{2}} = \\
 & = x \left( \frac{x^{\frac{2}{3}} + a^{\frac{2}{3}}}{a^{\frac{2}{3}}} \right)^{-\frac{1}{2}} + a \left( \frac{x^{\frac{2}{3}} + a^{\frac{2}{3}}}{x^{\frac{2}{3}}} \right)^{-\frac{1}{2}} = \frac{xa^{\frac{1}{3}} + x^{\frac{1}{3}}a}{\sqrt{x^{\frac{2}{3}} + a^{\frac{2}{3}}}} = x^{\frac{1}{3}}a^{\frac{1}{3}}(a^{\frac{2}{3}} + x^{\frac{2}{3}})^{\frac{1}{2}} \Rightarrow \text{мын } x = (1 - a^{-\frac{2}{3}})^{\frac{3}{2}} \\
 & = (1 - a^{-\frac{2}{3}})^{-\frac{1}{2}} \cdot a^{\frac{1}{3}} \left( (1 - a^{-\frac{2}{3}})^{-1} + a^{\frac{2}{3}} \right)^{\frac{1}{2}} = \left( \frac{a^{\frac{2}{3}} - 1}{a^{\frac{2}{3}}} \right)^{\frac{1}{2}} \cdot a^{\frac{1}{3}} \cdot \left( \frac{1}{1 - \frac{1}{a^{\frac{2}{3}}}} + a^{\frac{2}{3}} \right)^{\frac{1}{2}} = \frac{(a^{\frac{2}{3}} - 1)^{\frac{1}{2}}}{a^{\frac{1}{3}}} \cdot a^{\frac{1}{3}} \cdot \frac{a^{\frac{2}{3}} + a^{\frac{4}{3}} - a^{\frac{2}{3}}}{(a^{\frac{2}{3}} - 1)^{\frac{1}{2}}} = a^{\frac{2}{3}}
 \end{aligned}$$

$$\begin{aligned}
 2) \quad & \left( \frac{(x^2+1)^{-\frac{1}{2}} + (x^2-1)^{-\frac{1}{2}}}{(x^2+1)^{-\frac{1}{2}} + (x^2-1)^{-\frac{1}{2}}} \right)^{-2} = \left( \frac{\sqrt{x^2-1} + \sqrt{x^2+1}}{\sqrt{x^4-1}} \cdot \frac{\sqrt{x^4-1}}{\sqrt{x^2-1} - \sqrt{x^2+1}} \right)^{-2} = \left( \frac{\sqrt{x^2-1} + \sqrt{x^2+1}}{\sqrt{x^2-1} + \sqrt{x^2+1}} \right)^{-2} = \\
 & = \frac{x^2 - 1 - 2\sqrt{x^4-1} + x^2 + 1}{x^2 - 1 + 2\sqrt{x^4-1} + x^2 + 1} = \frac{2(x^2 - \sqrt{x^4-1})}{2(x^2 + \sqrt{x^4-1})} = \frac{x^2 - \sqrt{x^4-1}}{x^2 + \sqrt{x^4-1}} \Rightarrow \text{мын } x = \left( \frac{m^2 + n^2}{2mn} \right)^{\frac{1}{2}} \\
 & = \left( \frac{(m^2+n^2)^{\frac{1}{2}}}{2mn} \right)^{\frac{1}{2}} \cdot \left( \frac{(m^2+n^2)^{\frac{1}{2}}}{2mn} \right)^{\frac{1}{2}} - 1 = \frac{m^2+n^2}{2mn} - \frac{m^4+2m^2n^2+n^4-4m^2n^2}{4m^2n^2} = \frac{n}{m} = \frac{n^2}{m^2} \\
 & = \frac{(m^2+n^2)^{\frac{1}{2}}}{2mn} + \sqrt{\left( \frac{(m^2+n^2)^{\frac{1}{2}}}{2mn} \right)^2 - 1} = \frac{m^2+n^2}{2mn} + \sqrt{\frac{m^4+2m^2n^2+n^4-4m^2n^2}{4m^2n^2}} = \frac{n}{m} = \frac{n^2}{m^2}
 \end{aligned}$$

~108

$$\begin{aligned}
 1) \quad & \frac{x^{\frac{3}{p}} - x^{\frac{3}{q}}}{(x^{\frac{1}{p}} + x^{\frac{1}{q}})^2 - 2x^{\frac{1}{2}}(x^{\frac{1}{p}} + x^{\frac{1}{q}})} + \frac{x^{\frac{1}{p}}}{x^{\frac{3}{p}} + 1} = \frac{x^{\frac{3}{p}} - x^{\frac{3}{q}}}{x^{\frac{1}{p}} + 2x^{\frac{1}{2}}x^{\frac{1}{p}} + x^{\frac{2}{p}} - 2x^{\frac{1}{2}} - 2x^{\frac{1}{p}}x^{\frac{1}{q}} + x^{\frac{1}{p}}:x^{\frac{1}{q}} + 1} = \\
 & = \frac{x^{\frac{3}{p}} - x^{\frac{3}{q}}}{x^{\frac{1}{p}} - x^{\frac{2}{q}} + x^{\frac{1}{p}}x^{\frac{1}{q}} - (x^{\frac{1}{p}} - x^{\frac{1}{q}})(x^{\frac{2}{p}} + x^{\frac{q+p}{2p}} + x^{\frac{2}{q}})} + \frac{x^{\frac{q+p}{2p}}}{x^{\frac{1}{p}} + x^{\frac{1}{q}}} = \frac{x^{\frac{3}{p}} + 2x^{\frac{q+p}{2p}} + x^{\frac{2}{q}}}{x^{\frac{1}{p}} + x^{\frac{1}{q}}} = \frac{(x^{\frac{1}{p}} + x^{\frac{1}{q}})^2}{(x^{\frac{1}{p}} + x^{\frac{1}{q}})} = x^{\frac{1}{p}} + x^{\frac{1}{q}}
 \end{aligned}$$

$$\begin{aligned}
 2) \quad & \left( \frac{9-4a^{-2}}{3a^{-\frac{1}{2}} + 2a^{-\frac{3}{2}}} - \frac{1+a^{-1}-6a^{-2}}{a^{-\frac{1}{2}} + 3a^{-\frac{3}{2}}} \right) - 16a^2 = \left( \frac{9 - \frac{4}{a^2}}{\frac{3}{a^{\frac{1}{2}}} + \frac{2}{a^{\frac{3}{2}}}} - \frac{1 + \frac{1}{a} - \frac{6}{a^2}}{\frac{1}{a^{\frac{1}{2}}} + \frac{3}{a^{\frac{3}{2}}}} \right) - 16a^2 = \\
 & = \left( \frac{9a^2 - 4}{3a + 2} - \frac{a^2 + a - 6}{a + 3} \right) - 16a^2 = \frac{(3a-2)(3a+2)}{a^2} - \frac{a^3}{(3a+2)} - \frac{(a-2)(a+3)}{a^2} - \frac{a^{\frac{3}{2}}}{(a+3)} - 16a^2 =
 \end{aligned}$$

$$16 - (3a-2-a+2)^4 - 16a^2 - 16a^2 - 16a^2 - 0$$

~ 109

$$1) \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} + \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{10+6}{5-3} = 8$$

$$3) \frac{11+2\sqrt{30}}{11-2\sqrt{30}} - \frac{11-2\sqrt{30}}{11+2\sqrt{30}} = \frac{11+2\sqrt{30}-11+2\sqrt{30}}{121-4\cdot 30} = 4\sqrt{30}$$

$$2) \frac{11+\sqrt{21}}{11-\sqrt{21}} + \frac{11-\sqrt{21}}{11+\sqrt{21}} = \frac{2(121+21)}{121-21} = \frac{71}{25}$$

$$4) \frac{3-2\sqrt{2}}{3+2\sqrt{2}} + \frac{3+2\sqrt{2}}{3-2\sqrt{2}} = \frac{5(3-2\sqrt{2}+3+2\sqrt{2})}{9-4\cdot 2} = 30$$

~ 110

$$1) \sqrt[4]{6+\sqrt{20}} \cdot \sqrt[4]{6-\sqrt{20}} = \sqrt[4]{36-20} = \sqrt[4]{16} = 2$$

$$2) \sqrt[4]{4+\sqrt{15}} \cdot \sqrt[4]{4+\sqrt{15}} = \sqrt[4]{16-15} = 1$$

$$3) (\sqrt{14}-3\sqrt{2})^2 + 6\sqrt{28} = 14 - 6\sqrt{28} + 9\cdot 2 + 6\sqrt{28} = 32$$

$$4) (3\sqrt{5}+\sqrt{15})^2 - 10\sqrt{27} = 9\cdot 5 + 10\sqrt{27} + 15 - 10\sqrt{27} = 60$$

~ 111

$$1) \sqrt{5+\sqrt{24}} = \sqrt{\frac{5+\sqrt{25-24}}{2}} + \sqrt{\frac{5-\sqrt{25-24}}{2}} = \sqrt{3} + \sqrt{2}$$

$$2) \sqrt{6-\sqrt{20}} = \sqrt{\frac{6+\sqrt{36-20}}{2}} - \sqrt{\frac{6-\sqrt{36-20}}{2}} = \sqrt{5} - 1$$

$$3) \sqrt{7-\sqrt{13}} = \sqrt{\frac{7+\sqrt{49-13}}{2}} - \sqrt{\frac{7-\sqrt{49-13}}{2}} = \frac{\sqrt{13}}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}$$

$$4) \sqrt{8+\sqrt{28}} = \sqrt{\frac{8+\sqrt{64-28}}{2}} + \sqrt{\frac{8+\sqrt{64-28}}{2}} = \sqrt{7} + 1$$

~ 112

$$\left( \frac{\sqrt{m+3}}{\sqrt{m-3}} - \frac{\sqrt{m-3}}{\sqrt{m+3}} \right) : \frac{2m}{m-6m+9} = \frac{(m+3)\sqrt{m-3} - (m-3)\sqrt{m+3}}{m-9} : \frac{2m}{(m-3)^2} = \frac{2\sqrt{m}}{(m+3)(m-3)} - \frac{(m-3)^2}{2m} \cdot \frac{\sqrt{m-3}}{m+3\sqrt{m}}$$

~ 113

$$1) \sqrt{8-2\sqrt{15}} = \sqrt{\frac{8+\sqrt{64-60}}{2}} - \sqrt{\frac{8-\sqrt{64-60}}{2}} = \sqrt{5} - \sqrt{3}$$

$$2) \sqrt{6+3\sqrt{3}} = \sqrt{\frac{6+\sqrt{36-27}}{2}} + \sqrt{\frac{6-\sqrt{36-27}}{2}} = \sqrt{4.5} + \sqrt{1.5}$$

$$3) \sqrt{10-2\sqrt{21}} = \sqrt{\frac{10+\sqrt{100}}{2}} - \sqrt{\frac{10-\sqrt{100}}{2}} = \sqrt{7} - \sqrt{3}$$

$$4) \sqrt{11-2\sqrt{10}} = \sqrt{\frac{11+\sqrt{121}}{2}} - \sqrt{\frac{11-\sqrt{121}}{2}} = \sqrt{10} - 1$$

~ 114

$$1) \sqrt[3]{12-\sqrt{13}} \cdot \sqrt[3]{12-\sqrt{13}} = \sqrt[3]{144-19} = \sqrt[3]{125} = 5$$

$$2) \sqrt[3]{7+\sqrt{17}} \cdot \sqrt[3]{7-\sqrt{17}} = \sqrt[3]{49-17} = \sqrt[3]{32} = 2$$

$$3) \left( 2\sqrt{24} - \frac{1}{2}\sqrt{6} + 4\sqrt{3} \right) : \frac{1}{2}\sqrt{3} = \left( 2\cdot 3\sqrt{3} - \frac{\sqrt{6}}{2} + 4\sqrt{3} \right) : \frac{\sqrt{3}}{2} = \left( 10\sqrt{3} - \frac{\sqrt{6}}{2} \right) \cdot \frac{2}{\sqrt{3}} = 10\sqrt{3} \cdot \frac{2}{\sqrt{3}} - \frac{\sqrt{6}}{2} \cdot \frac{2}{\sqrt{3}} = 20 - \sqrt{2}$$

$$4) \left( 5\sqrt{8} - \frac{1}{3}\sqrt{10} - 2\sqrt{2} \right) : \frac{1}{3}\sqrt{2} = \left( 5\cdot 2\sqrt{2} - \frac{\sqrt{10}}{3} - 2\cdot 3\sqrt{2} \right) : \frac{\sqrt{2}}{3} = 4\sqrt{2} \cdot \frac{3}{\sqrt{2}} - \frac{\sqrt{10}}{3} \cdot \frac{3}{\sqrt{2}} = 12 - \sqrt{5}$$

~ 115

$$\sqrt{a+2\sqrt{a-1}} + \sqrt{a-2\sqrt{a-1}} = \frac{a+\sqrt{a^2-4a+4}}{2} + \frac{a-\sqrt{a^2-4a+4}}{2} + \frac{a+\sqrt{a^2-4a+4}}{2} - \frac{a-\sqrt{a^2-4a+4}}{2} = \frac{a-a+2}{2} + \frac{a+a-2}{2} = 2\sqrt{a-1}$$

~ 116

$$\sqrt{x+2\sqrt{x-1}} - \sqrt{x-2\sqrt{x-1}} = \sqrt{\frac{x+\sqrt{(x-2)^2}}{2}} + \sqrt{\frac{x-\sqrt{(x-2)^2}}{2}} - \sqrt{\frac{x+\sqrt{(x-2)^2}}{2}} + \sqrt{\frac{x-\sqrt{(x-2)^2}}{2}} = 2 \cdot \sqrt{\frac{x-(x-2)}{2}} = 2 \cdot \sqrt{\frac{x-x+2}{2}} = 2$$

1)  $a \geq 2$   $\sqrt{a} + \sqrt{\frac{a^2-4}{a}} + \sqrt{a} - \sqrt{\frac{a^2-4}{a}} = \frac{a + \sqrt{a^2-a^2+4}}{2\sqrt{a}} + \frac{a - \sqrt{a^2-a^2+4}}{2\sqrt{a}} + \frac{a + \sqrt{a^2-a^2+4}}{2\sqrt{a}} - \frac{a - \sqrt{a^2-a^2+4}}{2\sqrt{a}}$   
 $= 2 \cdot \sqrt{\frac{a+2}{2a}} = \sqrt{4 \cdot \frac{a+2}{2a}} = \sqrt{\frac{2a+4}{a}}$

2)  $x > 1$   $\left(\sqrt[3]{(x^2+1)\sqrt{1+\frac{1}{x^2}}} + \sqrt[3]{(x^2-1)\sqrt{1-\frac{1}{x^2}}}\right)^2 = \left(\sqrt[3]{\frac{(x^2+1)^3}{x^2}} + \sqrt[3]{\frac{(x^2-1)^3}{x^2}}\right)^2 = \left(\left(\frac{(x^2+1)^3}{x^2}\right)^{\frac{1}{3}} + \left(\frac{(x^2-1)^3}{x^2}\right)^{\frac{1}{3}}\right)^2$   
 $= \left(\frac{(x^2+1)^{\frac{1}{3}}}{x^{\frac{2}{3}}} + \frac{(x^2-1)^{\frac{1}{3}}}{x^{\frac{2}{3}}}\right)^2 = \frac{(x^2+1) + 2(x^2-1)^{\frac{1}{3}} + (x^2-1)}{x^{\frac{4}{3}}} = \frac{2(x^2 + \sqrt{x^2-1})}{\sqrt[3]{x^2}} = \frac{\sqrt[3]{x^2} \cdot (x^2 + \sqrt{x^2-1})}{2^{-1}}$

~118

$\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}-\sqrt{y}} \cdot \frac{(x+\sqrt{xy^3}-\sqrt{xy}) \cdot \sqrt{x+\sqrt{y}}}{(\sqrt{x+\sqrt{xy}}-\sqrt{xy})} = \frac{(\sqrt{x}-\sqrt{y})(\sqrt{x+\sqrt{y}})}{(\sqrt{x}-\sqrt{y})} = \frac{x+\sqrt{xy^3}-\sqrt{x^3y}-\sqrt{xy} \cdot \sqrt{x+\sqrt{y}}}{\sqrt{x+\sqrt{xy}}-\sqrt{xy}} = \frac{1}{(\sqrt{x}-\sqrt{y})}$   
 $= \sqrt{x} + \sqrt{y} - \frac{x^{\frac{1}{2}}y^{\frac{3}{2}}(y^{\frac{1}{2}}-x^{\frac{1}{2}}) + x^{\frac{1}{2}}(x^{\frac{3}{2}}-y^{\frac{1}{2}}) \cdot (\sqrt{x+\sqrt{y}})}{\sqrt{x}-\sqrt{y}} = \sqrt{x} + \sqrt{y} - \frac{\sqrt{x}(\sqrt{x}-\sqrt{y})(\sqrt{x}-\sqrt{y})}{\sqrt{x}-\sqrt{y}} = \frac{1}{(\sqrt{x}-\sqrt{y})}$   
 $= \sqrt{x} - \sqrt{y} - \sqrt{x} + \sqrt{y} = 2\sqrt{x}$

~119

1)  $\left(\frac{2\sqrt{x}}{x^2}\right)^3 \cdot \left((x\sqrt{x})^{-1}\right)^{\frac{1}{2}} = \frac{x^0}{2x^{\frac{3}{2}}} - x^{\frac{1}{2}} \cdot x^{\frac{1}{4}} = \frac{x^5}{2x^{\frac{3}{2}}} - x^{\frac{3}{4}} = \frac{x^5 - x^{\frac{3}{2}}}{2\sqrt{x}} = \frac{x^5 - x^{\frac{3}{2}}}{2\sqrt{x}}$

2)  $\left(\sqrt[5]{x}\right)^{\frac{6}{5}} - \left(\left(\frac{1}{\sqrt{x}}\right)^{-\frac{4}{5}}\right)^{\frac{3}{5}} = (x^{\frac{1}{5}})^{\frac{6}{5}} - \left(\frac{1}{x^{\frac{1}{2}}}\right)^{\frac{4}{5}} = \frac{1}{x^{\frac{3}{5}}} - \frac{1}{x^{\frac{2}{5}}} = 0$

3)  $\left(\frac{x^2}{\sqrt[5]{x^3}}\right)^{-\frac{4}{5}} - \left(\sqrt[5]{x^2\sqrt{x^3}}\right)^{\frac{6}{5}} = \left(\frac{x^{\frac{2}{5}}}{x^{\frac{3}{5}}}\right)^{-\frac{4}{5}} - \left(\sqrt[5]{x^2 \cdot x^{\frac{3}{2}}}\right)^{\frac{6}{5}} = \frac{x^{\frac{3}{5}}}{x^{\frac{2}{5}}} - \left(x^{\frac{7}{2} \cdot \frac{1}{5}}\right)^{\frac{6}{5}} = x^{\frac{3}{5} - \frac{8}{5}} - x^{\frac{4}{5} \cdot \frac{6}{5}} = \frac{1}{x} - x$

~120

1)  $\sqrt{x} = 3 \Rightarrow (\sqrt{x})^2 = (3)^2$   
 $x = 9$

3)  $\sqrt{x} = 2-x \Rightarrow (\sqrt{x})^2 = (2-x)^2$   
 $x = 4 - 4x + x^2 \Rightarrow x^2 - 5x + 4 = 0$   
 $D = 25 - 4 \cdot 4 = 9$   
 $x_1 = \frac{5-3}{2} = 1$   $x_2 = \frac{5+3}{2} = 4$  *өйткені*  
 $\begin{cases} x \geq 0 \\ 2-x \geq 0 \end{cases} \Rightarrow \begin{cases} x \geq 0 \\ x \leq 2 \end{cases}$  *[0, 2] аралықтағы нәтижелерді қабылд*

2)  $\sqrt{x-3} = 2 \Rightarrow (\sqrt{x-3})^2 = (2)^2$   
 $x-3=4 \Rightarrow x=7$

4)  $(\sqrt{x-2})^2 = \left(\frac{x}{3}\right)^2$   $\begin{cases} x-2 \geq 0 \\ \frac{x}{3} \geq 0 \end{cases} \Rightarrow \begin{cases} x \geq 2 \\ x \geq 0 \end{cases}$   $[2, +\infty)$   
 $x-2 = \frac{x^2}{9} \Rightarrow x^2 - 9x + 18 = 0$   
 $D = 81 - 4 \cdot 18 = 9$   
 $x_1 = \frac{9-3}{2} = 3$   $x_2 = \frac{9+3}{2} = 6$

~121

1)  $\sqrt[3]{x+2} = 3 \Rightarrow (\sqrt[3]{x+2})^3 = (3)^3$   
 $x+2=27 \Rightarrow x=25$

3)  $3 + \sqrt{x+3} = x \Rightarrow (\sqrt{x+3})^2 = (x-3)^2$   
 $x+3 = x^2 - 6x + 9 \Rightarrow x^2 - 7x + 6 = 0$   
 $D = 49 - 24 = 25$   
 $x_1 = \frac{7-5}{2} = 1$   $x_2 = \frac{7+5}{2} = 6$

2)  $\sqrt{x-3} = 2 \Rightarrow (\sqrt{x-3})^4 = (2)^4$   
 $x-3=16 \Rightarrow x=19$

4)  $5 + \sqrt{x+1} = x \Rightarrow (\sqrt{x+1})^2 = (x-5)^2$   
 $x+1 = x^2 - 10x + 25 \Rightarrow x^2 - 11x + 24 = 0$   
 $D = 121 - 96 = 25$   
 $x_1 = \frac{11-5}{2} = 3$   $x_2 = \frac{11+5}{2} = 8$   $\begin{cases} x \geq 3 \\ x \geq 5 \end{cases}$   $[5, +\infty)$

~122

$$1) \quad x - \sqrt{x} - 6 = 0 \quad \sqrt{x} = a \quad a^2 - a - 6 = 0$$

$$D = 1 + 24 = 25$$

$$a_1 = \frac{1-5}{2} = -2; \quad a_2 = \frac{1+5}{2} = 3 \quad \sqrt{x} = 3$$

$$x = 9$$

$$2) \quad x + \sqrt{2x} - 4 = 0 \quad \sqrt{x} = a \quad a^2 + \sqrt{2}a - 4 = 0$$

$$D = 2 + 4 \cdot 4 = 18$$

$$a_1 = \frac{-\sqrt{2} - 3\sqrt{2}}{2} = -\sqrt{2}; \quad a_2 = \frac{-\sqrt{2} + 3\sqrt{2}}{2} = \sqrt{2}$$

$$\sqrt{x} = \sqrt{2} \Rightarrow x = 2$$

$$4) \quad (x^2 - 9)\sqrt{x+5} = 0 \quad x+5 \geq 0 \quad x \geq -5$$

$$x^2 - 9 = 0 \quad \sqrt{x+5} = 0$$

$$x = \pm 3 \quad x = -5$$

~123

$$1) \quad \sqrt{x} + \sqrt{x} - 6 = 0 \quad \sqrt{x} = a \quad x \geq 0$$

$$a^2 + a - 6 = 0$$

$$D = 1 + 24 = 25$$

$$a_1 = \frac{-1-5}{2} = -3; \quad a_2 = \frac{-1+5}{2} = 2 \quad \sqrt{x} = 2$$

$$x = 4$$

$$2) \quad \sqrt[3]{x} + \sqrt[3]{x} - 2 = 0 \quad \sqrt[3]{x} = a \quad x \geq 0$$

$$a^2 + a - 2 = 0$$

$$D = 1 + 8 = 9$$

$$a_1 = \frac{-1-3}{2} = -2; \quad a_2 = \frac{-1+3}{2} = 1 \quad \sqrt[3]{x} = 1$$

$$x = 1$$

$$3) \quad \sqrt{x} - 3\sqrt{x} - 10 = 0 \quad \sqrt{x} = a \quad x \geq 0$$

$$a^2 - 3a - 10 = 0$$

$$D = 9 + 4 \cdot 10 = 49$$

$$a_1 = \frac{3-7}{2} = -2; \quad a_2 = \frac{3+7}{2} = 5 \quad \sqrt{x} = 5$$

$$x = 25$$

$$4) \quad \sqrt[3]{x} - 3\sqrt[3]{x} - 18 = 0 \quad \sqrt[3]{x} = a \quad x \geq 0$$

$$a^2 - 3a - 18 = 0$$

$$D = 9 + 4 \cdot 18 = 81$$

$$a_1 = \frac{3-9}{2} = -3; \quad a_2 = \frac{3+9}{2} = 6 \quad \sqrt[3]{x} = 6$$

$$x = 216$$

~124

$$1) \quad \begin{cases} \sqrt{x} + \sqrt{y} = 3 \\ \sqrt{x} - \sqrt{y} = 1 \end{cases} \Rightarrow \begin{cases} \sqrt{x} = 3 - \sqrt{y} \\ 3 - \sqrt{y} - \sqrt{y} = 1 \end{cases} \Rightarrow \begin{cases} x = 4 \\ y = 1 \end{cases}$$

$$\sqrt{y} = 1 \Rightarrow y = 1$$

$$2) \quad \begin{cases} \sqrt{x} + \sqrt{y} = 5 \\ x + y = 13 \end{cases} \Rightarrow \begin{cases} x = 25 - 10\sqrt{y} + y \\ 25 - 10\sqrt{y} + y = 13 \end{cases} \Rightarrow$$

$$y - 5\sqrt{y} - 6 = 0$$

$$D = 25 + 24 = 49$$

$$\sqrt{y}_1 = \frac{5-1}{2} = 2 \Rightarrow y_1 = 4 \quad x_1 = 9$$

$$\sqrt{y}_2 = \frac{5+1}{2} = 3 \Rightarrow y_2 = 9 \quad x_2 = 4$$

~125

$$1) \quad \sqrt{x^2 + 5x + 1} + 1 = 2x$$

$$(\sqrt{x^2 + 5x + 1})^2 = (2x - 1)^2$$

$$x^2 + 5x + 1 = 4x^2 - 4x + 1$$

$$3x^2 - 9x = 0 \Rightarrow 3x(x - 3) = 0$$

$$3x = 0 \quad x - 3 = 0$$

$$x_1 = 0 \quad x_2 = 3$$

$$2) \quad (\sqrt{x+2})^2 = (2 + \sqrt{x-6})^2 \quad \begin{cases} x+2 \geq 0 \\ x-6 \geq 0 \end{cases} \Rightarrow \begin{cases} x \geq -2 \\ x \geq 6 \end{cases}$$

$$x+2 = 4 + 4\sqrt{x-6} + x-6$$

$$4\sqrt{x-6} = 4$$

$$(\sqrt{x-6})^2 = (1)^2$$

$$x-6 = 1$$

$$x = 7$$

$$3) \quad (\sqrt{3x-2})^2 = (\sqrt{x-2} + 2)^2 \Rightarrow \begin{cases} 3x-2 \geq 0 \\ x-2 \geq 0 \end{cases} \Rightarrow \begin{cases} x \geq \frac{2}{3} \\ x \geq 2 \end{cases}$$

$$3x-2 = x-2 + 4\sqrt{x-2} + 4 \quad \begin{cases} x-2 \geq 0 \\ x \geq 2 \end{cases}$$

$$2\sqrt{x-2} = x-2 \Rightarrow (2)^2 = (\sqrt{x-2})^2$$

$$4 = x-2 \Rightarrow x = 6$$

$$4) \quad \sqrt{22-x} - \sqrt{10-x} = 2 \quad \begin{cases} 22-x \geq 0 \\ 10-x \geq 0 \end{cases} \Rightarrow \begin{cases} x \leq 22 \\ x \leq 10 \end{cases}$$

$$22-x = 4 + 4\sqrt{10-x} + 10-x$$

$$4\sqrt{10-x} = 8$$

$$10-x = 4 \Rightarrow x = 6$$

~126

$$1) \quad \sqrt{x} - \sqrt{x+3} = 1 \quad \begin{cases} x \geq 0 \\ x+3 \geq 0 \end{cases} \Rightarrow \begin{cases} x \geq 0 \\ x \geq -3 \end{cases}$$

$$(\sqrt{x})^2 = (\sqrt{x+3} + 1)^2$$

$$x = x+3 + 2\sqrt{x+3} + 1$$

$$2\sqrt{x+3} = -2$$

$$\sqrt{x+3} = -1$$

$$x \neq 1 \quad \text{⊗}$$

$$2) \quad \sqrt{x-5} + \sqrt{10-x} = 3 \quad \begin{cases} x-5 \geq 0 \\ 10-x \geq 0 \end{cases} \Rightarrow \begin{cases} x \geq 5 \\ x \leq 10 \end{cases}$$

$$(\sqrt{x-5})^2 = (3 - \sqrt{10-x})^2$$

$$x-5 = 9 - 6\sqrt{10-x} + 10-x$$

$$(3\sqrt{10-x})^2 = (12-x)^2$$

$$x^2 - 15x + 54 = 0$$

$$D = 225 - 4 \cdot 54 = 9 \Rightarrow x_1 = 6 \quad x_2 = 9$$



$$\sqrt{x-9} - \sqrt{x-16} = 1$$

$$(\sqrt{x-9})^2 = (1 + \sqrt{x-16})^2$$

$$x-9 = 1 + 2\sqrt{x-16} + x-16$$

$$(\sqrt{x-16})^2 = (3)^2$$

$$x-16 = 9$$

$$x = 25$$

$$4) \frac{2x-5}{\sqrt{x+2}} = \sqrt{x+2} \quad x \geq -2$$

$$2x-5 = x+2$$

$$x = 7$$

~127

$$1) (\sqrt{16 - \sqrt{x+1}})^2 = (1)^2$$

$$16 - \sqrt{x+1} = 1$$

$$(\sqrt{x+1})^2 = (15)^2$$

$$x = 224$$

$$2) (\sqrt[3]{5 - \sqrt{x+15}})^3 = (1)^3$$

$$5 - \sqrt{x+15} = 1$$

$$(\sqrt{x+15})^2 = (4)^2$$

$$x = 1$$

$$3) \left(\frac{x+3}{\sqrt{x-1}}\right)^2 = (3x+1)^2$$

$$x^2 + 8x + 9 = 3x^2 + x - 3x - 1$$

$$x^2 - 4x - 5 = 0 \Rightarrow x_1 = -1, x_2 = 5$$

$$4) \frac{2x-5}{\sqrt{x+2}} = \sqrt{x+2}$$

$$2x-5 = x+2$$

$$x = 7$$

~128

$$1) \frac{x-4}{\sqrt{x-2}} = x+2 \quad x \neq 4; \quad \frac{(\sqrt{x-2})(\sqrt{x+2})}{(\sqrt{x-2})} = x+2$$

$$x-4 = x\sqrt{x-2} + 2\sqrt{x-2} - 2x - 4$$

$$(3\sqrt{x})^2 = (x+2)^2$$

$$x^2 - 5x + 4 = 0$$

$$\Delta = 9 \Rightarrow x_1 = 4; x_2 = 1$$

$$2) \frac{x-9}{\sqrt{x+3}} = 24-x \quad x \geq 0$$

$$\frac{(\sqrt{x+3})(\sqrt{x-3})}{(\sqrt{x+3})} = 24-x \Rightarrow \sqrt{x} = 30-x$$

$$x^2 - 61x + 900 = 0 \Rightarrow \Delta = 121 \Rightarrow x_1 = \frac{61-11}{2} = 25; x_2 = \frac{61+11}{2} = 38$$

$$3) \frac{x+1}{\sqrt{x-1}} = (2x-1)^{\frac{1}{2}} \quad x > 1$$

$$(x+1)^2 = ((2x-1)^{\frac{1}{2}})^2$$

$$x^2 + 2x + 1 = 2x^2 - 3x + 1$$

$$x^2 - 5x = 0 \quad x(x-5) = 0$$

$$x_1 \neq 0 \quad x_2 = 5$$

$$4) \frac{x+6}{(x-6)^{\frac{1}{2}}} = \sqrt{3x+2} \quad x > 6$$

$$(x+6)^2 = (\sqrt{3x^2+2x-18x-12})^2$$

$$x^2 + 12x + 36 = 3x^2 - 16x - 12$$

$$x^2 - 14x - 24 = 0$$

$$\Delta = 292 \Rightarrow x = 4 + \sqrt{73}$$

~129

$$1) \sqrt{x^2+32} - 2\sqrt{x^2+32} = 3$$

$$(\sqrt{x^2+32} - 1)^2 - 1 = 3$$

$$\sqrt{x^2+32} - 1 = 2$$

$$(\sqrt{x^2+32})^4 = (3)^4$$

$$x^2 + 32 = 81$$

$$x^2 = 49$$

$$x = \pm 7$$

$$2) \sqrt[3]{x^3 - 5\sqrt{x-1}} = 2$$

$$3x^{\frac{1}{3}} - \frac{5}{x^{\frac{1}{3}}} - 2 = 0$$

$$3x^{\frac{2}{3}} - 2x^{\frac{1}{3}} - 5 = 0$$

$$x^{\frac{1}{3}} = t$$

$$3t^2 - 2t - 5 = 0$$

$$\Delta = 64$$

$$t_1 = -1 \Rightarrow x_1 = -1$$

$$t_2 = \frac{5}{3} \Rightarrow x_2 = \frac{125}{27}$$

$$3) \sqrt{3x^2+13} - \sqrt{3x^2+13} = 2$$

$$\sqrt{3x^2+13} = t$$

$$t^2 - t - 2 = 0$$

$$\Delta = 1 + 4 \cdot 2 = 9$$

$$t_1 = \frac{1+3}{2} = 2$$

$$(\sqrt{3x^2+13})^2 = (2)^2 \Rightarrow x = \pm 1$$

$$t_2 = \frac{1-3}{2} = -1 \quad \sqrt{3x^2+13} \neq -1$$

$$4) \sqrt{5+\sqrt{x}} + \sqrt{5-\sqrt{x}} = \sqrt[3]{x}$$

$$\sqrt[3]{x} = a$$

$$(\sqrt{5+a} + \sqrt{5-a})^2 = a^2$$

$$2\sqrt{25-a^2} = a^2 - 10$$

$$16a^2 - a^4 = a$$

$$a_1 = -4 \quad a_2 = 4$$

$$\sqrt[3]{x} = 4$$

$$x = 64$$

~130

$$1) \begin{cases} \sqrt[3]{x} + \sqrt[3]{y} = 4 \\ x + y = 28 \end{cases} \quad \begin{matrix} \sqrt[3]{x} = a \\ \sqrt[3]{y} = b \end{matrix} \Rightarrow \begin{cases} a = 4 - b \\ (4-b)^3 + b^3 = 28 \end{cases}$$

$$64 - 48b + 12b^2 - b^3 + b^3 = 28$$

$$b^2 - 4b + 3 = 0$$

$$\Delta = 16 - 4 \cdot 3 = 4$$

$$b_1 = \frac{4-2}{2} = 1 \Rightarrow \sqrt[3]{y} = 1; a_1 = 3 \Rightarrow \sqrt[3]{x} = 3$$

$$y_1 = 1; x_1 = 27$$

$$b_2 = \frac{4+2}{2} = 3 \Rightarrow \sqrt[3]{y} = 3; a_2 = 1 \Rightarrow \sqrt[3]{x} = 1$$

$$y_2 = 27; x_2 = 1$$

$$2) \begin{cases} x + y = 42 \\ \sqrt[3]{x} + \sqrt[3]{y} = 6 \end{cases} \quad \begin{matrix} \sqrt[3]{x} = a \\ \sqrt[3]{y} = b \end{matrix} \Rightarrow \begin{cases} b = 6 - a \\ a^3 + (6-a)^3 = 42 \end{cases}$$

$$a^3 + 216 - 108a + 18a^2 - a^3 = 42$$

$$a^2 - 6a + 8 = 0$$

$$\Delta = 36 - 32 = 4$$

$$a_1 = \frac{6-2}{2} = 2 \Rightarrow \sqrt[3]{x} = 2; b_1 = 4 \Rightarrow \sqrt[3]{y} = 4$$

$$x_1 = 8; y_1 = 64$$

$$a_2 = \frac{6+2}{2} = 4 \Rightarrow \sqrt[3]{x} = 4; b_2 = 2 \Rightarrow \sqrt[3]{y} = 2$$

$$x_2 = 64; y_2 = 8$$



$$1) \sqrt{x} - \sqrt{y} = 2 \quad \sqrt[3]{x} = a \Rightarrow \begin{cases} a - b = 2 \\ (ab)^3 = 27 \end{cases}$$

$$\begin{cases} a^2 = 2 + b \\ b(2 + b) = 3 \end{cases}$$

$$b^2 + 2b - 3 = 0$$

$$b = 4 + 12 = 16$$

$$b_1 = \frac{-2 - 4}{2} = -3 \quad \sqrt[3]{y} = 3, a = -1 \quad \sqrt[3]{x} = -1$$

$$b_2 = \frac{-2 + 4}{2} = 1 \quad \sqrt[3]{y} = 1, a = 3 \quad \sqrt[3]{x} = 3$$

$$y = 27, x = -1$$

$$y = 1, x = 27$$

$$2) \begin{cases} \sqrt{x} + \sqrt{y} = 10 \\ \sqrt[4]{x} + \sqrt[4]{y} = 4 \end{cases} \quad \sqrt[4]{x} = a \Rightarrow \begin{cases} a^2 + b^2 = 10 \\ a + b = 4 \end{cases}$$

$$b = 4 - a$$

$$a^2 + (4 - a)^2 = 10$$

$$a^2 - 4a + 3 = 0$$

$$a = 16 - 4 \cdot 3 = 4$$

$$a_1 = \frac{4 + 2}{2} = 3 \Rightarrow \sqrt[4]{x} = 3, b_1 = 1 \Rightarrow \sqrt[4]{y} = 1$$

$$x_1 = 81, y_1 = 1$$

$$a_2 = \frac{4 - 2}{2} = 1 \Rightarrow \sqrt[4]{x} = 1, b_2 = 3 \Rightarrow \sqrt[4]{y} = 3$$

$$x_2 = 1, y_2 = 81$$

-132

$$1) \sqrt{x+6} + \sqrt{x+1} = (\sqrt{7x+4})^2$$

$$x+6 + 2\sqrt{x^2+x+6x+6} + x+1 = 7x+4$$

$$(2\sqrt{x^2+x+6x+6})^2 = (5x-3)^2$$

$$4x^2 + 28x + 24 = 25x^2 - 30x + 9$$

$$21x^2 - 58x - 15 = 0$$

$$D = 3364 + 4 \cdot 15 \cdot 21 = 4624$$

$$x_1 = \frac{58 - 68}{2 \cdot 21} = -\frac{5}{21}$$

$$x_2 = \frac{58 + 68}{2 \cdot 21} = 3$$

$$2) \sqrt{x\sqrt{x}} - \sqrt[5]{x\sqrt{x}} = 56 \quad x \geq 0$$

$$(x \cdot x^{\frac{1}{2}})^{\frac{1}{2}} - (x \cdot x^{\frac{1}{2}})^{\frac{1}{5}} = 56$$

$$x^{\frac{3}{4}} - x^{\frac{3}{10}} = 56 \quad x^{\frac{3}{10}} = t$$

$$t^2 - t - 56 = 0$$

$$D = 1 + 224 = 15$$

$$t_1 = \frac{1 - 15}{2} = -7$$

$$t_2 = \frac{1 + 15}{2} = 8 \Rightarrow x^{\frac{3}{10}} = 8$$

$$x = 1024$$

$$3) \sqrt[3]{24 + \sqrt{x}} - \sqrt[3]{5 + \sqrt{x}} = 1 \quad x \geq 0$$

$$(\sqrt[3]{24 + \sqrt{x}})^3 = (\sqrt[3]{5 + \sqrt{x}} + 1)^3$$

$$24 + \sqrt{x} = 5 + \sqrt{x} + 3(\sqrt[3]{5 + \sqrt{x}})^2 + 3\sqrt[3]{5 + \sqrt{x}} + 1$$

$$\sqrt[3]{5 + \sqrt{x}} = a \quad \text{generec} \quad 3a^2 + 3a - 18 = 0$$

$$a^2 + a - 6 = 0$$

$$D = 1 + 24 = 25$$

$$a_1 = \frac{-1 + 5}{2} = 2 \Rightarrow \sqrt[3]{5 + \sqrt{x}} = 2$$

$$5 + \sqrt{x} = 8 \Rightarrow x = 9$$

$$a_2 = \frac{-1 - 5}{2} = -3 \Rightarrow \sqrt[3]{5 + \sqrt{x}} \neq -3$$

$$4) \sqrt[3]{x+2} - \sqrt[3]{x+17} = 1$$

$$(\sqrt[3]{x+2})^3 = (\sqrt[3]{x+17} + 1)^3$$

$$x+2 = x+17 + 1 + 3\sqrt[3]{x+17}(\sqrt[3]{x+17} + 1)$$

$$3\sqrt[3]{x^2+19x+34} = -16$$

$$27(x^2+19x+34) = -4096$$

$$27x^2 + 513x + 5014 = 0$$

$$D < 0$$

$$\emptyset$$

-133

$$1) \sqrt[5]{x-3} + \sqrt[5]{5-x} = 2 \quad \sqrt[5]{x-3} = t$$

$$\sqrt[5]{5-x} = 2 - t$$

$$t + \frac{1}{t} = 2 \Rightarrow t^2 + 1 = 2t$$

$$t^2 - 2t + 1 = 0$$

$$D = 4 - 4 = 0$$

$$t = \frac{2}{2} = 1 \Rightarrow \left(\sqrt[5]{\frac{x-3}{5-x}}\right)^5 = (1)^5$$

$$x-3 = 5-x$$

$$x = 4$$

$$2) \sqrt{x+3-2\sqrt{x+2}} + \sqrt{x+27-10\sqrt{x+2}} = 4$$

$$\sqrt{x+2-2\sqrt{x+2}+1} + \sqrt{x+2-10\sqrt{x+2}+25} = 4$$

$$\sqrt{(\sqrt{x+2}-1)^2} + \sqrt{(\sqrt{x+2}-5)^2} = 4$$

$$2\sqrt{x+2} - 6 = 4$$

$$2\sqrt{x+2} = 10$$

$$(\sqrt{x+2})^2 = 5^2$$

$$x+2 = 25$$

$$x = 25 - 2$$

$$x = 23$$

$$3) \sqrt[5]{5x+2} - \frac{16}{\sqrt[5]{5x+2}} = 6 \quad \sqrt[5]{5x+2} = t \quad x > -0,4$$

$$t - \frac{16}{t} = 6 \Rightarrow t^2 - 16 = 6t$$

$$t^2 - 6t - 16 = 0$$

$$D = 36 + 4 \cdot 16 = 100$$

$$t_1 = \frac{6 - 10}{2} = -2 \quad t_2 = \frac{6 + 10}{2} = 8 \quad \sqrt[5]{5x+2} = 8$$

$$x = 6$$

$$4) \frac{(5-x)^{1,5} + (x-3)^{1,5}}{\sqrt{5-x} + \sqrt{x-3}} = 2$$

$$(5-x)^{1,5} + (x-3)^{1,5} - 2\sqrt{5-x} - 2\sqrt{x-3} = 0$$

$$\sqrt{5-x}(5-x-2) + \sqrt{x-3}(x-3-2) = 0$$

$$-\sqrt{5-x}(x-3) - \sqrt{x-3}(5-x) = 0$$

$$\sqrt{(5-x)(x-3)}(\sqrt{x-3} + \sqrt{5-x}) = 0$$

$$(5-x)(x-3) = 0 \quad \sqrt{x+3} + \sqrt{5-x} \neq 0$$

$$x_1 = 5 \quad x_2 = 3$$

$$1) \sqrt{x-9} + \sqrt{x} = \frac{36}{\sqrt{x-9}}$$

$$x-9 + \sqrt{x^2-9x} = 36$$

$$(\sqrt{x^2-9x})^2 = (45-x)^2$$

$$x^2-9x = 2025-90x+x^2$$

$$81x = 2025$$

$$x = 25$$

$$3) \sqrt{2+x} + \sqrt{x} = \frac{4}{\sqrt{2+x}} \quad x > -2$$

$$2+x + \sqrt{2x+x^2} = 4$$

$$(\sqrt{2x+x^2})^2 = (4-x)^2$$

$$2x+x^2 = 4-4x+x^2$$

$$6x = 4$$

$$x = \frac{2}{3}$$

~135

$$1) \begin{cases} \frac{1}{\sqrt{x}} + \frac{1}{\sqrt{y}} = \frac{4}{3} \\ x \cdot y = 9 \end{cases} \Rightarrow \begin{cases} \frac{\sqrt{y} + \sqrt{x}}{\sqrt{xy}} = \frac{4}{3} \\ \sqrt{xy} = 3 \end{cases}$$

$$\begin{cases} \sqrt{y} = 4 - \sqrt{x} \\ \sqrt{x}(4 - \sqrt{x}) = 3 \end{cases} \Rightarrow \begin{cases} x_1 = 1 \\ y_1 = 9 \end{cases} \text{ н/е } \begin{cases} x_2 = 9 \\ y_2 = 1 \end{cases}$$

$$x - 4\sqrt{x} + 3 = 0$$

$$D = 16 - 12 = 4$$

$$\sqrt{x}_1 = \frac{4-2}{2} = 1$$

$$\sqrt{x}_2 = \frac{4+2}{2} = 3$$

$$x_1 = 1$$

$$x_2 = 9$$

$$y_1 = 9$$

$$y_2 = 1$$

~136

$$1) \begin{cases} \sqrt{\frac{2x-1}{y+2}} + \sqrt{\frac{y+2}{2x-1}} = 2 \\ \frac{2x-1}{y+2} = a \\ xy - x - y = 0 \end{cases}$$

$$a + \frac{1}{a} = 2 \Rightarrow a^2 + 1 = 2a \Rightarrow (a-1)^2 = 0 \Rightarrow a = 1 \text{ генер}$$

$$\sqrt{\frac{2x-1}{y+2}} = 1 \Rightarrow \begin{cases} 2x-1 = y+2 \\ y = 2x-3 \end{cases}$$

$$\begin{cases} y = 2x-3 \\ x + 2x - 3 = 12 \end{cases} \Rightarrow \begin{cases} y = 7 \\ x = 5 \end{cases}$$

$$2) \sqrt{9-5x} = \sqrt{3-x} + \frac{6}{\sqrt{3-x}} \quad x \leq 1,8$$

$$(\sqrt{27-15x-9x+5x^2})^2 = (3-x+6)^2$$

$$27-24x+5x^2-81+10x-x^2 = 0$$

$$2x^2-3x-24=0$$

$$D = 9-216 = -207$$

$$x_1 = \frac{3-15}{4} = -3 \quad x_2 = \frac{3+15}{4} = 4,5$$

$$4) \sqrt{4x+20} = \frac{4-\sqrt{x}}{\sqrt{x}}$$

$$\frac{4+\sqrt{x}}{\sqrt{x}} = \frac{4-\sqrt{x}}{\sqrt{x}}$$

$$(\sqrt{4x^2+20x})^2 = (4-\sqrt{x})^2$$

$$4x^2+20x = 256-32\sqrt{x}+x^2$$

$$3x^2+52x-256=0$$

$$D = 5476$$

$$x_1 = \frac{-52+46}{2 \cdot 3} = -1; \quad x_2 = \frac{-52-46}{2 \cdot 3} = -16$$

$$2) \begin{cases} \sqrt{x} + \sqrt{y} = \frac{5}{2} \\ x+y = 10 \\ a^2 + b^2 = 10 \\ a^2 + b^2 = \frac{5}{2} ab \end{cases} \quad \begin{cases} \sqrt{x} = a \\ \sqrt{y} = b \end{cases} \Rightarrow \begin{cases} \frac{a}{b} + \frac{b}{a} = \frac{5}{2} \\ a^2 + b^2 = 10 \\ a^2 + b^2 = \frac{5}{2} ab \end{cases}$$

$$16 + a^2 = 10$$

$$a^2 - 10a^2 + 16 = 0$$

$$D = 100 - 4 \cdot 16 = 36$$

$$a_1 = \frac{10-6}{2} = 2 \quad a_2 = \sqrt{2}$$

$$a_2 = \frac{10+6}{2} = 8 \quad a_2 = \sqrt{8}$$

буған

$$\begin{cases} a_1 = \sqrt{2} \\ b_1 = \sqrt{8} \end{cases} \Rightarrow \begin{cases} x_1 = 2 \\ y_1 = 8 \end{cases}; \text{ н/е } \begin{cases} a_2 = \sqrt{8} \\ b_2 = \sqrt{2} \end{cases} \Rightarrow \begin{cases} x_2 = 8 \\ y_2 = 2 \end{cases}$$

$$2) \begin{cases} \sqrt{\frac{6x}{x+y}} + \sqrt{\frac{x+y}{6x}} = \frac{5}{2} \\ \frac{6x}{x+y} = a \\ xy - x - y = 0 \end{cases}$$

$$a + \frac{1}{a} = \frac{5}{2} \Rightarrow 2a^2 + 2 = 5a$$

$$2a^2 - 5a + 2 = 0$$

$$D = 25 - 16 = 9$$

$$a_1 = \frac{5-3}{2 \cdot 2} = \frac{1}{2}; \quad a_2 = \frac{5+3}{2 \cdot 2} = 2$$

$$\left(\frac{6x}{x+y}\right)^2 = \left(\frac{1}{2}\right)^2 \Rightarrow \begin{cases} 24x = x+y \\ y = 23x \end{cases} \text{ генер } \begin{cases} y = 23x \\ xy - x - y = 0 \end{cases}$$

$$\begin{cases} y = 23x \\ 23x^2 - x - 23x = 0 \end{cases} \Rightarrow \begin{cases} y = 24 \\ x = \frac{24}{23} \end{cases}$$

$$\left(\frac{6x}{x+y}\right)^2 = (2)^2 \Rightarrow \begin{cases} 6x = 4x + 4y \\ x = 2y \end{cases} \text{ генер } \begin{cases} x = 2y \\ xy - x - y = 0 \end{cases}$$

$$\begin{cases} x = 2y \\ 2y^2 - 2y - y = 0 \end{cases} \Rightarrow \begin{cases} x = 3 \\ y = 1,5 \end{cases}$$

~150

1)  $f(x) = x^5$   
 $x \in \mathbb{R}$

2)  $f(x) = x^{-7}$   
 $x \in (-\infty; 0) \cup (0; +\infty)$

3)  $f(x) = x^{\frac{1}{5}}$   
 $x \in \mathbb{R}$

4)  $f(x) = x^{\frac{9}{10}}$   
 $x \in [0; +\infty)$

5)  $f(x) = x^{\frac{4}{7}}$   
 $x \in \mathbb{R}$

6)  $f(x) = x^{\frac{11}{13}}$   
 $x \in \mathbb{R}$

7)  $f(x) = x^{-\frac{3}{4}}$   
 $x \in (0; +\infty)$

8)  $f(x) = x^{-\frac{2}{3}}$   
 $x \in (-\infty; 0) \cup (0; +\infty)$

9)  $f(x) = x^{-\frac{5}{7}}$   
 $x \in (-\infty; 0) \cup (0; +\infty)$

~151

1)  $f(x) = x^{11}$   
 $(-\infty; 0)$  аралықта  $f(x) < 0$   
 $(0; +\infty)$  аралықта  $f(x) > 0$

2)  $f(x) = x^{\frac{1}{5}}$   
 $(-\infty; 0)$  аралықта  $f(x) < 0$   
 $(0; +\infty)$  аралықта  $f(x) > 0$

3)  $f(x) = x^{-8}$   
 $(-\infty; 0) \cup (0; +\infty)$  аралықта  $f(x) > 0$

4)  $f(x) = x^{\frac{11}{12}}$   
 $(0; +\infty)$  аралықта  $f(x) > 0$

5)  $f(x) = x^{\frac{12}{13}}$   
 $(-\infty; +\infty)$  аралықта  $f(x) > 0$

6)  $f(x) = x^{\frac{15}{17}}$   
 $(-\infty; 0)$  ар  $f(x) < 0$ ;  $(0; +\infty)$  ар  $f(x) > 0$

7)  $f(x) = x^{-\frac{7}{10}}$   
 $(0; +\infty)$  аралықта  $f(x) > 0$

8)  $f(x) = x^{-\frac{8}{13}}$   
 $(-\infty; 0) \cup (0; +\infty)$  аралықта  $f(x) > 0$

9)  $f(x) = x^{-\frac{11}{13}}$   
 $(-\infty; 0)$  аралықта  $f(x) < 0$   
 $(0; +\infty)$  аралықта  $f(x) > 0$

~152

1)  $f(x) = x^3$  таңу

2)  $f(x) = x^{-4}$  жұп

3)  $f(x) = x^{\frac{1}{7}}$  таңу

4)  $f(x) = (1+x)^{\frac{4}{5}}$  жстф

5)  $f(x) = x^{\frac{5}{8}} + 2$  жстф

6)  $f(x) = x^{\frac{6}{5}} - 1$  жұп

7)  $f(x) = (3-x)^{\frac{5}{6}}$  жстф

8)  $f(x) = 1 - x^{-\frac{4}{7}}$  жұп

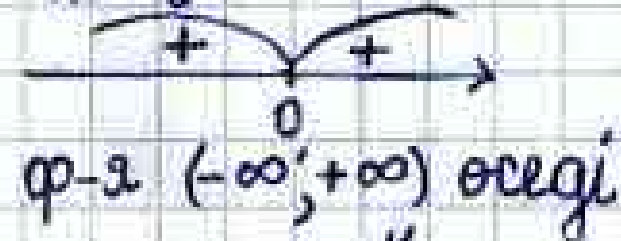
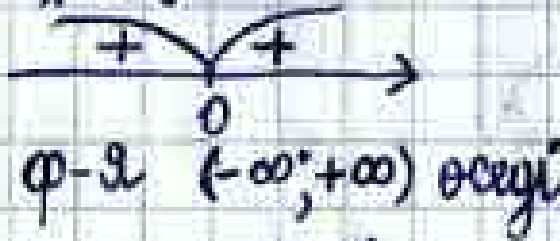
9)  $f(x) = (x+2)^{-\frac{3}{5}}$  жстф

~153

1)  $f(x) = 1 + x^7$   
 $f'(x) = 7x^6 = 0$   
 $x = 0$

2)  $f(x) = 2 - x^{-10}$   
 $f'(x) = +10x^{-11} = 0$

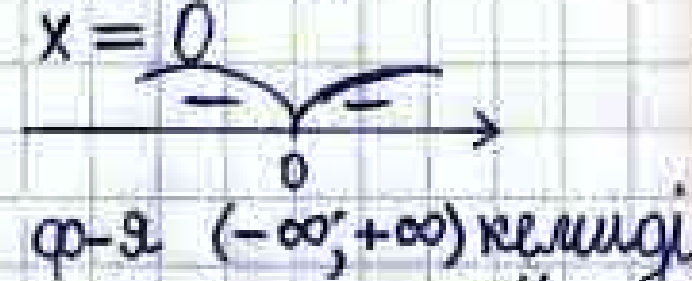
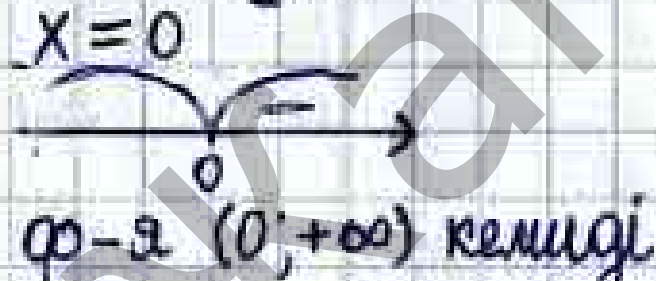
3)  $f(x) = 3 + x^{\frac{1}{9}}$   
 $f'(x) = \frac{1}{9}x^{-\frac{8}{9}} = 0$   
 $x = 0$



4)  $f(x) = 4 - x^{\frac{11}{16}}$   
 $f'(x) = -\frac{11}{16}x^{-\frac{5}{16}} = 0$   $x \geq 0$   
 $x = 0$

5)  $f(x) = x - 5^{\frac{13}{15}}$   
 $f'(x) = 1$   
φ-я өспеді  
 $(-\infty; +\infty)$

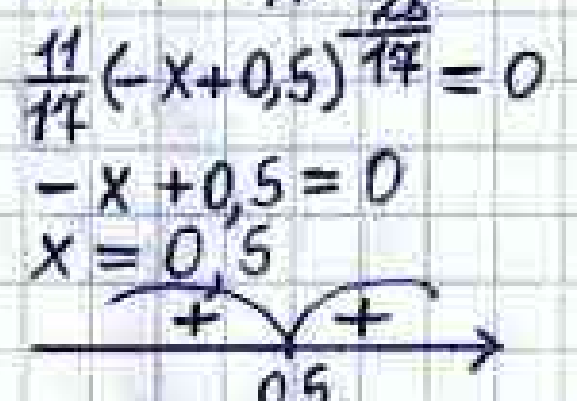
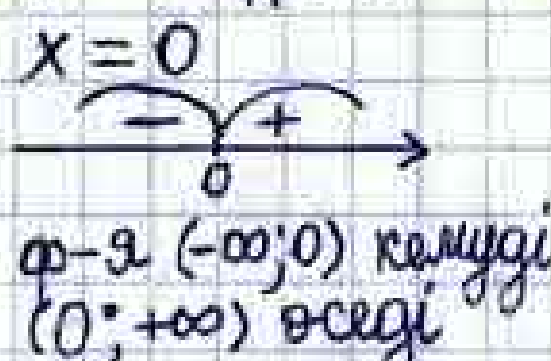
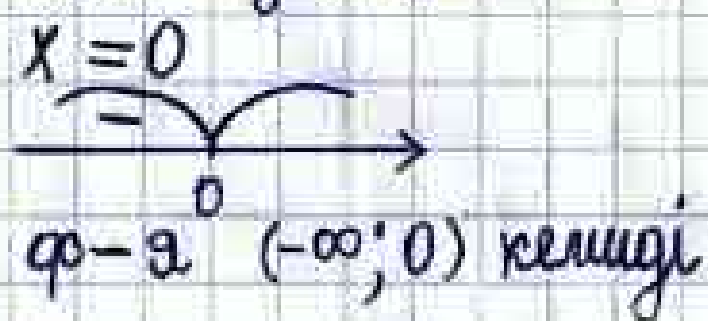
6)  $f(x) = (-x)^{\frac{11}{13}}$   
 $f'(x) = -\frac{11}{13}x^{-\frac{2}{13}} = 0$   
 $x = 0$



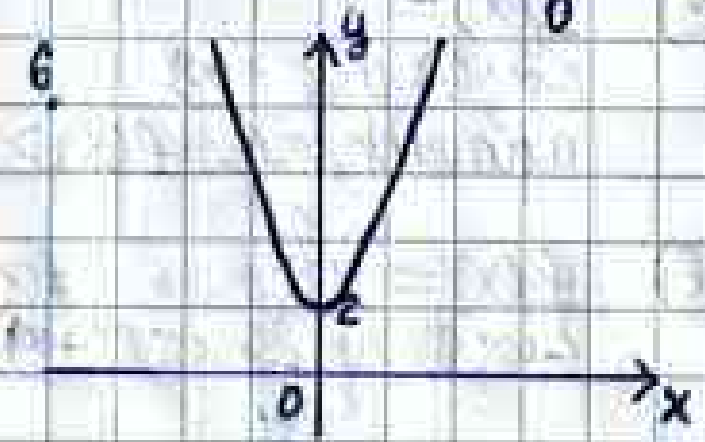
7)  $f(x) = (-x)^{\frac{7}{8}}$   
 $f'(x) = -\frac{7}{8}(-x)^{-\frac{1}{8}} = 0$   $x < 0$   
 $x = 0$

8)  $f(x) = (-x)^{-\frac{8}{11}}$   
 $f'(x) = -\frac{8}{11}(-x)^{-\frac{18}{11}} = 0$   
 $x = 0$

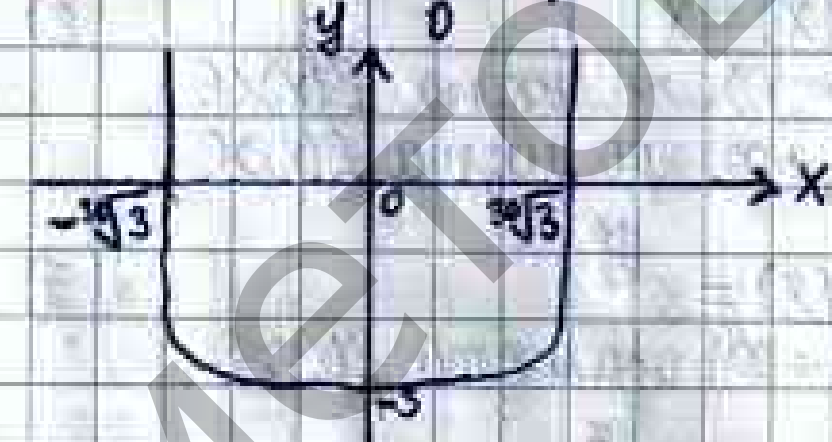
9)  $f(x) = (-x+0,5)^{-\frac{11}{14}}$   
 $f'(x) = -\frac{11}{14}(-x+0,5)^{-\frac{25}{14}} \cdot (-1)$   
 $\frac{11}{14}(-x+0,5)^{\frac{25}{14}} = 0$   
 $-x+0,5 = 0$   
 $x = 0,5$



- 1)  $f(x) = x^4 + 2$
- $x \in \mathbb{R}$
  - $f(-x) = x^4 + 2$  пар
  - $x=0, y=2$  Oy осімен қиылу нүктесі  $(0; 2)$
  - $x^4 + 2 = 0$   
 $x^4 = -2$   $\infty - 2$  оң
  - $f'(x) = 4x^3 = 0$   
 $x=0$



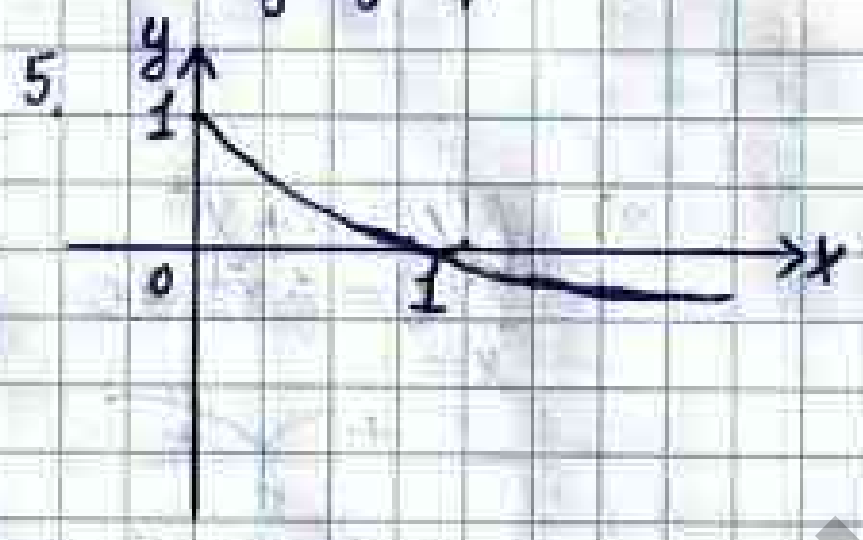
- 2)  $f(x) = x^{30} - 3$
- $x \in \mathbb{R}$
  - $f(-x) = x^{30} - 3$  пар
  - $x=0, y=-3$  Oy осімен  $(0; -3)$  қиылу нүктесі
  - $y=0, x = \pm \sqrt[30]{3}$  Ox осімен қиылу нүктесі  $(-\sqrt[30]{3}; 0)$  және  $(\sqrt[30]{3}; 0)$
  - $f'(x) = 30x^{29} = 0$   
 $x=0$



- 3)  $f(x) = 1 - x^{\frac{1}{2}}$
- $x \in [0; +\infty)$
  - $x=0, y=1$  Oy осімен  $(0; 1)$   
 $y=0, x=1$  Ox осімен  $(1; 0)$
  - $1 - x^{\frac{1}{2}} = 0, x \geq 0$   
 $x=1$   $(0; 1)$  оң  
 $(1; +\infty)$  теріс



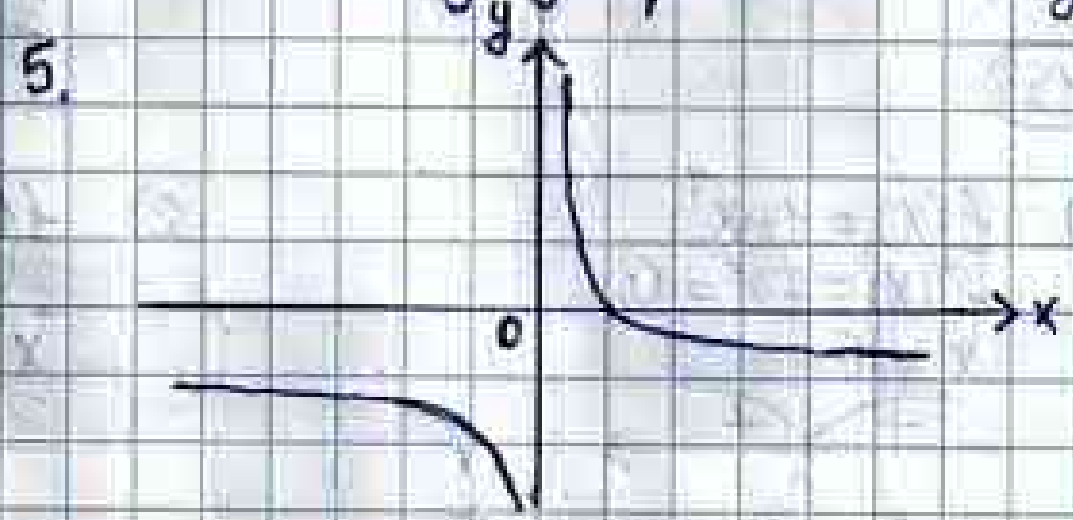
4.  $f'(x) = \frac{1}{2\sqrt{x}} \neq 0, x \geq 0$   
Сындық нүктесі болмайды



- 4)  $f(x) = -1 + x^{-\frac{1}{3}}$
- $x \in \mathbb{R}$  жТФ
  - $y=0, x=-1$  Ox осімен  $(0; -1)$
  - $-1 + x^{-\frac{1}{3}} = 0$   
 $x=1$   $(-\infty; 0)$  теріс  
 $(0; 1)$  теріс  
 $(1; +\infty)$  оң



4.  $f'(x) = -\frac{1}{3}x^{-\frac{4}{3}} \neq 0$   
 $\infty - 2$  сындық нүктесі болмайды



~158

1)  $f(x) = x^9$   
 $f'(x) = 9x^8$

2)  $f(x) = x^{-1}$   
 $f'(x) = -1x^{-2}$

3)  $f(x) = \frac{1}{7}x^7$   
 $f'(x) = x^6$

4)  $f(x) = x^{\frac{11}{6}}$   
 $f'(x) = \frac{11}{6}x^{\frac{5}{6}}$

~159

1)  $f(x) = 2x^4$   
 $f'(1) = 8x^3 = 8$

2)  $f(x) = x^{-3}$   
 $f'(1) = -3x^{-4} = 3$

3)  $f(x) = \frac{1}{3}x^3$   
 $f'(1) = 3x^2 = 3$

4)  $f(x) = x^{-2,5}$   
 $f'(1) = -2,5$

~160

1)  $f(x) = \frac{1}{2\sqrt{x}}$   $F(x) = \frac{1}{2} \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = \sqrt{x} + C$

2)  $f(x) = \frac{2}{3\sqrt{x^3}}$   $F(x) = \frac{2}{3} \frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} = -\frac{4}{3\sqrt{x}}$

3)  $f(x) = \frac{3}{4}x^{\frac{4}{5}}$   $F(x) = \frac{3}{4} \frac{x^{\frac{4}{5}+1}}{\frac{4}{5}+1} = \frac{5}{12}x^{\frac{9}{5}} + C$

4)  $f(x) = x^{\frac{7}{8}}$   $F(x) = \frac{x^{-\frac{7}{8}+1}}{-\frac{7}{8}+1} + C = 8x^{\frac{1}{8}} + C$

~161

1)  $f(x) = \sqrt[3]{x}$   $x_0 = 8$   
 $f'(8) = \frac{1}{3\sqrt[3]{x^2}} = \frac{1}{3 \cdot 4} = \frac{1}{12}$

2)  $f(x) = \frac{1}{\sqrt{x}}$   $x_0 = 9$

$f'(9) = -\frac{1}{2\sqrt{x^3}} = -\frac{1}{2 \cdot 27} = -\frac{1}{54}$

3)  $f(x) = -\frac{3}{x^2}$   $x_0 = 6$   
 $f'(6) = \frac{6}{x^3} = \frac{6}{36 \cdot 6} = \frac{1}{36}$

4)  $f(x) = x^{\frac{1}{3}}$   $x_0 = 1$   
 $f'(1) = \frac{1}{3\sqrt[3]{x^2}} = -\frac{1}{3}$

~162

1)  $f(x) = x^{-\frac{3}{4}}$   $x_0 = 1$   
 $f(1) = 1$   
 $f'(1) = -\frac{3}{4}x^{-\frac{7}{4}} = -\frac{3}{4}$   
 $y = 1 - \frac{3}{4}(x-1) = 1 - \frac{3}{4}x + \frac{3}{4}$

2)  $f(x) = x^{\frac{4}{5}}$   $x_0 = -1$   
 $f'(-1) = \frac{4}{5}x^{-\frac{1}{5}} = -\frac{4}{5}$   
 $f(-1) = 1$

$y = 1 - \frac{4}{5}(x - (-1)) = \frac{1}{5} - \frac{4}{5}x = \frac{1-4x}{5}$

~163

1)  $y = \sqrt{x}$   $y=1$   $x=9$   
 $\sqrt{x}=1 \Rightarrow x=1$   
 $S = \int_1^9 \sqrt{x} dx = \frac{2}{3}\sqrt{x^3} \Big|_1^9 = \frac{2}{3} \cdot 27 - \frac{2}{3} = 17 \frac{1}{3}$

2)  $y = \frac{1}{x^2}$   $y=1$   $x=-3$   $x=-2$

$S_1 = \int_{-3}^{-2} 1 dx = x \Big|_{-3}^{-2} = -2 - (-3) = 1$  кв. дп.

$S_2 = \int_{-3}^{-2} \frac{1}{x^2} dx = -\frac{1}{x} \Big|_{-3}^{-2} = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$  кв. дп.

$S_1 - S_2 = 1 - \frac{1}{6} = \frac{5}{6}$  кв. дп. и т.д.

~164

1)  $f(x) = x\sqrt{x}$   
 $f'(x) = (x^{\frac{3}{2}})' = \frac{3}{2}\sqrt{x}$

2)  $f(x) = x^{\sqrt{3}}$   
 $f'(x) = \sqrt{3}x^{\sqrt{3}-1} = \frac{\sqrt{3}x^{\sqrt{3}}}{x}$

3)  $f(x) = \frac{1}{x^{\frac{7}{4}}}$   
 $f'(x) = -\frac{3}{4}x^{-\frac{7}{4}-1} = -\frac{3}{4\sqrt[4]{x}}$

4)  $f(x) = \frac{3}{\sqrt[3]{x}}$   
 $f'(x) = -\frac{1}{x\sqrt[3]{x}}$

5)  $f(x) = x\sqrt[3]{x^2}$   
 $f'(x) = (x^{\frac{5}{3}})' = \frac{5}{3}x^{\frac{2}{3}}$

6)  $f(x) = \frac{x+5}{x^4}$   
 $f'(x) = \frac{x^4 - 4x^3 - 20x^3}{x^8}$



~165

$$f(x) = 5x^{-\frac{4}{5}+1} = 5x^{\frac{1}{5}} = 5\sqrt[5]{x}$$

$$F(x) = \frac{5x^{\frac{1}{5}+1}}{-\frac{4}{5}+1} = 25x^{\frac{1}{5}} = 25\sqrt[5]{x}$$

$$f(x) = \frac{1}{x^{\frac{3}{2}+1}} = x^{-\frac{5}{2}}$$

$$F(x) = \frac{x^{-\frac{5}{2}+1}}{-\frac{3}{2}+1} = -\frac{3}{\sqrt{x}}$$

$$f(x) = \frac{2x^{-1}+3x}{4x^3} = \frac{2x^{-4}+12x^{-2}}{4x^3}$$

$$F(x) = \frac{8x^{-3}+12x^{-1}}{-3} = -\frac{8x^{-3}}{3} - 12x^{-1}$$

$$f(x) = (x^5+x)^2 = x^{10}+2x^6+x^2$$

$$F(x) = \frac{x^{11}}{11} + \frac{2x^7}{7} + \frac{x^3}{3}$$

~166

$$1) \int_1^9 (\sqrt{x}+x) dx = \left[ \frac{2}{3}x\sqrt{x} + \frac{1}{2}x^2 \right]_1^9 = \frac{2}{3} \cdot 9 \cdot 3 + \frac{1}{2} \cdot 81 - \frac{2}{3} - \frac{1}{2} = 57\frac{1}{3} \checkmark$$

$$2) \int_1^4 (0,25x+3)^3 dx = \left[ \frac{(0,25x+3)^4}{4} \right]_1^4 = \frac{(0,25+3)^4}{4} - \frac{(-0,25+3)^4}{4} = 3,25 - 2,45$$

~167

$$f(x) = \frac{1}{2} \sin \frac{x}{6} \cdot \cos \frac{x}{6} \quad f'(x) = \left( \frac{1}{4} \cdot 2 \sin \frac{x}{6} \cos \frac{x}{6} \right)' = \left( \frac{1}{4} \sin \frac{x}{3} \right)' = \frac{1}{4 \cdot 3} \cos \frac{x}{3} = \frac{1}{12} \cos \frac{x}{3}$$

~168

$$f(x) = \left( \frac{x+3}{2} \right)^3 \quad F(x) = \frac{1}{8} \cdot \frac{(x+3)^4}{4 \cdot 1} + c = \frac{(x+3)^4}{32} + c \quad M(0; 0)$$

$$0 = \frac{(0+3)^4}{32} + c \Rightarrow c = -\frac{81}{32} \quad \text{H/e} \quad c = -2\frac{19}{32}$$

~169

$$1) \int_{-3}^2 3x^{-2} dx = \left[ -3x^{-1} \right]_{-3}^2 = -3 \cdot \frac{1}{2} - \left( -3 \right) \cdot \frac{1}{-3} = -\frac{3}{2} - \frac{1}{1} = -2,5 \checkmark$$

$$2) \int_1^{32} \frac{1}{\sqrt{x}} dx = \left[ \frac{5}{4} x^{\frac{4}{5}} \right]_1^{32} = \frac{5}{4} \cdot 32^{\frac{4}{5}} - \frac{5}{4} = 18\frac{3}{4}$$

$$3) \int_1^3 \left( \frac{1}{3}x+x \right)^2 dx = \int_1^3 \frac{16}{9}x^2 dx = \left[ \frac{16}{9} \cdot \frac{x^3}{3} \right]_1^3 = \frac{16 \cdot 3^3}{27} - \frac{16}{27} = 15\frac{11}{27} \text{ rb dipnik} \checkmark$$

~170

$$1) \begin{cases} y = -x^2 + 2x \\ y = -4 \end{cases}$$

$$-x^2 + 2x = -4$$

$$x^2 - 2x - 4 = 0$$

$$D = 4 + 16 = 20$$

$$x_1 = \frac{2 - 2\sqrt{5}}{2} = 1 - \sqrt{5} \quad x_2 = 1 + \sqrt{5}$$

$$S_1 = \int_{1-\sqrt{5}}^{1+\sqrt{5}} (-x^2 + 2x) dx = \left[ -\frac{x^3}{3} + x^2 \right]_{1-\sqrt{5}}^{1+\sqrt{5}} = -\frac{(1+\sqrt{5})^3}{3} + (1+\sqrt{5})^2 + \frac{(1-\sqrt{5})^3}{3} - (1-\sqrt{5})^2 = -\frac{(1-\sqrt{5})^3 - (1+\sqrt{5})^3}{3} + (1+\sqrt{5})^2 - (1-\sqrt{5})^2 = \frac{-16\sqrt{5}}{3} + 4\sqrt{5} = \frac{-16\sqrt{5} + 12\sqrt{5}}{3} = \frac{4\sqrt{5}}{3}$$

$$S_2 = \int_{1-\sqrt{5}}^{1+\sqrt{5}} -4 dx = \left[ -4x \right]_{1-\sqrt{5}}^{1+\sqrt{5}} = -4 - 4\sqrt{5} - (-4 + 4\sqrt{5}) = 8\sqrt{5}$$

$$1) \begin{cases} y = -x^2 + 2x \\ y = -4 \end{cases}$$

$$-x^2 + 2x = -4$$

$$x^2 - 2x - 4 = 0$$

$$D = 4 + 16 = 20$$

$$x_1 = 1 - \sqrt{5} \quad x_2 = 1 + \sqrt{5}$$

$$S = \int_{1-\sqrt{5}}^{1+\sqrt{5}} (-x^2 + 2x + 4) dx = \left[ -\frac{x^3}{3} + x^2 + 4x \right]_{1-\sqrt{5}}^{1+\sqrt{5}} = -\frac{(1+\sqrt{5})^3}{3} + (1+\sqrt{5})^2 + 4(1+\sqrt{5}) - \left( -\frac{(1-\sqrt{5})^3}{3} + (1-\sqrt{5})^2 + 4(1-\sqrt{5}) \right) = \frac{-(1-\sqrt{5})^3 + (1+\sqrt{5})^3}{3} + (1+\sqrt{5})^2 - (1-\sqrt{5})^2 + 4(1+\sqrt{5}) - 4(1-\sqrt{5}) = \frac{-16\sqrt{5}}{3} + 4\sqrt{5} + 8\sqrt{5} = \frac{-16\sqrt{5} + 36\sqrt{5}}{3} = \frac{20\sqrt{5}}{3} \text{ rb dipnik}$$

$$2) \quad y = \sqrt{x} \quad y = 2 \quad x = 9$$

$$\frac{2}{2} = \sqrt{x}$$

$$x = 4$$

$$S_1 = \int_4^9 \sqrt{x} dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} \Big|_4^9 = \frac{2}{3} x\sqrt{x} \Big|_4^9 = \frac{2}{3} \cdot 9 \cdot 3 - \frac{2}{3} \cdot 4 \cdot 2 = 18 - 5 \frac{1}{3} = 12 \frac{2}{3} \quad \checkmark$$

$$S_2 = \int_4^9 2 dx = \frac{2x}{1} \Big|_4^9 = 18 - 8 = 10 \quad S_1 - S_2 = 12 \frac{2}{3} - 10 = 2 \frac{2}{3} = \frac{8}{3} \text{ кв. единиц}$$

~171

$$1) \quad y = \sqrt{x\sqrt{x}} = (x \cdot x^{\frac{1}{2}})^{\frac{1}{2}} = x^{\frac{3}{4}}$$

$$y' = \frac{3}{4} x^{\frac{3}{4}-1} = \frac{3}{4} x^{-\frac{1}{4}} = \frac{3}{4\sqrt[4]{x}}$$

$$2) \quad y = \frac{1}{x\sqrt[3]{2x}} = \frac{1}{\sqrt[3]{2} x^{\frac{4}{3}}}$$

$$y' = -\frac{4}{3} \cdot \frac{1}{\sqrt[3]{2}} x^{-\frac{4}{3}-1} = -\frac{4}{3x^2\sqrt[3]{2x}}$$

$$3) \quad y = \frac{1+2x-x^4}{x\sqrt{x}}$$

$$y' = \frac{(1+2x-x^4)'x^{\frac{3}{2}} - (1+2x-x^4) \cdot (\frac{3}{2}x^{\frac{1}{2}})}{(x^{\frac{3}{2}})^2} = \frac{(2-4x^3)x^{\frac{3}{2}} - (1+2x-x^4) \cdot \frac{3}{2}x^{\frac{1}{2}}}{x^3}$$

$$= \frac{2x^{\frac{3}{2}} - 4x^{\frac{3}{2}} - \frac{3}{2}x^{\frac{1}{2}} - 3x^{\frac{3}{2}} + 6x^{\frac{1}{2}}}{x^3} = \frac{-4x^{\frac{3}{2}} - X\sqrt{x} + 4,5\sqrt{x}}{x^3}$$

$$4) \quad y = x^{-\sqrt{4}} \quad y' = -\sqrt{4} x^{-\sqrt{4}-1} = \frac{-\sqrt{4}}{x^{\sqrt{4}+1}}$$

~173

$$1) \quad f(x) = \sqrt[3]{2x\sqrt{3x}} + \pi = \sqrt[3]{12} x^{\frac{1}{2}} + \pi \quad F(x) = \sqrt[3]{12} \cdot \frac{2}{3} x^{\frac{3}{2}} + \pi K + C$$

$$2) \quad f(x) = \frac{3x^2-x+1}{\sqrt{x}} = 3x^{\frac{3}{2}} - x^{\frac{1}{2}} + x^{\frac{1}{2}} + C \quad F(x) = \frac{2}{5} \cdot 3x^{\frac{5}{2}} - \frac{2}{3} \cdot x\sqrt{x} + 2\sqrt{x} + C$$

~174

$$1) \quad \int \frac{dx}{4\cos^2(3-x)} = \frac{1}{4} \operatorname{tg}(3-x) + C$$

$$2) \quad \int \frac{\cos^2 x dx}{1-\sin x} = \int \frac{(1-\sin x)(1+\sin x)}{-(1-\sin x)} dx = x - \cos x + C$$

~175

$$f(x) = \sqrt{x} + 2\sqrt[3]{x} \quad M(1, 1,5)$$

$$F(x) = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + 2 \cdot \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + C = \frac{2}{3} x\sqrt{x} + \frac{3}{4} \cdot 2 x\sqrt[3]{x} + C = \frac{2}{3} x\sqrt{x} + \frac{3}{2} x\sqrt[3]{x} + C$$

$$\frac{2}{3} \cdot 1 + \frac{3}{2} \cdot 1 + C = 1,5 \Rightarrow 4 + 9 + 6C = 9 \Rightarrow C = -\frac{2}{3} \quad \text{генер. } F(x) = \frac{2}{3} x\sqrt{x} + \frac{3}{2} x\sqrt[3]{x} - \frac{2}{3}$$

~176

$$1) \quad \int_1^8 \frac{5}{2x^{\frac{2}{3}}} dx = \int_1^8 \frac{5}{2} x^{-\frac{2}{3}} dx = \frac{5}{2} \cdot 3 x^{\frac{1}{3}} \Big|_1^8 = \frac{15}{2} \sqrt[3]{x} \Big|_1^8 = \frac{15}{2} \cdot \sqrt[3]{8} - \frac{15}{2} = 15 - 7,5 = 7,5 \text{ кв. единиц}$$

$$2) \quad \int_4^9 \frac{3}{x^{-\frac{1}{2}}} dx = 3 \cdot \frac{2}{-1} x^{\frac{1}{2}} \Big|_4^9 = 2x\sqrt{x} \Big|_4^9 = 2 \cdot 9 \cdot \sqrt{9} - 2 \cdot 4 \cdot \sqrt{4} = 54 - 16 = 38 \text{ кв. единиц}$$

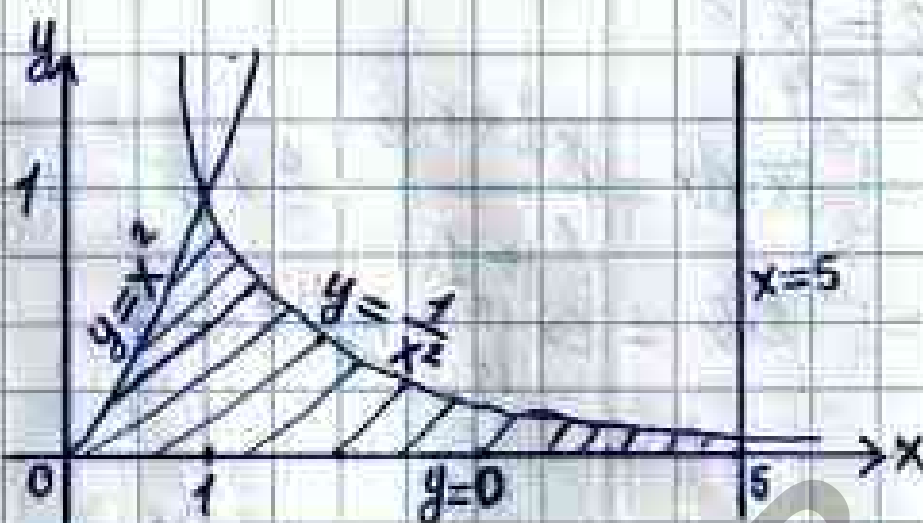
$$3) \quad \int_{\frac{1}{2}}^1 \sqrt[5]{2x-1} dx = \frac{(2x-1)^{\frac{5}{4}+1}}{\frac{5}{4} \cdot 2} \Big|_{\frac{1}{2}}^1 = \frac{2}{5} \sqrt[5]{(2x-1)^5} \Big|_{\frac{1}{2}}^1 = \frac{2}{5} \sqrt[5]{(2-1)^5} - \frac{2}{5} \sqrt[5]{(2 \cdot \frac{1}{2} - 1)^5} = \frac{2}{5} \text{ кв. единиц}$$

1)  $y = x^2, y = 0, x = 5, y = \frac{1}{x^2} (x \geq 0)$

$$S_1 = \int_0^5 x^2 dx = \frac{x^3}{3} \Big|_0^5 = \frac{125}{3}$$

$$S_2 = \int_1^5 \frac{1}{x^2} dx = -\frac{1}{x} \Big|_1^5 = -\frac{1}{5} + 1 = \frac{4}{5}$$

$$S_1 + S_2 = \frac{125}{3} + \frac{4}{5} = \frac{625 + 12}{15} = \frac{637}{15} \text{ кв. диметр}$$

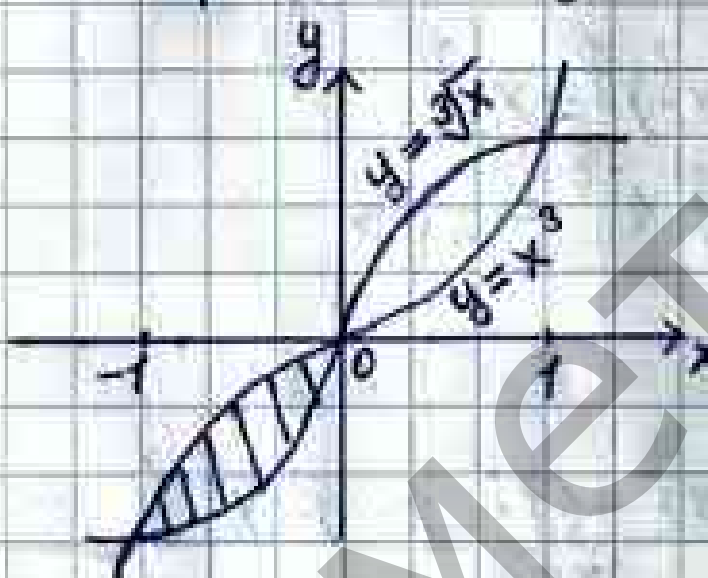


2)  $y = x^3, y = x^{\frac{1}{3}}, x = -1, x = 0$

$$S_1 = \int_{-1}^0 x^3 dx = \frac{x^4}{4} \Big|_{-1}^0 = 0 - \frac{1}{4} = -\frac{1}{4}$$

$$S_2 = \int_{-1}^0 x^{\frac{1}{3}} dx = \frac{3}{4} x^{\frac{4}{3}} \Big|_{-1}^0 = 0 + \frac{3}{4} = \frac{3}{4}$$

$$S_2 - S_1 = \frac{3}{4} - (-\frac{1}{4}) = \frac{4}{4} = 1 \text{ кв. диметр}$$



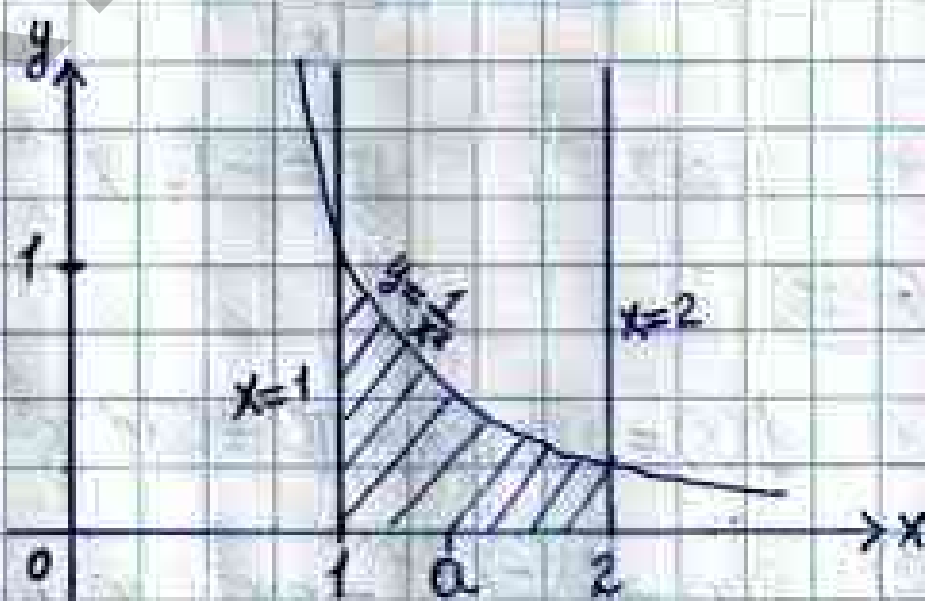
178)  $y = \frac{1}{x^2}, x = 1, x = 2, y = 0$

$$S = \int_1^2 \frac{1}{x^2} dx = -\frac{1}{x} \Big|_1^2 = -\frac{1}{2} + 1 = \frac{1}{2}$$

$$\frac{S}{2} = \frac{1/2}{2} \Rightarrow S = \frac{1}{4}$$

$$\int_1^a \frac{1}{x^2} dx = -\frac{1}{x} \Big|_1^a = -\frac{1}{a} + 1 = \frac{1}{4}$$

$$\frac{1}{a} = 1 - \frac{1}{4} \Rightarrow a = \frac{4}{3}$$



Визуалі мескер!

1)  $y = \sqrt{x^3 - 5x^2 + 6x}, x^3 - 5x^2 + 6x \geq 0$

$$x(x^2 - 5x + 6) = 0$$

$$x_1 = 0, x^2 - 5x + 6 = 0$$

$$D = 25 - 24 = 1$$

$$x = \frac{5 \pm 1}{2} = 2, x_2 = \frac{5 + 1}{2} = 3$$



М:  $[0, 2] \cup [3, +\infty)$

2)  $\left(\frac{125}{512}\right)^{\frac{1}{3}} = \frac{\sqrt[3]{125}}{\sqrt[3]{512}} = \frac{\sqrt[3]{8^3}}{\sqrt[3]{8^3}} = \frac{8}{8} = 1,6$  (D)

3)  $\frac{x^{\frac{1}{2}} - x^{\frac{5}{8}}}{x^{\frac{1}{2}} - x^{\frac{5}{8}}} = \frac{x^{\frac{1}{2}}(1 - x^{\frac{1}{4}})}{x^{\frac{1}{2}}(1 + x^{\frac{1}{4}})} = \frac{1 - \sqrt[4]{x}}{1 + \sqrt[4]{x}}$   $x = 0,008$ ;  $\frac{1 - \sqrt[4]{0,008}}{1 + \sqrt[4]{0,008}} = \frac{1 - 0,2}{1 + 0,2} = \frac{0,8}{1,2} = \frac{2}{3}$  (C)

4)  $\left(\frac{b^{\frac{1}{3}} + b}{b - 1 + b^{\frac{1}{3}} - b^{\frac{2}{3}}}\right) \cdot (b^{\frac{1}{3}} - 1) \cdot \frac{b - 1}{b^{\frac{1}{3}}} = \frac{b + b}{b^{\frac{1}{3}}(b - 1)} \cdot (b^{\frac{1}{3}} - 1) \cdot \frac{(b - 1)}{b^{\frac{1}{3}}} = \frac{2b(b^{\frac{1}{3}} - 1)}{b} = 2(b^{\frac{1}{3}} - 1)$  (D)

5)  $(x^{\frac{1}{3}} + q^{\frac{1}{3}})(x^{\frac{2}{3}} + q^{\frac{2}{3}} - (xq)^{\frac{1}{3}}) = x + q$  (B)

6)  $\frac{27^{\frac{1}{3}} \cdot 4^{-\frac{1}{2}} \cdot 2^{-1}}{625^{-\frac{1}{4}}} = \frac{2^{-1}(3 - 1)}{5^{-1}} = \frac{2^{-1} \cdot 2}{5^{-1}} = 5$  (C)

7)  $\sqrt{\frac{x}{x-2}} + 6\sqrt{\frac{x-2}{x}} = 5 \Rightarrow \frac{x}{x-2} + 12 + 36 \cdot \frac{x-2}{x} = 25 \Rightarrow \frac{x}{x-2} + 36 \frac{x-2}{x} - 13 = 0 \Rightarrow$   
 $x^2 + 36x^2 - 144x + 144x + 144 - 13x^2 + 26x = 0 \Rightarrow 12x^2 - 59x + 72 = 0 \Rightarrow$   
 $D = 59^2 - 4 \cdot 12 \cdot 72 = 25 \Rightarrow x_1 = \frac{59 - 5}{2 \cdot 12} = \frac{54}{24} = \frac{9}{4}, x_2 = \frac{59 + 5}{2 \cdot 12} = \frac{64}{24} = \frac{8}{3}$  (C)



$$8) \sqrt{4-3x} < 2 \Rightarrow 4-3x < 4 \Rightarrow -3x < 0 \Rightarrow x > 0 \quad (C)$$

$$9) \begin{cases} \sqrt{x+y} = 4 \\ x^2 - y + 5x = 0 \end{cases} \Rightarrow \begin{cases} x+y = 16 \\ x^2 - y + 5x = 0 \end{cases} \Rightarrow \begin{cases} y = 16-x \\ x^2 - 16 + x + 5x = 0 \end{cases} \quad \begin{cases} x_1 = 2 \\ y_1 = 14 \end{cases} \quad \begin{cases} x_2 = -8 \\ y_2 = 24 \end{cases} \quad (C)$$

$$x^2 + 6x - 16 = 0$$

$$D = 36 + 64 = 100$$


$$x_1 = \frac{-6+10}{2} = 2 \quad y_1 = 16-2 = 14; \quad x_2 = \frac{-6-10}{2} = -8 \quad y_2 = 16-(-8) = 24$$

$$10) \sqrt{x-3} \leq 4 \Rightarrow x-3 \leq 16 \Rightarrow x \leq 19 \quad (C)$$

$$11) (x-5) \cdot \sqrt{9-x^2} = 0$$

$$\begin{cases} x-5 = 0 \\ 9-x^2 = 0 \end{cases} \quad (A)$$

$$\begin{cases} x = 5 \\ x = \pm 3 \end{cases}$$

$$12) \begin{cases} \sqrt{x-1} < 2 \\ 10-x \leq 8 \end{cases} \Rightarrow \begin{cases} x-1 < 4 \\ -x \leq -2 \end{cases} \Rightarrow \begin{cases} x < 5 \\ x \geq 2 \end{cases} \Rightarrow [2; 5) \quad (C)$$


$$13) f(x) = x^{\frac{2}{3}} + 5 \quad x = 8$$

$$f'(8) = \frac{2}{3\sqrt[3]{x}} = \frac{2}{3\sqrt[3]{8}} = \frac{2}{3 \cdot 2} = \frac{1}{3} \quad (D)$$



$$14) y = x^{-\frac{1}{3}} + 1 \quad x_0 = \frac{1}{27}$$

$$y\left(\frac{1}{27}\right) = \left(\frac{1}{27}\right)^{-\frac{1}{3}} + 1 = 4; \quad y'\left(\frac{1}{27}\right) = -\frac{1}{3} \cdot x^{-\frac{4}{3}} = -\frac{1}{3} \cdot 27 \cdot 3 = -27$$

$$y = f(x_0) + f'(x_0)(x-x_0) \quad y = 4 - 27\left(x - \frac{1}{27}\right) = -27x + 5$$


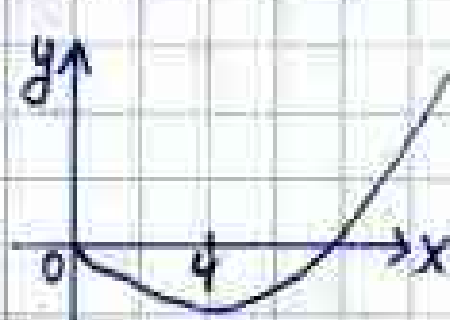
$$15) y = \frac{2}{3}x^{\frac{3}{2}} - x \quad x \geq 0$$

$$y' = \frac{2}{3} \cdot \frac{3}{2} \sqrt{x} - 1 = 0$$

$$x = 1 \text{ min}$$



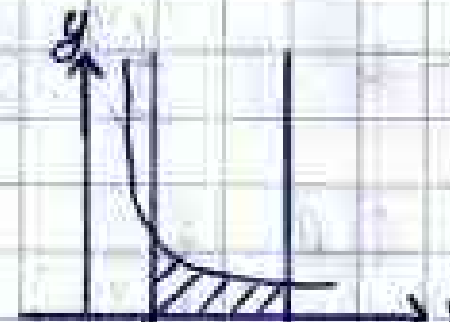
$$16) y = \frac{2}{3}x^{\frac{3}{2}} - 2x \quad x \geq 0$$

$$y' = \frac{2}{3}x^{\frac{1}{2}} - 2 = 0$$

$$x = 4$$



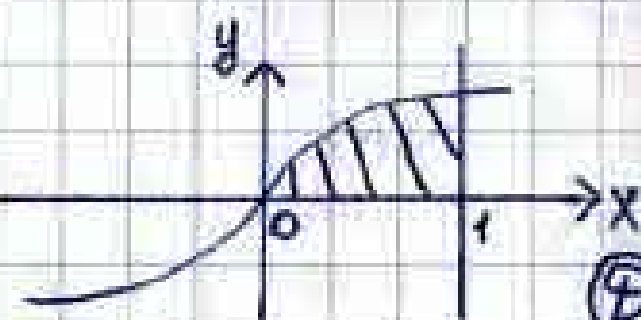
(0; 4)  
(4; +∞) ocregi

$$18) y = \frac{1}{x^4}, \quad x=1, \quad x=3, \quad y=0$$

$$S = \int_1^3 \frac{dx}{x^4} = -\frac{1}{3x^3} \Big|_1^3 = -\frac{1}{3 \cdot 27} + \frac{1}{3} = \frac{26}{81} \text{ kb dipirin} \quad (A)$$


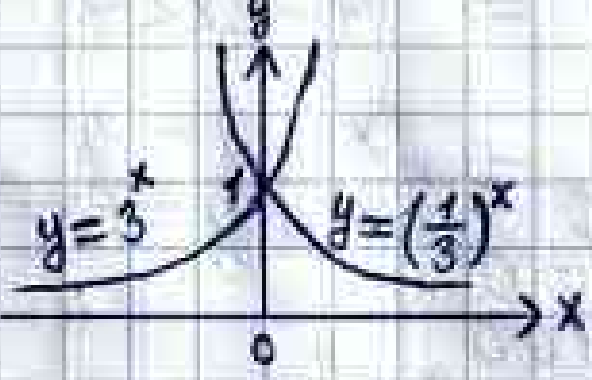
$$19) \int_0^{64} \left(\frac{3}{4}x^{\frac{1}{3}} + \frac{3}{2}x^{\frac{1}{2}}\right) dx = \frac{3}{4} \cdot \frac{3\sqrt[3]{x}}{4} + \frac{3}{2} \cdot \frac{2}{3} x^{\frac{3}{2}} \Big|_0^{64} = \frac{9}{16} x \sqrt[3]{x} + x \sqrt{x} = \frac{9}{16} \cdot 64 \cdot 4 + 64 \cdot 8 = 656 \quad (C)$$

$$20) y = \frac{3}{\sqrt{10}} x^{\frac{1}{3}} \quad x=0 \quad x=1$$

$$V = \int_0^1 Q(x) dx = \pi \int_0^1 y^2 dx \quad V = \pi \int_0^1 \frac{9}{10} x^{\frac{2}{3}} dx = \frac{27\pi}{50} \Big|_0^1 = \frac{27\pi}{50}$$




9)  $y = 3^x$      $y = \left(\frac{1}{3}\right)^x$   
 $\frac{y}{3} = \left(\frac{1}{3}\right)^x \Rightarrow 3^x = 3^{-x} \Rightarrow X = -X$   
 $2X = 0 \Rightarrow X = 0$



~180  
1)

- 1)  $f(x) = 4^{\frac{1}{x}}$     2)  $f(x) = \left(\frac{1}{3}\right)^{\sqrt{x}}$     3)  $f(x) = \frac{1}{7^x}$     4)  $f(x) = 0,35^x$   
 D:  $(-\infty, 0) \cup (0, +\infty)$     D:  $[0, +\infty)$     D:  $(-\infty, +\infty)$     D:  $(-\infty, +\infty)$

- ~181  
1)  $f(x) = \left(\frac{1}{5}\right)^x - 2$     2)  $f(x) = 6^{x+2} + \frac{1}{4}$     3)  $f(x) = 2,5^x + 3$     4)  $f(x) = 0,7^{x-1}$   
 D:  $(-2, +\infty)$     D:  $(\frac{1}{4}, +\infty)$     D:  $(3, +\infty)$     D:  $(-1, +\infty)$

~182

1)  $y = 3^x$

x	0	2	3	4	1
y	1	9	27	81	3

2)  $y = 3^x$

x	-1	-2	-3	-4
y	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$	$\frac{1}{81}$

3)  $y = 3^x$

x	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{3}{4}$
y	$\sqrt{3}$	$\sqrt[3]{3}$	$\sqrt[3]{8}$	$\sqrt{3}$	$\sqrt[3]{27}$

- ~183  
1)  $y = 4^x$  өспей    2)  $y = 10^x$  өспей    3)  $y = \left(\frac{1}{4}\right)^x$  кемей    4)  $y = (\sqrt{2})^x$  өспей

~184  
1)  $y = 2^x$  өр-а  $y = (\sqrt{2})^x$  қарағанда жылдам өседі. Мысалы,

x	0	2	4
y	1	4	16

x	0	2	4
y	1	2	4

2)  $y = \left(\frac{1}{3}\right)^x$  өр-а  $y = \left(\frac{1}{2}\right)^x$  қарағанда жылдам кемиді. Мысалы

x	0	1	2	3
y	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$

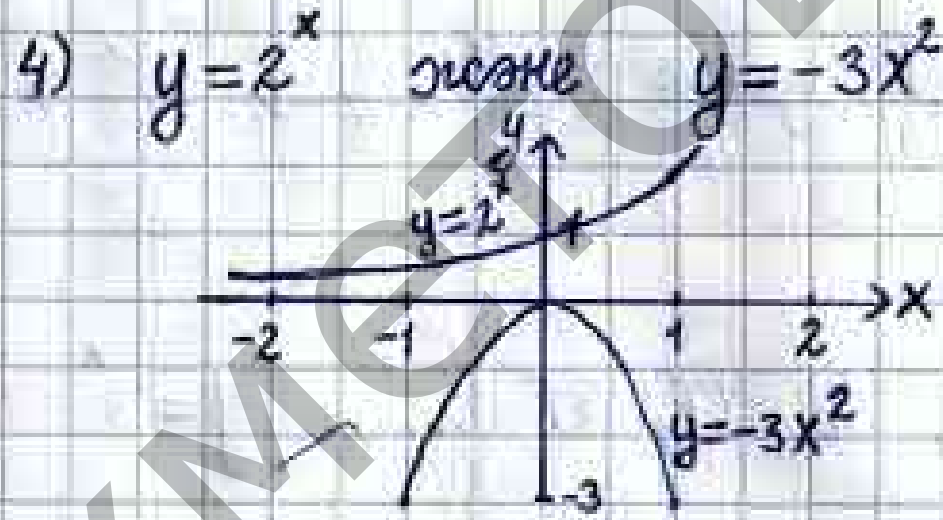
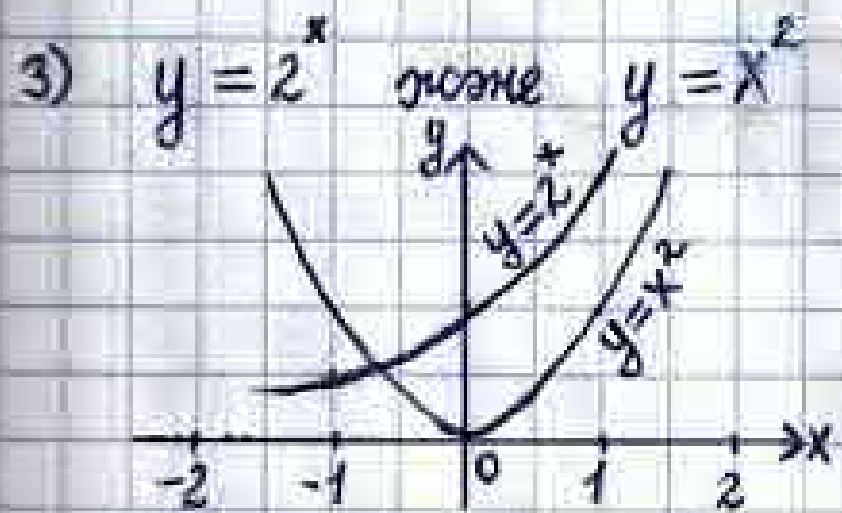
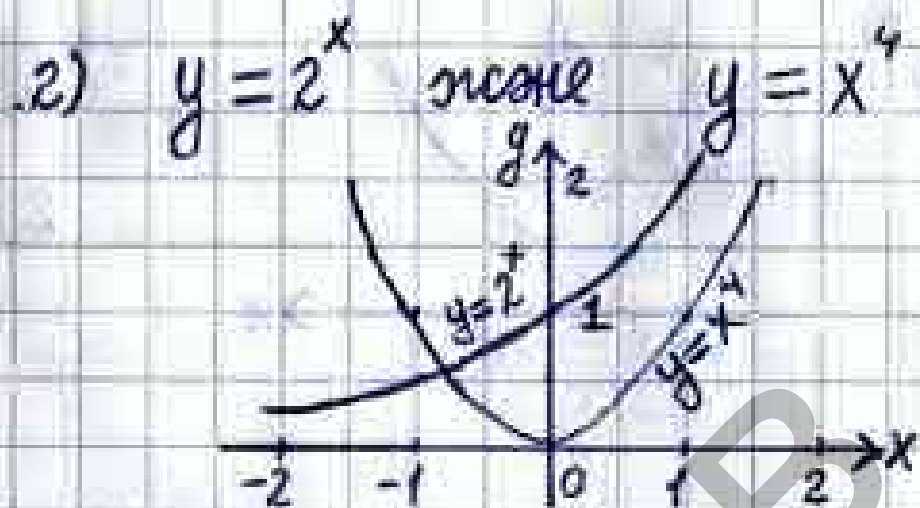
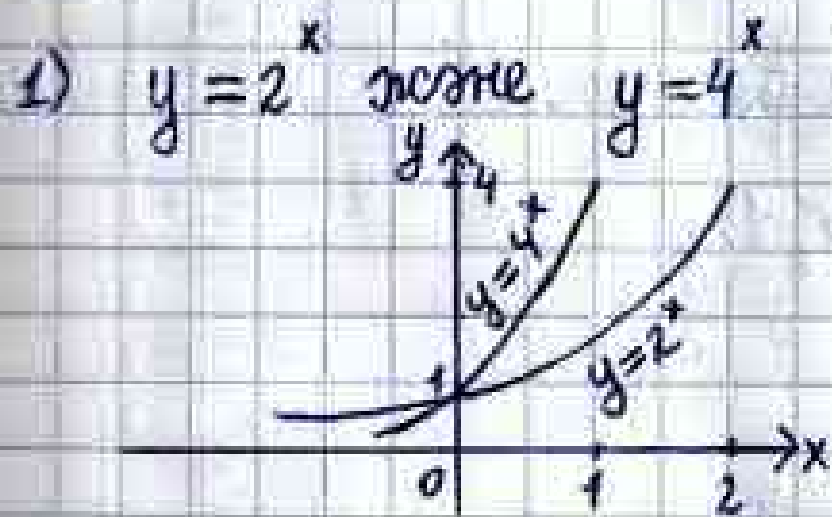
x	0	1	2	3
y	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

- ~185  
1)  $11^{-5} < 1$     2)  $\left(\frac{5}{6}\right)^{\frac{2}{3}} < 1$     3)  $(0,15)^{-3} > 1$     4)  $(1,2)^{-2} < 1$

- ~186  
1)  $(3,5)^{-2} < \left(\frac{1}{3,5}\right)^{\sqrt{2}}$     2)  $\left(\frac{3}{4}\right)^{1+\sqrt{3}} < \left(\frac{3}{4}\right)^2$     3)  $(\sqrt{5})^{\sqrt{2}-\sqrt{5}} < (\sqrt{5})^{\sqrt{5}-2}$     4)  $\left(\frac{1}{\sqrt{3}}\right)^{-2\sqrt{3}} = 3^{\sqrt{3}}$

~187  
1)  $y = 9^x$ ;  $y = 4^x$     2)  $y = \left(\frac{1}{2}\right)^x$ ;  $y = \left(\frac{1}{3}\right)^x$

~188

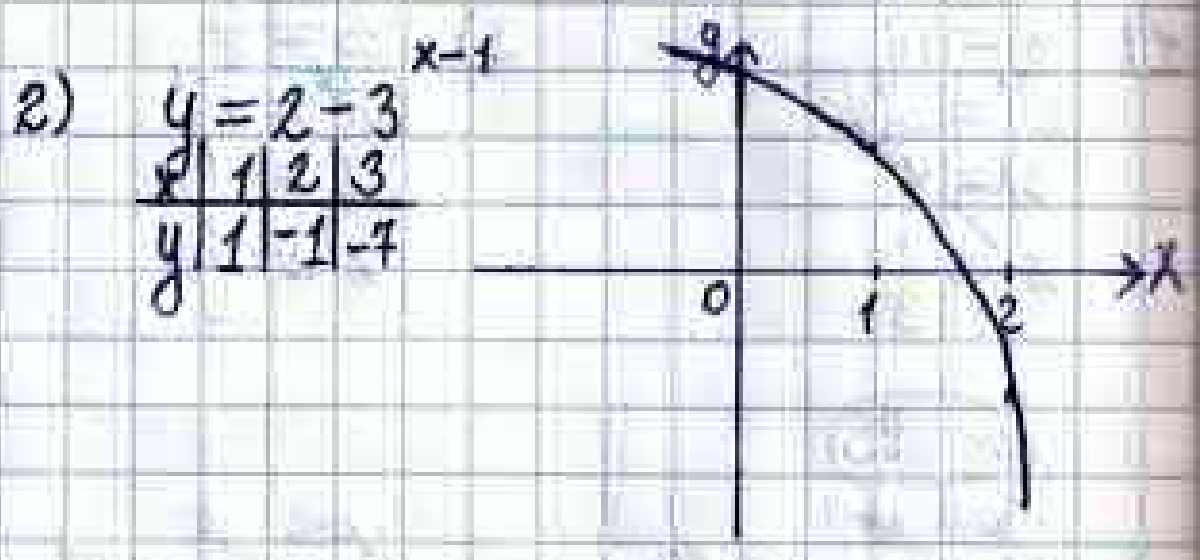
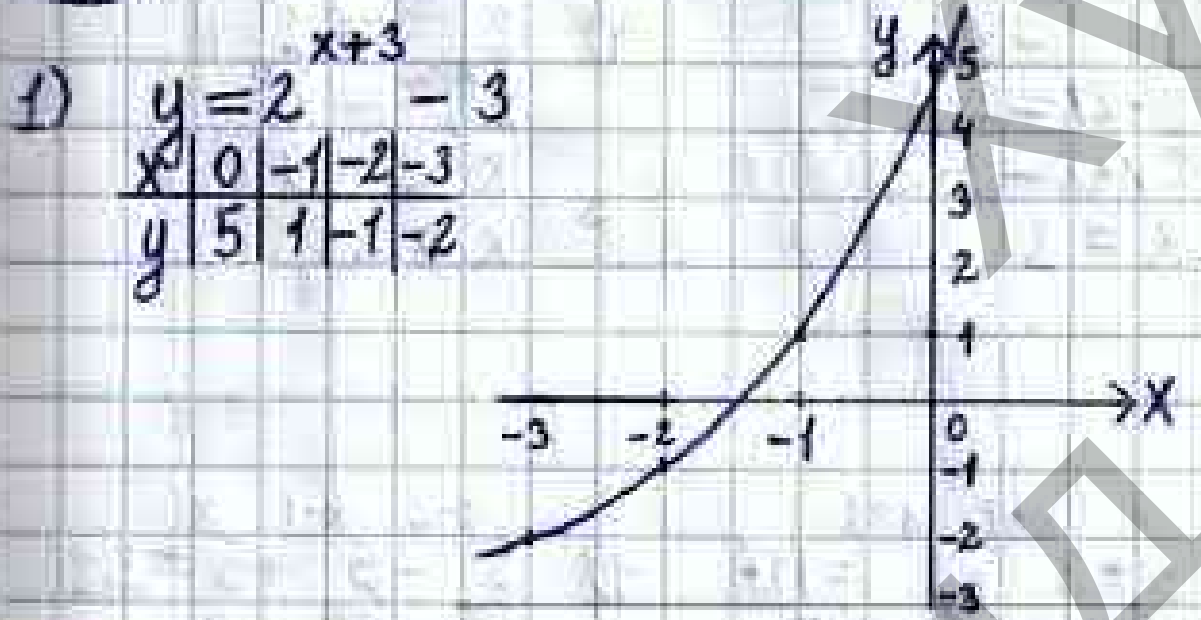


~189

$y = \left(\frac{1}{3}\right)^x$

x	1	2	3	4
y	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$	$\frac{1}{81}$

~190



~191

$a > 1$

1) егер  $x < 0$  болса  $a^x < 1$   
 егер  $x = 0$  болса  $a^x = 1$   
 егер  $x > 0$  болса  $a^x > 1$

$0 < a < 1$

2) егер  $x < 0$  болса  $a^x > 1$   
 егер  $x = 0$  болса  $a^x = 1$   
 егер  $x > 0$  болса  $a^x < 1$

~192



~193

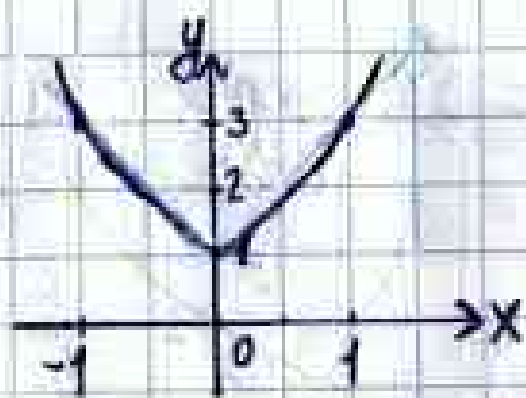
1)  $a^x > a^3$   
 $x > 3$  егер  $a > 1$   
 и: ақиқат

2)  $\left(\frac{1}{9}\right)^x > \left(\frac{1}{3}\right)^{x-1}$   
 $\left(\frac{1}{3}\right)^{2x} > \left(\frac{1}{3}\right)^{x-1}$   
 $2x < x-1$   
 и: ақиқат

3)  $7^{x^2} > 7^x$   
 $x^2 > x$   
 и: ақиқат емес

194

$$y = 3^{|x|}$$



1) ең үлкен мәні болмайды

2) ең кіші мәні (0; 1)

195

$$y > \frac{1}{4} \text{ геленек } 2^{2x} > \frac{1}{4} \Rightarrow 2^{2x} > 2^{-2} \Rightarrow 2x > -2 \Rightarrow x > -1$$

196

$$1, 3, 9, 27, 81, \dots \quad y = 3^x \quad x \in \mathbb{N}$$

197

1)  $3^x = 81$   
 $3^x = 3^4$   
 $x = 4$

2)  $4^x = 256$   
 $4^x = 4^4$   
 $x = 4$

3)  $2^x = \frac{1}{32}$   
 $2^x = 2^{-5}$   
 $x = -5$

4)  $5^{x+1} = 125$   
 $5^{x+1} = 5^3$   
 $x = 2$

198

1)  $8^x = 16$   
 $2^{3x} = 2^4$   
 $3x = 4$   
 $x = \frac{4}{3}$

2)  $25^x = \frac{1}{5}$   
 $5^{2x} = 5^{-1}$   
 $2x = -1$   
 $x = -\frac{1}{2}$

3)  $4^{3-2x} = 4^{2-x}$   
 $3-2x = 2-x$   
 $2x-x = 3-2$   
 $x = 1$

4)  $2^{x-2} = 1$   
 $2^{x-2} = 2^0$   
 $x-2 = 0$   
 $x = 2$

199

1)  $2^x + 2^{x+1} = 12$   
 $2^x + 2 \cdot 2^x = 12$   
 $3 \cdot 2^x = 12$   
 $2^x = 2^2$   
 $x = 2$

2)  $7^{x+2} - 7^x = 336$   
 $49 \cdot 7^x - 7^x = 336$   
 $48 \cdot 7^x = 336$   
 $7^x = 7$   
 $x = 1$

3)  $3^x + 3^{x+1} + 3^{x+2} = 117$   
 $9^x + 12 \cdot 3^x = 117$   
 $13 \cdot 3^x = 117$   
 $3^x = 3^2$   
 $x = 2$

4)  $5^{x-2} - 5^{x-1} + 5^x = 21$   
 $\frac{5^x}{25} - \frac{5^x}{5} + 5^x = 21$   
 $19 \cdot 5^x = 525$   
 $x = 2$

200

1)  $3^{2x+1} = 9^{2x}$   
 $3^{2x+1} = 3^{4x}$   
 $2x+1 = 4x$   
 $x = \frac{1}{2}$

2)  $2 \cdot 2^{2x} - 3 \cdot 2^x - 2 = 0$   
 $2^x = t \quad 2^{2x} = t^2$   
 $2t^2 - 3t - 2 = 0$   
 $D = 9 + 4 \cdot 2 \cdot 2 = 25$   
 $t_1 = \frac{3-5}{4} = -\frac{1}{2} \quad 2^x \neq -\frac{1}{2}$   
 $t_2 = \frac{3+5}{4} = 2 \quad 2^x = 2$   
 $x = 1$

3)  $2 \cdot 9^x - 3 \cdot 9^{x+1} - 9 = 0$   
 $2 \cdot 3^{2x} - 3 \cdot 3^{2x+1} - 9 = 0$   
 $3^x = t \quad 3^{2x} = t^2$   
 $2t^2 - 3t - 9 = 0$   
 $D = 9 + 4 \cdot 2 \cdot 9 = 81$

4)  $25^x - 26 \cdot 5^x + 25 = 0$   
 $5^{2x} - 26 \cdot 5^x + 25 = 0$   
 $5^x = t \quad 5^{2x} = t^2$   
 $t^2 - 26t + 25 = 0$   
 $D = 676 - 4 \cdot 25 = 576$   
 $t_1 = \frac{26-24}{2} = 1 \quad 5^x = 1$   
 $t_2 = \frac{26+24}{2} = 25 \quad 5^x = 25$   
 $x = 0$   
 $x = 2$

3.2  $t_1 = \frac{3-9}{4} = -\frac{3}{2} \quad 3^x \neq -\frac{3}{2}$   
 $t_2 = \frac{3+9}{4} = 3 \quad 3^x = 3$

~201

$$1) \begin{cases} 5^x + 5^y = 30 \\ 5^x - 5^y = 20 \end{cases} \Rightarrow \begin{cases} 5^x = 30 - 5^y \\ 30 - 5^y - 5^y = 20 \end{cases} \Rightarrow \begin{cases} x = 2 \\ y = 1 \end{cases}$$

$$-2 \cdot 5^y = -10$$

$$5^y = 5 \Rightarrow \begin{cases} 5^x = 30 - 5 \\ 5^x = 5^2 \\ x = 2 \end{cases}$$

$$y = 1$$

$$2) \begin{cases} x - y = 1 \\ 2^x + 2^y = 12 \end{cases} \Rightarrow \begin{cases} x = 1 + y \\ 2^{1+y} + 2^y = 12 \end{cases} \Rightarrow \begin{cases} x = 3 \\ y = 2 \end{cases}$$

$$2 \cdot 2^y + 2^y = 12$$

$$3 \cdot 2^y = 12$$

$$2^y = 2^2 \Rightarrow \begin{cases} x = 1 + 2 \\ y = 2 \\ x = 3 \end{cases}$$

~202

$$1) \begin{cases} 3 \cdot 2^x + 2 \cdot 3^y = 12 \\ 2^x - 3^y = -1 \end{cases} \Rightarrow \begin{cases} 3^y = 1 + 2^x \\ 3 \cdot 2^x + 2(1 + 2^x) = 12 \end{cases}$$

$$5 \cdot 2^x + 2 = 12$$

$$5 \cdot 2^x = 10$$

$$2^x = 2 \Rightarrow \begin{cases} 3^y = 1 + 2 \\ x = 1 \\ y = 1 \end{cases}$$

$$x = 1$$

$$2) \begin{cases} 2^x \cdot 4^y = 32 \\ x - y = 2 \end{cases} \Rightarrow \begin{cases} 2^{x+2y} = 2^5 \\ x = 2 + y \end{cases} \Rightarrow \begin{cases} x = 3 \\ y = 1 \end{cases}$$

$$4 \cdot 2^{2y} \cdot 2^{2y} = 32$$

$$2^{4y} = 2^5$$

$$4y = 5 \Rightarrow \begin{cases} x = 2 + 1 \\ y = 1 \\ x = 3 \end{cases}$$

~203

$$1) \begin{matrix} 4x^2 - 2x - 2 & & 2x - 3 \\ (0, 1) & = & (0, 1) \\ 4x^2 - 2x - 2 - 2x + 3 = 0 \\ 4x^2 - 4x + 1 = 0 \\ D = 16 - 16 = 0 \\ x = \frac{4 \pm 0}{2 \cdot 4} = \frac{1}{2} \end{matrix}$$

$$2) \begin{matrix} x^2 - 2x + 2 \\ (0, 3) & = & 0, 0, 9 \\ (0, 3) & = & 0, 3^2 \\ x^2 - 2x + 2 = 2 \\ x(x - 2) = 0 \\ x_1 = 0 & x_2 = 2 \end{matrix}$$

$$3) \begin{matrix} x+2 & x+3 & x+4 & x+1 & x+2 \\ 2 & -2 & -2 & = & 5 & -5 \\ 4 \cdot 2^x & -8 \cdot 2^x & -16 \cdot 2^x & = & 5 \cdot 5^x & -25 \cdot 5^x \\ -20 \cdot 2^x & = & -20 \cdot 5^x & & 5^x \neq 0 \end{matrix}$$

$$\frac{2^x}{5^x} = \frac{5^x}{5^x}$$

$$\left(\frac{2}{5}\right)^x = 1$$

$$x = 0$$

$$4) \begin{matrix} x+2 & & x+2 \\ 3 & - & 7 \\ 3^{x+2} & - & 7^{x+2} = 0 \\ \frac{3^{x+2}}{7^{x+2}} = \frac{7^{x+2}}{7^{x+2}} & & 7^{x+2} \neq 0 \\ \left(\frac{3}{7}\right)^{x+2} = 1 \\ x+2 = 0 \\ x = -2 \end{matrix}$$

~204

$$1) \begin{matrix} x^2 - 4 & & x^2 - 1 \\ (0, 25) & = & 2 \\ (1) & & 2^{x^2 - 1} \\ (4) & & 2^{x^2 - 1} \\ 2^{3 - 2x^2} & = & 2^{x^2 - 1} \\ 3x^2 = 9 \\ x = \pm \sqrt{3} \end{matrix}$$

$$2) \begin{matrix} -1 & & 2x \\ 27 & \cdot & 9 \\ 3^{-3} \cdot 3^{4x} & = & 3^5 = 243 \\ 4x - 3 = 5 \\ 4x = 8 \\ x = 2 \end{matrix}$$

$$3) \begin{matrix} 3x \\ \sqrt{5 \cdot 5} = 125 \\ 5^{3x+4} = 5^3 \\ 3x = 3 - \frac{1}{4} \\ x = \frac{11}{12} \end{matrix}$$

$$4) \begin{matrix} x+1 \\ 6^{x+1} \cdot \sqrt[3]{6} = 216 \\ 6^{x+1+\frac{1}{3}} = 6^3 \\ x+1\frac{1}{3} = 3 \\ x = 1\frac{2}{3} \end{matrix}$$

~205

$$1) \begin{matrix} 2 & x & x+1 \\ x^2 \cdot 3 - 3 & = & 0 \\ 3^x(x^2 - 3) = 0 \\ 3^x \neq 0 & & x^2 - 3 = 0 \\ & & x = \pm \sqrt{3} \end{matrix}$$

$$2) \begin{matrix} x & & 2+x \\ x^2 \cdot 5^x - 5 & = & 0 \\ 5^x(x^2 - 25) = 0 \\ 5^x \neq 0 & & x^2 - 25 = 0 \\ & & x = \pm 5 \end{matrix}$$

$$3) \begin{matrix} x & & x-2 \\ 2 \cdot 3^x + 3 & = & 54 \\ 19 \cdot 3^x = 513 \\ 3^x = 3^3 \\ x = 3 \end{matrix}$$

$$4) \begin{matrix} 3 & x & x+1 \\ x^3 \cdot 8 - 8 & = & 0 \\ 8^x(x^3 - 8) = 0 \\ 8^x \neq 0 \\ x^3 - 8 = 0 \\ x = 2 \end{matrix}$$



~206

$$\begin{cases} 2 \cdot 4^x + 3 \cdot 5^y = 11 \\ 5 \cdot 4^x + 4 \cdot 5^y = 24 \end{cases}$$

$$\begin{cases} 4^x = \frac{11 - 3 \cdot 5^y}{2} \\ 5 \cdot \frac{11 - 3 \cdot 5^y}{2} + 4 \cdot 5^y = 24 \end{cases}$$

$$55 - 15 \cdot 5^y + 8 \cdot 5^y = 48$$

$$-7 \cdot 5^y = -7$$

$$5^y = 1$$

$$y = 0$$

$$4^x = \frac{11 - 3 \cdot 5^0}{2}$$

$$4^x = 4$$

$$x = 1$$

~207

$$1) \begin{cases} 2^x \cdot 3^y = 648 \\ 3^x \cdot 2^y = 432 \end{cases}$$

$$\begin{cases} 2^x \cdot 3^y = 2^3 \cdot 3^4 \\ 3^x \cdot 2^y = 2^4 \cdot 3^3 \end{cases}$$

$$\begin{cases} x = y - 1 \\ 3^{y+3} = 2^{4-y} \end{cases}$$

$$\left(\frac{3}{2}\right)^{y-4} = \left(\frac{1}{2}\right)^{y-4}$$

$$3^{y-4} = 1$$

$$y = 4 \Rightarrow x = 4 - 1 = 3$$

$$2) \begin{cases} 2^x - 2^y = 1 \\ 2^{3x} - 2^{3y} = 7 \end{cases}$$

$$\begin{cases} 2^x = 2^y + 1 \\ (2^y + 1)^3 - 2^{3y} = 7 \end{cases}$$

$$2^{3y} + 3 \cdot 2^{2y} + 3 \cdot 2^y + 1 - 2^{3y} = 7$$

$$2^{2y} + 2^y - 2 = 0$$

$$2^y = t \quad 2^{2y} = t^2$$

$$t^2 + t - 2 = 0$$

$$\Delta = 1 + 8 = 9$$

$$t_1 = \frac{-1-3}{2} = -2 \quad 2^y \neq -2$$

$$t_2 = \frac{-1+3}{2} = 1 \quad 2^y = 1 \Rightarrow x = 2 + 1 = 3$$

~208

$$1) \left(\frac{1}{3}\right)^{\sqrt{x}} \cdot \left(\frac{1}{3}\right)^x = 1$$

$$\left(\frac{1}{3}\right)^{x+\sqrt{x}} = \left(\frac{1}{3}\right)^0$$

$$x + \sqrt{x} = 0$$

$$\sqrt{x}(\sqrt{x} + 1) = 0$$

$$\sqrt{x} = 0 \quad \sqrt{x} + 1 = 0$$

$$x = 0 \quad \sqrt{x} \neq -1$$

$$2) \begin{cases} u - x = 4 \\ 3^x \cdot 2^y = 576 \end{cases}$$

$$\begin{cases} u = 4 + x \\ 3^x \cdot 2^{4+x} = 576 \\ 3^x \cdot 16 \cdot 2^x = 576 \\ 6^x = 6^2 \\ x = 2 \\ y = 4 + 2 = 6 \end{cases}$$

$$\begin{cases} x = 2 \\ y = 6 \end{cases}$$

~209

$$1) 8^x + 3 \cdot 4^x = 12 + 2^{x+2}$$

$$2^{3x} + 3 \cdot 2^{2x} - 3 \cdot 4 - 2^x \cdot 4 = 0$$

$$2^x(2^{2x} - 4) + 3(2^{2x} - 4) = 0$$

$$(4^x - 4)(2^x + 3) = 0$$

$$4^x - 4 = 0 \quad 2^x + 3 = 0$$

$$4^x = 4 \quad 2^x \neq -3$$

$$x = 1$$

$$2) \sqrt{6-x} (5^{x^2-7,2x+3,4} - 25) = 0 \quad x \geq 6$$

$$5^{x^2-7,2x+3,4} = 5^2 \quad 6-x=0$$

$$x^2 - 7,2x + 1,4 = 0 \quad x_3 = 6$$

$$\Delta = 51,84 - 4 \cdot 1,4 = 46,24$$

$$x_1 = \frac{7,2 - 6,8}{2} = 0,2$$

$$x_2 = \frac{7,2 + 6,8}{2} \neq 7$$

$$3) 16^x + 8^x - 4 \cdot 4^x + 2^x + 1 = 0$$

$$2^{4x} + 2^{3x} - 4 \cdot 2^{2x} + 2^x + 1 = 0$$

$$2^x = a \quad a^4 + a^3 - 4a^2 + a + 1 = 0$$

$$(a-1)^2(a^2+3a+1) = 0$$

$$a-1=0 \quad a^2+3a+1=0$$

$$a=1 \quad \Delta = 9-4=5$$

$$a = \frac{-3 \pm \sqrt{5}}{2} \quad 2^x \neq \frac{-3 \pm \sqrt{5}}{2}$$

$$2) \begin{cases} 1+3x & x & x+2 \\ 3 & -9=3 & -3 \end{cases}$$

$$3 \cdot 3^{3x} - 3^{2x} = 9 \cdot 3^x - 3$$

$$3^{2x}(3 \cdot 3^x - 1) = 3(3 \cdot 3^x - 1)$$

$$3^{2x} = 3 \quad 3 \cdot 3^x - 1 = 0$$

$$2x = 1 \quad x + 1 = 0$$

$$x_1 = \frac{1}{2} \quad x_2 = -1$$

$$4) 3 \cdot 8^x + 4 \cdot 12^x - 18^x - 2 \cdot 27^x = 0$$

$$3 \cdot 2^{3x} + 6 \cdot 2^{2x} \cdot 3^x + 3 \cdot 2^x \cdot 3^{2x} - 2 \cdot 3^x \cdot 2^{2x} - 4 \cdot 2^x \cdot 3^{3x} - 2 \cdot 3^{3x} = 0$$

$$3 \cdot 2^x(2^{2x} + 2 \cdot 2^x \cdot 3^x + 3^{2x}) - 2 \cdot 3^x(2^{2x} + 2 \cdot 2^x \cdot 3^x - 3^{2x}) = 0$$

$$(2^x + 3^x)^2 - (3 \cdot 2^x - 2 \cdot 3^x) = 0$$

$$2^x + 3^x = 0 \quad 3 \cdot 2^x - 2 \cdot 3^x = 0$$

$$\left(\frac{2}{3}\right)^x \neq -1 \quad \left(\frac{3}{2}\right)^x = \left(\frac{3}{2}\right)^x \Rightarrow x = 1$$

~210

$$1) 5^x \cdot 3^{8^{x-1}} = 500$$

$$5^x \cdot 2^{x-1} = 500$$

$$34 \quad 5^x \cdot 2^x = 1000$$

$$10 = 10^3$$

$$x = 3$$

$$2) x^2 + 4x + 2^{\sqrt{x+2}} + 3 = 0 \quad x \geq -2$$

$$x^2 + 4x + 4 - 1 = -2^{\sqrt{x+2}}$$

$$(x+2)^2 = 1 - 2^{\sqrt{x+2}} \Rightarrow x = 2 \quad (-2+2)^2 = 1-2$$

$$0=0$$

3)  $\sqrt{4^{2x} - 3 \cdot 2^{2x}} = 10 - 2$        $2^{2x} = a$

$$\sqrt{a^2 - 3a} = 10 - 2a$$

$$a^2 - 3a = 100 - 40a + 4a^2$$

$$3a^2 - 37a + 100 = 0$$

$$\Delta = 1369 - 1200 = 169$$

$$a_1 = \frac{37 - 13}{6} = 4 \quad 2^{2x} = 4$$

$$a_2 = \frac{37 + 13}{6} = \frac{25}{3} \quad 2^{2x} \neq \frac{25}{3}$$

$$x = 1$$

4)  $(\sqrt{3-2\sqrt{2}})^x + (\sqrt{3+2\sqrt{2}})^x = 6$

$$\left(\frac{\sqrt{3+\sqrt{9-8}}}{2} - \frac{\sqrt{3-\sqrt{9-8}}}{2}\right)^x + \left(\frac{\sqrt{3+\sqrt{9-8}}}{2} + \frac{\sqrt{3-\sqrt{9-8}}}{2}\right)^x = 6$$

$$(\sqrt{2}-1)^x + (\sqrt{2}+1)^x = 6$$

$$\frac{(\sqrt{2}-1)^x (\sqrt{2}+1)^x + (\sqrt{2}+1)^x}{(\sqrt{2}+1)^x} = 6$$

$$\frac{1}{(\sqrt{2}+1)^x} + (\sqrt{2}+1)^x = 6 \quad (\sqrt{2}+1)^x = a$$

$$\frac{1}{a} + a = 6 \Rightarrow 1 + a^2 = 6a$$

$$a^2 - 6a + 1 = 0$$

$$\Delta = 36 - 4 = 32$$

$$a = \frac{6 \pm 4\sqrt{2}}{2} = 3 \pm 2\sqrt{2}$$

$$(\sqrt{2}+1)^x = 3 + 2\sqrt{2} \quad (\sqrt{2}+1)^x \neq 3 - 2\sqrt{2}$$

$$(\sqrt{2}+1)^x = (2 + 2\sqrt{2} + 1)$$

$$(\sqrt{2}+1)^x = (2+1)^2$$

$$x = 2$$

~211

1)  $9 \cdot \left(\frac{1}{27}\right)^{|1+\frac{1}{2}x|} = \left(\frac{1}{81}\right)^x$

$$2 - 3|1+\frac{1}{2}x| = -4x$$

$$\frac{3}{2} - 3 - \frac{3}{2}x = \frac{-4x}{2}$$

$$-2 - 3x = -8x$$

$$5x = 2$$

$$x = \frac{2}{5}$$

2)  $2^{|x-1|} = 0.5^{1-x}$

$$2^{|x-1|} = 2^{x-1}$$

$$|x-1| = x-1 \quad x-1 \geq 0$$

$$x-x = -1 \quad x \geq 1$$

$$0x = 0$$

$$x \in [1, +\infty)$$

3)  $27^{|x+2|} = 81^{x^2-1}$

$$3^{3|x+2|} = 3^{4x^2-4}$$

$$3|x+2| = 4x^2-4$$

$$3x+6 = 4x^2-4$$

$$4x^2-3x-10=0$$

$$\Delta = 9+4 \cdot 40 = 169$$

$$x_1 = \frac{3-13}{8} = -\frac{5}{4}$$

$$x_2 = \frac{3+13}{8} = 2$$

4)  $(0.2)^{|x+3|} = \left(\frac{1}{5}\right)^{x+1}$

$$(0.2)^{|x+3|} = (0.2)^{x+1}$$

$$|x+3| = x+1$$

$$x+3 = x+1$$

$$3 \neq 1 \quad \emptyset$$

~212

1)  $\begin{cases} x - \sqrt{49} = y - \sqrt{343} \\ 3^y = 9^{2x-y} \end{cases}$

$$49^{\frac{1}{x-1}} = 343^{\frac{1}{y-1}}$$

$$7^{\frac{2}{x-1}} = 7^{\frac{3}{y-1}}$$

$$\frac{2}{x-1} = \frac{3}{y-1}$$

$$2y-2 = 3x-3$$

$$x = \frac{2y+1}{3}$$

$$\begin{cases} x = \frac{2y+1}{3} \\ 4x-2y=0 \end{cases} \Rightarrow \begin{cases} x = \frac{2y+1}{3} \\ 4 \cdot \frac{2y+1}{3} - 2y = 0 \end{cases} \Rightarrow \begin{cases} x = 3 \\ y = 4 \end{cases}$$

2)  $\begin{cases} 5 \cdot 3^{x+1} - 3 \cdot 2^y = -1 \\ 3^{x+1} + 5 \cdot 2^{y-1} = 14 \end{cases} \quad \begin{cases} 3^x = a \\ 2^y = b \end{cases}$

$$\begin{cases} 5 \cdot \frac{a}{3} - 3b = -1 \\ 3a + 5 \cdot \frac{b}{2} = 14 \end{cases} \Rightarrow \begin{cases} 5a - 9b = -3 \\ 6a + 5b = 28 \end{cases} \Rightarrow$$

$$\begin{cases} a = \frac{9b-3}{5} \\ 6 \cdot \frac{9b-3}{5} + 5b = 28 \end{cases} \Rightarrow \begin{cases} 54b - 18 + 25b = 140 \\ 79b = 158 \\ b = 2 \text{ generik} \\ a = \frac{9 \cdot 2 - 3}{5} = 3 \end{cases}$$

$$\begin{cases} 3^x = 3 \\ 2^y = 2 \\ x = 1 \\ y = 1 \end{cases} \Rightarrow \begin{cases} x = 1 \\ y = 1 \end{cases}$$

~213

1)  $\begin{cases} 3^{2x} - 2^y = 725 \\ 3^x - 2^{\frac{y}{2}} = 25 \end{cases} \quad \begin{cases} 3^x = a \\ 2^{\frac{y}{2}} = b \end{cases}$

$$\begin{cases} a^2 - b^2 = 725 \\ a - b = 25 \end{cases} \Rightarrow \begin{cases} a = 25 + b \\ (25+b)^2 - b^2 = 725 \end{cases}$$

$$625 + 50b + b^2 - b^2 = 725$$

$$50b = 100$$

$$b = 2 \Rightarrow a = 25 + 2 = 27$$

$$\begin{cases} 3^x = 27 \\ 2^{\frac{y}{2}} = 2 \\ x = 3 \\ y = 2 \end{cases}$$

2)  $\begin{cases} 16^y - 4^x = 12 \\ 2^{x+1} - 4^y = 0 \end{cases} \Rightarrow \begin{cases} 4^{2y} - 2^{2x} = 12 \\ 2 \cdot 2^x - 4^y = 0 \end{cases} \quad \begin{cases} 4^y = a \\ 2^x = b \end{cases}$

$$\begin{cases} a^2 - b^2 = 12 \\ 2b - a = 0 \end{cases} \Rightarrow \begin{cases} a = 2b \\ 4b^2 - b^2 = 12 \\ 3b^2 = 12 \\ b^2 = 4 \\ b = \pm 2 \Rightarrow a = \pm 4 \end{cases}$$

$$\begin{cases} 2^x = 2 \\ 4^y = 4 \\ x = 1 \\ y = 1 \\ 2^x \neq -2 \\ 4^y \neq -4 \end{cases} \Rightarrow \begin{cases} x = 1 \\ y = 1 \end{cases}$$



~214

$$1) \begin{cases} 3^x > \frac{1}{27} \\ 3^x > 3^{-3} \\ x > -3 \end{cases}$$

$$2) \begin{cases} 2^x < \frac{1}{8} \\ 2^x < 2^{-3} \\ x < -3 \end{cases}$$

$$3) \begin{cases} \left(\frac{2}{5}\right)^{x+2} > \left(\frac{2}{5}\right)^{-1} \\ x+2 < -1 \\ x < -3 \end{cases}$$

$$4) \begin{cases} \left(\frac{1}{4}\right)^{x^2-x} < \frac{1}{16} \\ \left(\frac{1}{4}\right)^{x^2-x} < \left(\frac{1}{4}\right)^2 \\ x^2-x > 2 \\ x^2-x-2 > 0 \\ D = 1+8 = 9 \\ x_1 = \frac{1-3}{2} = -1 \\ x_2 = \frac{1+3}{2} = 2 \end{cases}$$

$$5) \begin{cases} \left(\frac{1}{5}\right)^{3-x} < 25 \\ 5^{-3+x} < 5^2 \\ x-3 < 2 \\ x < 5 \\ MC: (-\infty; 5) \end{cases}$$

$$6) \begin{cases} \left(\frac{1}{3}\right)^{x+2} < 9 \\ 3^{-x-2} < 3^2 \\ -x-2 < 2 \\ -x < 4 \\ x > -4 \\ MC: (-4; +\infty) \end{cases}$$

$$MC: (-\infty; -1) \cup (2; +\infty)$$

~215

$$1) \begin{cases} 5^{x-1} < 25 \\ 5^{x-1} < 5^2 \\ x-1 < 2 \\ x < 3 \\ MC: 2 \end{cases}$$

$$2) \begin{cases} 3^{3-x} \geq 9 \\ 3^{3-x} \geq 3^2 \\ 3-x \geq 2 \\ x \leq 1 \\ MC: 1 \end{cases}$$

$$3) \begin{cases} 6^{2x} \leq 6^{-2} \\ 2x \leq -2 \\ x \leq -1 \end{cases}$$

$$4) \begin{cases} \left(\frac{1}{2}\right)^{2x-2} \geq 4 \\ 2^{2-2x} \geq 2^2 \\ 2-2x \geq 2 \\ -2x \geq 0 \\ x \leq 0 \\ MC: 0 \end{cases}$$

$$5) \begin{cases} \left(\frac{1}{3}\right)^{5-3x} \leq 81 \\ 3^{3x-5} \leq 3^4 \\ 3x-5 \leq 4 \\ 3x \leq 9 \\ x \leq 3 \\ MC: 3 \end{cases}$$

$$6) \begin{cases} \left(\frac{1}{2}\right)^{2x-3} > \left(\frac{1}{2}\right)^2 \\ 2x-3 < 2 \\ 2x < 5 \\ x < 2,5 \\ MC: 2 \end{cases}$$

~216

$$1) \begin{cases} 5^x > 25 \\ \left(\frac{1}{3}\right)^{x-8} < \frac{1}{27} \end{cases} \Rightarrow \begin{cases} 5^x > 5^2 \\ \left(\frac{1}{3}\right)^{x-8} < \left(\frac{1}{3}\right)^3 \end{cases} \Rightarrow \begin{cases} x > 2 \\ x > 11 \\ MC: (11; +\infty) \end{cases}$$

$$2) \begin{cases} 8 > \left(\frac{1}{2}\right)^{6-x} \\ 3^{4x} > 81 \end{cases} \Rightarrow \begin{cases} 2^3 > 2^{x-6} \\ 3^{4x} > 3^4 \end{cases} \Rightarrow \begin{cases} x < 9 \\ x > 1 \\ MC: (1; 9) \end{cases}$$

~217

$$1) \begin{cases} 3^{-2x} < \sqrt{3} \\ 3^{-2x} < 3^{\frac{1}{2}} \\ -2x < \frac{1}{2} \\ x > -\frac{1}{4} \end{cases}$$

$$2) \begin{cases} \left(\frac{1}{5}\right)^{\frac{2x}{3}} > 25 \\ 5^{\frac{2x}{3}} > 5^2 \\ \frac{2x}{3} > 2 \\ x > 3 \end{cases}$$

$$3) \begin{cases} \left(\frac{1}{9}\right)^{-3x+1} > \sqrt{3} \\ 3^{6x-2} > 3^{\frac{1}{2}} \\ 6x-2 > \frac{1}{2} \\ x > \frac{5}{12} \end{cases}$$


$$4) \begin{cases} 2^{\frac{3x}{2}+3} < 16 \\ 2^{\frac{3x}{2}+3} < 2^4 \\ \frac{3x}{2}+3 < 4 \\ x < \frac{2}{3} \end{cases}$$


$$5) \begin{cases} 5^{\frac{x+1}{3}} \geq \frac{1}{\sqrt{5}} \\ 5^{\frac{x+1}{3}} \geq 5^{-\frac{1}{3}} \\ \frac{x+1}{3} \geq -\frac{1}{3} \\ x \geq -2 \end{cases}$$


$$6) \begin{cases} \left(\frac{2}{3}\right)^{\frac{1}{x-3}} > \frac{9}{4} \\ \left(\frac{2}{3}\right)^{\frac{1}{x-3}} > \left(\frac{2}{3}\right)^{-2} \\ \frac{1}{x-3} > -2 \Rightarrow (-\infty; 0) \cup (0; 4) \end{cases} \quad \begin{matrix} x \neq 0 \\ x < 4 \end{matrix}$$



~218

1)  $\left(\frac{3}{7}\right)^{x^2} > \left(\frac{9}{49}\right)^{x+1,5}$   
 $\left(\frac{3}{7}\right)^{x^2} > \left(\frac{3}{7}\right)^{2x+3}$   
 $x^2 - 2x - 3 < 0$   
 $D = 4 + 4 \cdot 3 = 16$   
 $x_1 = \frac{2-4}{2} = -2$   
 $x_2 = \frac{2+4}{2} = 3$   
  
 $M: (-2; 3)$

2)  $\left(\frac{2}{3}\right)^{x^2+4x} \geq \left(\frac{8}{27}\right)^{x+2}$   
 $\left(\frac{2}{3}\right)^{x^2+4x} \geq \left(\frac{2}{3}\right)^{3x+6}$   
 $x^2 + 4x \leq 3x + 6$   
 $x^2 + x - 6 \leq 0$   
 $x_1 = \frac{-1-5}{2} = -3$   
 $x_2 = \frac{-1+5}{2} = 2$   
  
 $M: [-3; 2]$

3)  $\left(\frac{1}{27}\right)^{x^2+1} > \left(\frac{1}{9}\right)^{-x^2+8x}$   
 $\left(\frac{1}{3}\right)^{3x^2+3} > \left(\frac{1}{3}\right)^{-2x^2+8x}$   
 $3x^2 + 3 < -2x^2 - 8x$   
 $5x^2 + 8x + 3 < 0$   
 $D = -4 \cdot 5 \cdot 3 = -14^2$   
 $x_1 = \frac{-8-14}{10} = -\frac{22}{10} = -\frac{11}{5}$   
 $x_2 = \frac{-8+14}{10} = \frac{6}{10} = \frac{3}{5}$   
  
 $M: \left[-\frac{11}{5}; \frac{3}{5}\right]$

4)  $\left(\frac{6x-1}{3-x}\right)^2 < \left(\frac{1}{8}\right)^2$   
 $\frac{6x-1}{3-x} > 2 \quad x \neq 3$   
 $6x-1 > 6-2x$   
 $8x > 7$   
 $x > \frac{7}{8}$   
 $M: \left(\frac{7}{8}; 3\right)$

5)  $\left(\frac{1}{7}\right)^{\frac{x}{x-4}} > 49$   
 $7^{\frac{x}{x-4}} > 7^2$   
 $\frac{x}{x-4} > 2 \quad x \neq 4$   
 $x > 2x - 8$   
 $x < 8$   
 $M: (4; 8)$

6)  $\left(\frac{1}{2}\right)^{\frac{x-1}{x+2}} \leq 4$   
 $2^{\frac{1-x}{x+2}} \leq 2^2$   
 $\frac{1-x}{x+2} \leq 2 \quad x \neq -2$   
 $1-x \leq 2x+4$   
 $x \geq -1$   
 $M: (-2; -1]$

~219


1)  $2 < \sqrt[3]{2}$   
 $3x < \frac{1}{5}$   
 $x < \frac{1}{15}$   
 $M: 0$


2)  $\left(\frac{1}{8}\right)^{\frac{x+1}{2}} > 4$   
 $\frac{-3x-3}{2} > 2$   
 $\frac{-3x-3}{2} > 2$   
 $-3x-3 > 4$   
 $x < -\frac{7}{3}$   
 $M: -3$

3)  $\left(\frac{1}{49}\right)^{\frac{x}{2}} \leq 7$   
 $7^x \leq 7$   
 $x \leq 1$   
 $M: 1$

4)  $3^{\frac{2x+1}{5}} < \sqrt[3]{3}$   
 $\frac{2x+1}{5} < \frac{1}{3}$   
 $\frac{2x+1}{5} < -\frac{1}{3}$   
 $2x+1 < -\frac{5}{3}$   
 $2x < -\frac{8}{3}$   
 $x < -\frac{4}{3} \quad M: -2$

~220

1)  $5^{2x+1} - 5^{x+2} \leq 5^x - 5 \quad 5^x = a$   
 $5a^2 - 25a \leq a - 5$   
 $5a^2 - 26a + 5 \leq 0$   
 $D = 676 - 4 \cdot 25 = 576$   
 $a_1 = \frac{26-24}{2 \cdot 5} = \frac{1}{5} \quad 5^x = \frac{1}{5} \quad x_1 = -1$   
 $a_2 = \frac{26+24}{2 \cdot 5} = 5 \quad 5^x = 5 \quad x_2 = 1$   
  
 $M: 1$

2)  $2^{2x} - 3 \cdot 2^x + 2 \leq 0$   
 $D = 9 - 4 \cdot 2 = 1$   
 $2^x = \frac{3-1}{2} = 1 \Rightarrow x_1 = 0$   
 $2^x = \frac{3+1}{2} = 2 \Rightarrow x_2 = 1$   
  
 $M: 1$

3)  $250 \cdot 5^{3-x} - 2 \cdot 5^{x-3} > 0 \quad 5^{x-3} = a$   
 $\frac{250}{a} - 2a > 0 \quad a \neq 0$   
 $250 - 2a^2 > 0$   
 $-2a^2 > -250$   
 $a < \pm 5\sqrt{5}$   
 $5^{x-3} < 5\sqrt{5}$   
 $x-3 < \frac{3}{2}$   
 $x < 4,5 \quad M: 4$

4)  $147 \cdot 7^{x-2} - 3 \cdot 7^{2-x} \leq 0 \quad 7^{x-2} = a$   
 $147a - \frac{3}{a} \leq 0 \quad a \neq 0$   
 $147a^2 - 3 \leq 0$   
 $a \leq \pm \frac{1}{7}$   
 $7^{x-2} \leq \frac{1}{7}$   
 $x-2 \leq -1$   
 $x \leq 1 \quad M: 1$

~224

1)  $2^{x+3} + 3 \cdot 5^x < 3 \cdot 2^x + 5^{x+1}$   
 $8 \cdot 2^x - 3 \cdot 2^x < 5 \cdot 5^x - 3 \cdot 5^x$   
 $5 \cdot 2^x < 2 \cdot 5^x$   
 $2^{x-1} < 5^{x-1}$   
 $\left(\frac{2}{5}\right)^{x-1} < 1$   
 $x-1 > 0$   
 $x > 1$   
 MC: 2

2)  $2^{2x+1} - 3^{2x+1} < 3^{2x} - 7 \cdot 2^{2x}$   
 $2 \cdot 2^{2x} + 7 \cdot 2^{2x} < 3^{2x} + 3 \cdot 3^{2x}$   
 $3^2 \cdot 2^{2x} < 2^2 \cdot 3^{2x}$   
 $2^{2x-2} < 3^{2x-2}$   
 $\left(\frac{2}{3}\right)^{2x-2} < 1$   
 $2x-2 > 0$   
 $x > 1$   
 MC: 2

3)  $5^{x+1} - 3^{x+2} > 2 \cdot 5^x - 2 \cdot 3^{x-1}$   
 $5 \cdot 5^x - 2 \cdot 5^x > 9 \cdot 3^x - 2 \cdot \frac{1}{3} \cdot 3^x$   
 $3 \cdot 5^x > \frac{25}{3} \cdot 3^x$   
 $5^{x-2} > 3^{x-2}$   
 $\left(\frac{5}{3}\right)^{x-2} > 1$   
 $x-2 > 0$   
 $x > 2$   
 MC: 3

4)  $3^x + 10^{x-2} > 19 \cdot 3^{x-2} + 10^{x-3}$   
 $3^x - \frac{19 \cdot 3^x}{3^2} > \frac{10^x}{10^3} - \frac{10^x}{10^2}$   
 $\frac{10^x \cdot 3^2}{3^2} > \frac{-9 \cdot 10^x}{10^3}$   
 $10^x \cdot 3^x < 3^4 \cdot 10^x$   
 $\left(\frac{3}{10}\right)^{x-4} < 1$   
 $x > 4$   
 MC: 5

~225


1)  $2^{\sqrt{x+1}} - 1 < 3 \cdot 2^{2-\sqrt{x+1}}$   $2^{\sqrt{x+1}} = t$   
 $t - \frac{3 \cdot 4}{t} - 1 < 0$   $x \geq 1$   
 $t^2 - t - 12 < 0$   
 $D = 1 + 4 \cdot 12 = 49$   
 $t_1 = \frac{1-7}{2} = -3$   $2^{\sqrt{x+1}} \neq -3$   
 $t_2 = \frac{1+7}{2} = 4$   $2^{\sqrt{x+1}} = 2^2$   
 $x = 3$   
 MC: [1; 3)


2)  $2 \cdot 3^{\sqrt{x+1}} - 5 > 3^{1-\sqrt{x+1}}$   $3^{\sqrt{x+1}} = t$   
 $2t - \frac{3}{t} - 5 > 0$   $x > 1$   
 $2t^2 - 5t - 3 > 0$   
 $D = 25 + 24 = 49$   
 $t_1 = \frac{5-7}{4} = -\frac{1}{2}$   
 $t_2 = \frac{5+7}{4} = 3$   $3^{\sqrt{x+1}} = 3$   
 $x = 0$   
 MC: [1; +∞)

3)  $5^{\sqrt{x-2}} > 5^{1-\sqrt{x-2}} + 4$   $5^{\sqrt{x-2}} = t$   
 $t - \frac{5}{t} - 4 > 0$   $x > 2$   
 $t^2 - 4t - 5 > 0$   
 $D = 16 + 20 = 36$   
 $t_1 = \frac{4-6}{2} = -1$   $5^{\sqrt{x-2}} \neq -1$   
 $t_2 = \frac{4+6}{2} = 5$   $5^{\sqrt{x-2}} = 5$   
 $x = 3$   
 MC: (3; +∞)

4)  $2 \cdot 7^{\sqrt{2x-5}} > 7^{1-\sqrt{2x-5}} + 13$   $7^{\sqrt{2x-5}} = t$   
 $2t - 13 - \frac{7}{t} > 0$   $x \geq 2,5$   
 $2t^2 - 13t - 7 > 0$   
 $D = 169 + 56 = 225$   
 $t_1 = \frac{13-15}{4} = -\frac{1}{2}$   
 $t_2 = \frac{13+15}{4} = 7$   $7^{\sqrt{2x-5}} = 7$   
 $x = 3$   
 MC: (3; +∞)

~226

1)  $(x-3)^{x^2-9} > 1$   
 $x^2 > 9$   
 $x > \pm 3$   
  
 MC: (-∞; 3) ∪ (3; +∞)

2)  $(x-2)^{x^2-1} > 1$   
 $x^2 - 1 > 0$   
 $x = \pm 1$   
  
 MC:

3)  $(x-1)^{\frac{2x-4}{x+1}} \geq 1$   
 $\frac{2x-4}{x+1} \geq 0$   
 $2x-4=0$   $x+1 \neq 0$   
 $x = 2$   $x \neq -1$



~ 227

$$1) \begin{cases} 2^{x+2} - 0,75 \cdot 2^{x+2} > 1 \\ 0,2^x \leq 0,04^{x^2} \end{cases} \begin{cases} 4 \cdot 2^x - 3 \cdot 2^x > 1 \\ 0,2^x \leq 0,2^{2x^2} \end{cases}$$

$$\begin{cases} 2^x > 1 \\ x \geq 2x^2 \end{cases} \begin{cases} x > 0 \\ 2x^2 - x \leq 0 \end{cases}$$

$x_1 = 0 \quad x_2 = 0,5$

MC:  $(0; 0,5]$

$$2) \begin{cases} (x-2)^{2x^2-11x+9} < 1 \\ \sqrt{5x^2-3x+2} > \sqrt{x} \end{cases}$$

$$\begin{cases} (0,3)^{4x^2-3x+2} > (0,3)^x \\ 4x^2-3x+2-x > 0 \\ 4x^2-4x+2 > 0 \end{cases}$$

$\Delta = 16 - 4 \cdot 4 \cdot 2 = -16 < 0$  gener  $\emptyset$

MC:  $\emptyset$

~ 228

1)  $\log_3 1 = 0$    2)  $\log_3 9 = 2$    3)  $\log_3 81 = 4$    4)  $\log_3 243 = 4$    5)  $\log_3 \frac{1}{3} = -1$

~ 229

1)  $\log_2 16 = 4$    2)  $\log_{0,2} 0,04 = 2$    3)  $\log_3 \frac{1}{81} = -4$

4)  $\log_{\frac{1}{3}} 9 = -2$    5)  $\log_{23} 1 = 0$    6)  $\log_5 \frac{1}{125} = -3$

~ 230

1)  $\log_5 22 - \log_5 11 - \log_5 10 =$   
 $= \log_5 \frac{22}{11} - \log_5 10 = \log_5 \frac{1}{5} = -1$

2)  $\log_2 7 - \log_2 63 + \log_2 36 =$   
 $= \log_2 \frac{7}{63} + \log_2 36 = \log_2 \left( \frac{1}{9} \cdot 36 \right) = 2$

3)  $\log_3 8 - \log_3 4 + \log_3 \frac{9}{2} =$   
 $= \log_3 \frac{8}{4} + \log_3 \frac{9}{2} = \log_3 \left( 2 \cdot \frac{9}{2} \right) = 2$

4)  $\log_7 64 - \log_7 256 + \log_7 28 =$   
 $\log_7 \frac{64}{256} + \log_7 28 = \log_7 \left( \frac{1}{4} \cdot 28 \right) = 1$

~ 231

1)  $3^6 = 729 \quad \log_3 729 = 6$    2)  $4^5 = 1024 \quad \log_4 1024 = 5$    3)  $10^4 = 10000 \quad \log_{10} 10000 = 4$

4)  $\left(\frac{1}{2}\right)^5 = \frac{1}{32} \quad \log_{\frac{1}{2}} \frac{1}{32} = 5$    5)  $\left(\frac{2}{3}\right)^3 = \frac{8}{27} \quad \log_{\frac{2}{3}} \frac{8}{27} = 3$    6)  $10^{-3} = 0,001 \quad \log_{10} 0,001 = -3$

~ 232

1)  $\log_2 64 = 6 \quad 2^6 = 64$    2)  $\log_3 81 = 4 \quad 3^4 = 81$    3)  $\log_5 125 = 3 \quad 5^3 = 125$

4)  $\lg_{10} 100000 = 5 \quad 10^5 = 100000$    5)  $\lg 0,01 = -2 \quad 10^{-2} = 0,01$    6)  $\log_{\frac{2}{3}} \frac{27}{64} = 3 \quad \left(\frac{3}{4}\right)^3 = \frac{27}{64}$

~ 233

1)  $\lg 100 = 2$    2)  $\lg 0,001 = -3$    3)  $\lg 10^2 = 2$    4)  $\lg \sqrt{10} = 0,5$    5)  $\lg \sqrt{10^2} = \frac{2}{2}$

~ 234

1)  $\lg 10000 = 4$    2)  $\lg 0,1 = -1$    3)  $\lg 0,0001 = -5$    6)  $\lg \sqrt{10} = \frac{1}{2}$

~ 235

1)  $\ln e = 1$    2)  $\ln e^{\frac{1}{3}} = \frac{1}{3}$

3)  $\ln \sqrt{e} = \frac{1}{2}$    4)  $\ln(\lg 10) = \ln 1 = 0$

~236

1)  $\lg 5 \approx 0,699 \Rightarrow \lg \frac{1}{5} = \lg 1 - \lg 5 = -0,699$ ;  $\lg 0,05 = \lg 5 - \lg 100 = 0,699 - 2 = -1,301$   
 $-\lg 0,005 = -(\lg 5 - \lg 1000) = -(0,699 - 3) = 2,301$

2)  $\lg 29 \approx 1,462 \Rightarrow \lg 29000 = \lg 29 + \lg 1000 = 1,462 + 3 = 3,462$   
 $\lg 2,9 = \lg \frac{29}{10} = \lg 29 - \lg 10 = 1,462 - 1 = 0,462$   $\lg 0,29 = \lg 29 - \lg 100 = 1,462 - 2 = -0,538$

~237

1)  $\log_{\frac{1}{5}} 9 + 2 \log_{\frac{1}{5}} \frac{5}{3} = \log_{\frac{1}{5}} (9 \cdot \frac{25}{9}) = \log_{\frac{1}{5}} 25 = -2$       2)  $\log_3 8 + 3 \log_3 \frac{3}{2} = \log_3 (8 \cdot \frac{27}{8}) = \log_3 27 = 3$

3)  $\log_7 196 - 2 \log_7 2 = \log_7 (\frac{196}{4}) = \log_7 49 = 2$       4)  $\log_2 \sqrt{3} + \frac{1}{2} \log_2 \frac{4}{3} = \log_2 (\sqrt{3} \cdot \frac{2}{\sqrt{3}}) = 1$

~238

1)  $\lg(a^2 \cdot b^3) = \lg a^2 + \lg b^3 = 2 \lg a + 3 \lg b$       2)  $\lg(5a^2 x^2) = \lg 5 + 2 \lg a + 2 \lg x$

3)  $\lg(mn)^3 = 3 \lg mn = 3(\lg m + \lg n)$       4)  $\lg \sqrt[3]{7a^3 b} = \frac{1}{3} \lg 7 + \lg a + \frac{1}{3} \lg b$

5)  $\lg(4 \cdot \sqrt[5]{2a^8 b^3}) = \lg 4 + \frac{1}{5}(\lg 2 + \lg a + 3 \lg b)$       6)  $\lg(7a^8 b^3 \sqrt{c}) = \lg 7 + 8 \lg a + \lg b + \frac{1}{2} \lg c$

~239

1)  $x = \log_3 27$   
 $27 = 3^x$   
 $3^3 = 3^x$   
 $x = 3$

2)  $y = \log_2 16$   
 $16 = 2^y$   
 $2^4 = 2^y$   
 $y = 4$

3)  $z = \log_5 625$   
 $625 = 5^z$   
 $5^4 = 5^z$   
 $z = 4$

4)  $x = \log_2 0,125$   
 $0,125 = 2^x$   
 $2^{-3} = 2^x$   
 $x = -3$

5)  $\log_2 y = 2$   
 $y = 2^2$   
 $y = 4$

6)  $\log_{\frac{1}{2}} z = -3$   
 $z = (\frac{1}{2})^{-3}$   
 $z = 8$

~240

1)  $\frac{25^{\log_5 2} + 1}{49^{\log_7 4}} = \frac{5^{2 \log_5 2} + 1}{7^{2 \log_7 4}} = \frac{4 + 1}{16} = \frac{5}{16}$       2)  $\frac{16^{0,5 \log_4 10}}{10^{\log_4 1} + 1} = \frac{4^{\log_4 10}}{4 + 1} = \frac{10}{5} = 2$

3)  $\frac{25^{-\log_5 2} + 7^{-\log_7 3}}{5} = \frac{2(\log_5 25 - \log_5 2)}{5} + 7^{\log_7 3^{-1}} = \frac{2 \log_5 \frac{25}{2}}{5} + 3^{-1} = \frac{2 \log_5 \frac{625}{4}}{5} + \frac{1}{3} = \frac{625}{4} + \frac{1}{3} = \frac{1875}{12} + \frac{4}{12} = \frac{1879}{12}$

4)  $\log_4 \frac{1}{5} + \log_4 36 + \frac{1}{2} \log_4 \frac{25}{81} = \log_4 (\frac{1}{5} \cdot 36) + \log_4 (\frac{25}{81})^{\frac{1}{2}} = \log_4 (\frac{36}{5} \cdot \frac{5}{9}) = \log_4 4 = 1$

~241

1)  $\log_2 12 + \log_2 \frac{5}{3} + \log_2 \frac{4}{5} = \log_2 16 = 4$

2)  $(\log_5 128) \cdot (\log_2 \frac{1}{125}) = (\log_5 \frac{1}{125}) \cdot (\log_2 128) =$

3)  $3^{2 - \log_3 5} + (\frac{1}{3})^{\log_3 5} = 3^{\log_3 9 - \log_3 5} + 3^{-\log_3 5} = 3^{\log_3 \frac{9}{5}} + 3^{-\log_3 5} = 3 \log_3 \frac{9}{5} + 3 \log_3 \frac{1}{5} = \frac{9}{5} + \frac{1}{5} = \frac{10}{5} = 2$

4)  $9^{3 - \log_3 54} + 7^{-\log_7 5} = 3^{2(\log_3 27 - \log_3 54)} + 7^{\log_7 3^{-1}} = 3 \log_3 \frac{1}{2} + 7 \log_7 \frac{1}{3} = \frac{1}{4} + \frac{1}{3} = \frac{7}{12}$

~242

1)  $9^{\log_{\frac{1}{3}} (\frac{2}{3})} < \sqrt{5}$        $9^{\log_{\frac{1}{3}} (\frac{2}{3})} = 9^{-\log_3 (\frac{2}{3})} = \frac{3}{2}$

2)  $\sqrt[3]{3} > \frac{1}{36} \log_6 2$        $6^{-2 \log_6 2} = 6^{\log_6 2^{-2}} = \frac{1}{4}$

~243

1)  $\log_x 36 = 0,5$   
 $x^{\frac{1}{2}} = 36$   
 $x = 1296$

2)  $\log_x 27 = \frac{3}{2}$   
 $x^{\frac{3}{2}} = 27$   
 $x = 27^{\frac{2}{3}}$   
 $x = 9$

3)  $\log_x 64 = 1,2$   
 $x^{1,2} = 64$   
 $x = 64^{\frac{5}{6}}$   
 $x = 32$

4)  $\log_x 2 = -0,5$   
 $x^{-0,5} = 2$   
 $x = 2$   
 $x = 0,25$

~244

$\lg 2 = a; \lg 3 = b; \lg(4) = 2a; \lg 6 = a+b; \lg 8 = 3a; \lg 9 = 2b; \lg 10 = 1;$   
 $\lg 5 = 1-a; \lg 12 = 2a+b; \lg 50 = 2-a; \lg 72 = 3a; \lg 50 = 2b+1; \lg 96 = 5a+b$

~245

1)  $\lg a - \lg b = 1$   
 $\lg \frac{a}{b} = 1$   
 $\frac{a}{b} = 10^1$   
 $\frac{a}{b} = 10$

2)  $\lg a - \lg b = 2$   
 $\lg \frac{a}{b} = 2$   
 $\frac{a}{b} = 10^2$   
 $\frac{a}{b} = 100$

3)  $\lg a - \lg b = 3$   
 $\lg \frac{a}{b} = 3$   
 $\frac{a}{b} = 10^3$   
 $\frac{a}{b} = 1000$

~246

1)  $\lg 2 = a$ , onga  $\lg 25 = \lg \frac{100}{4} = \lg 100 - \lg 4 = 2 - (\lg 2 + \lg 2) = 2 - 2a$

2)  $\lg 5 = a$  onle  $\lg 2 = c$ , onga  $\log_{50} 8 = \frac{\lg 8}{\lg 50} = \frac{3 \lg 2}{\lg 10 + \lg 5} = \frac{3c}{1+a}$

~247

1)  $343^{\frac{2 \log_{49} 2}{7}} = 7^{\frac{3 \cdot 2 \log_{49} 2}{7}} = 7^{\frac{3 \log_{49} 2}{7}} = 8$

2)  $4^{\frac{2 \log_{32} 10}{4}} = 2^{\frac{4 \log_{25} 10}{2}} = 2^{\frac{4}{3} \log_2 10} = \sqrt[3]{10^4}$

3)  $9^{\frac{\log_{27} \sqrt{5}}{3}} = 3^{\frac{2 \log_{27} \sqrt{5}}{3}} = 3^{\frac{2}{3} \log_3 \sqrt{5}} = \sqrt[3]{5}$

4)  $\sqrt{5}^{\frac{2 \log_5 3}{5}} = 5^{\frac{\log_5 3}{5}} = 3$

5)  $\left(\frac{1}{27}\right)^{\frac{\log_3 4}{3}} = 3^{\frac{2}{3} \log_3 4} = \sqrt[3]{4^3} = 8$

6)  $4^{\frac{\log_8 125}{4}} = 2^{\frac{2}{3} \log_2 125} = \sqrt[3]{125^2} = 25$

~248

1)  $\log_2 \log_5 \sqrt{5} = \log_2 \frac{1}{8} = -3$

2)  $\log_3^2 \log_{\frac{1}{3}} \frac{1}{125} = 2 \log_3 3 = 2 \cdot 1 = 2$

3)  $\log_4 \log_3 \sqrt{81} = \log_4 \log_3 9 = \log_4 2 = 0,5$

4)  $\log_{\sqrt{5}} \log_{\frac{1}{5}} \frac{1}{125} = \log_{\sqrt{5}} \log_5 125 = \log_{\sqrt{5}} 3 = 2$

5)  $\log_{\frac{8}{27}} \log_{25} 125 = \log_{\frac{8}{27}} (\log_{25} 25 + \log_{25} 5) = \log_{\frac{8}{27}} (1 + \frac{1}{2}) = \log_{\frac{8}{27}} \frac{3}{2} = -\log_{\frac{8}{27}} \frac{2}{3} = -\frac{1}{3}$

~249

1)  $\log_2 7 = b$   
 $2^b = 7$   
 $2^2 < 2^b < 2^3$   
 on: (2; 3)

2)  $\log_2 30 = b$   
 $2^b = 30$   
 $2^4 < 2^b < 2^5$   
 on: (4; 5)

3)  $\log_2 120 = b$   
 $2^b = 120$   
 $2^6 < 2^b < 2^7$   
 on: (6; 7)

4)  $\log_2 495 = b$   
 $2^b = 495$   
 $2^8 < 2^b < 2^9$   
 on: (8; 9)

5)  $\log_3 3 = b$   
 $10^b = 3$   
 $10^0 < 10^b < 10^1$   
 on: (0; 1)

6)  $\log_3 18 = b$   
 $10^b = 18$   
 $10 < 10^b < 10^2$   
 on: (1; 2)

7)  $\log_3 134 = b$   
 $10^b = 134$   
 $10^2 < 10^b < 10^3$   
 on: (2; 3)

8)  $\log_3 1782 = b$   
 $10^b = 1782$   
 $10^3 < 10^b < 10^4$   
 on: (3; 4)

~250

1)  $\lg 0,07 = b$   
 $10^b = 0,07$   
 $10^{-3} < 10^b < 10^{-2}$   
 mc: (-3; -2)

$\lg 0,018 = b$   
 $10^b = 0,018$   
 $10^{-3} < 10^b < 10^{-2}$   
 mc: (-3; 2)

$\lg 0,00125 = b$   
 $10^b = 0,00125$   
 $10^{-4} < 10^b < 10^{-3}$   
 mc: (-4; 3)

$\lg 0,00005 = b$   
 $10^b = 0,00005$   
 $10^{-6} < 10^b < 10^{-5}$   
 mc: (-6; -5)

2)  $\log_2 \frac{1}{15} = b$   
 $2^b = \frac{1}{15}$   
 $2^{-4} < 2^b < 2^{-3}$   
 mc: (-4; -3)

$\log_2 \frac{3}{80} = b$   
 $2^b = \frac{3}{80}$   
 $2^{-7} < 2^b < 2^{-6}$   
 mc: (-7; -6)

$\log_2 \frac{1}{120} = b$   
 $2^b = \frac{1}{120}$   
 $2^{-7} < 2^b < 2^{-6}$   
 mc: (-7; -6)

~251

1)  $\log_2 \sqrt[5]{8} = x$   
 $2^x = \sqrt[5]{8}$   
 $x = \frac{3}{5}$

$\log_4 \sqrt[5]{8} = x$   
 $(1) \sqrt[5]{8} = 2^{\frac{3}{5}}$   
 $(2) 4^x = 2^{\frac{3}{5}}$   
 $x = \frac{1}{3}$

$\log_{16} \sqrt[5]{8} = x$   
 $4^{2x} = \sqrt[5]{8}$   
 $2^{2x} = 2^{\frac{3}{5}}$   
 $x = \frac{3}{10}$

$\log_{16} \sqrt[5]{8} = x$   
 $16^{2x} = \sqrt[5]{8}$   
 $2^{4x} = 2^{\frac{3}{5}}$   
 $x = \frac{3}{20}$

$\log_{64} \sqrt[5]{8} = x$   
 $64^{2x} = \sqrt[5]{8}$   
 $8^{2x} = 8^{\frac{1}{5}}$   
 $x = \frac{1}{10}$

2)  $\log_x \sqrt[3]{27} = \frac{3}{2}$   
 $x^{\frac{3}{2}} = 27^{\frac{1}{2}}$   
 $x = \sqrt[3]{27}$   
 $x = 3$

$\log_x \sqrt[3]{27} = \frac{2}{3}$   
 $x^{\frac{2}{3}} = 27^{\frac{1}{3}}$   
 $x = \sqrt[3]{27^3}$

$\log_x \sqrt[3]{27} = -\frac{1}{2}$   
 $x^{-\frac{1}{2}} = 27^{\frac{1}{3}}$   
 $x = \frac{1}{27}$

$\log_x \sqrt[3]{27} = -\frac{3}{4}$   
 $x^{-\frac{3}{4}} = 27^{\frac{1}{3}}$   
 $x = \frac{1}{9}$

~252

$b_1; b_1 q; b_1 q^2; b_1 q^3; b_1 q^4; \dots b_1 q^{n-1}$

$\log_c b_1 q^{n-1} = \log_c b_1 + \log_c q^{n-1} = \log_c b_1 + (n-1) \log_c q$

$\log_c b_1; \log_c b_1 + \log_c q; \log_c b_1 + 2 \log_c q; \dots \log_c b_1 + (n-1) \log_c q$  ( $a_n = a_1 + (n-1)d$ )

$a_2 - a_1 = d$   
 $a_3 - a_2 = d$

$\log_c b_1 + \log_c q - \log_c b_1 = \log_c q$  w  
 $\log_c b_1 + 2 \log_c q - \log_c b_1 - \log_c q = \log_c q$  w

Математик:  $\log_c b_1 q^{n-1} = \log_c b_1 + (n-1) \log_c q$

~253

1)  $0,25(1+4^{\log_2 5}) \log_2 5^4 = 0,25(1+2^{\log_2 5}) \log_2 5^4 = 1$

2)  $81^{\log_3 2 - 0,25 \log_3 2} = 81^{\log_3 2 - \log_3 \sqrt{2}} = 81^{\log_3 \frac{2}{\sqrt{2}}} = 9^{2 \log_3 \sqrt{2}} = 9^{\log_3 2} = 2$

3)  $\frac{\log_2^2 14 + (\log_2 14)(\log_7 14) - 2 \log_2^2 7}{\log_2 14 - \log_2 56}$   $\log_2 14 = a$   
 $\log_2 56 = \log_2 (14 \cdot 4) = \log_2 14 + \log_2 4 = a + 2$   
 $\frac{a^2 + a(a+2) - 2a^2}{a - (a+2)} = \frac{a^2 + a^2 + 2a - 2a^2}{a - a - 2} = \frac{2a}{-2} = -a = -\log_2 14 = -(\log_2 7 + 1)$

4)  $\frac{\log_5^2 7\sqrt{5} + 2 \log_5^2 7 - 3(\log_5 7\sqrt{5})(\log_5 7)}{\log_5 7\sqrt{5} - \log_5 49}$   $\log_5 7 = x$   
 $\log_5 7\sqrt{5} = \log_5 7 + \log_5 \sqrt{5} = x + \frac{1}{2}$   
 $\frac{(x+\frac{1}{2})^2 + 2x^2 - 3x(x+\frac{1}{2})}{x+\frac{1}{2} - x - x} = \frac{x^2 + x + \frac{1}{4} + 2x^2 - 3x^2 - \frac{3}{2}x}{\frac{1}{2} - x} = \frac{-x^2 - \frac{1}{2}x + \frac{1}{4}}{\frac{1}{2} - x} = \frac{1}{2}$

$$5) \frac{\log_4^2 12 + 3 \log_4^2 \frac{1}{3} + 4(\log_4 12)(\log_4 \frac{1}{3})}{\log_4 12 + 3 \log_4 \frac{1}{3}}$$

$$\log_4 \frac{1}{3} = -\log_4 3 = a \Rightarrow \log_4 \frac{1}{3} = -a$$

$$\log_4 12 = \log_4(4 \cdot 3) = \log_4 4 + \log_4 3 = 1 - a$$

$$\frac{(1-a)^2 + 3a^2 + 4(1-a)(-a)}{1-a-3a} = \frac{1-2a+a^2-4a+4a^2+3a^2-8a^2-6a+1}{1-4a} = \frac{1-4a-1-4a}{1-4a} = \frac{-8a}{1-4a} = 1-2a = 1+2 \log_4 \frac{1}{3} = 1 - \log_4 9$$

~254

$$1) 27^{\log_5 \sqrt[6]{3}} + 4 \cdot 5^{\log_{0,04} 9} - 2^{\log_8 125} \cdot \log_{32} 16 = 27^{\frac{1}{6} \log_5 3} + 4 \cdot 5^{\log_{\frac{1}{25}} 9} - 2^{\frac{1}{3} \log_2 125} \cdot \log_5 2^4 =$$

$$= 3^{\frac{1}{3} + 4 \cdot 5^{-\frac{1}{2} \log_5 9}} - 2^{\frac{1}{3} \log_2 125} \cdot \frac{4}{5} \log_2 2 = 3^{\frac{1}{3} + \frac{4}{5}} - 2^{\frac{1}{3} \log_2 125} \cdot \frac{4}{5} = 3^{\frac{1}{3} + \frac{4}{5}} - 2^{\frac{1}{3} \log_2 125} \cdot \frac{4}{5} = 3^{\frac{13}{15}} - \frac{8}{5}$$

$$2) 4^{\frac{2}{\log_2 7} \cdot 4} \cdot \log_4 6 + 4 \cdot 6^{\frac{1}{\log_4 6}} + (3^5)^{\log_3 27} = 7^{\frac{\log_2 4}{\log_2 7}} \cdot 6 + 4 \cdot 6^{\frac{\log_4 6}{\log_4 6}} + (3^5)^{\log_3 27} = 4^{\frac{2}{\log_2 7}} \cdot 6 + 4 \cdot 6 + 6^{\log_4 6} \cdot 4 + 5 = 45$$

$$3) (3^{\frac{2}{\log_5 3} - 3} + 0,008^{\frac{1}{\log_5 3}})^{\frac{1}{2}} = (3^{\frac{2}{\log_5 3} - 3} + 0,008^{\frac{1}{\log_5 3}})^{\frac{1}{2}} = (5^{\frac{2}{\log_5 3} - 3} + 0,008^{\frac{1}{\log_5 3}})^{\frac{1}{2}} = 5$$

$$4) (3^{\frac{2}{\log_5 2} - 4} \log_5^2 2)^{-3} \frac{1}{\log_5 3}$$

~255

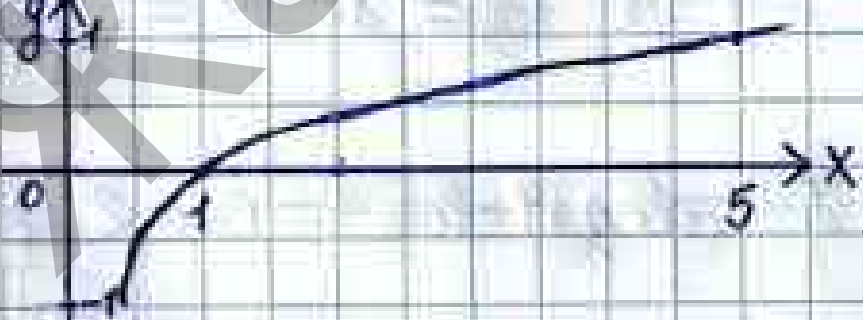
$$1) (\log_3 2 + \log_2 8 + 4)(\log_3 2 - 2 \log_8 2) \log_2 3 - \log_3 2 = \left(\frac{1}{\log_2 3} + 4 \log_2 3 + 4\right) \left(\frac{1}{\log_2 3} - \frac{2}{2 \log_2 3 + 1}\right) \cdot \log_2 3 - \frac{1}{\log_3 2} = \left(\frac{1}{a} + 4a + 4\right) \left(\frac{1}{a} - \frac{2}{2a+1}\right) \cdot a - \frac{1}{a} = \frac{(2a+1)^2}{a} \cdot \frac{1}{(2a+1)a} \cdot a - \frac{1}{a} = 2$$

$$2) (\log_2 7 + \log_7 16 + 4)(\log_2 7 - 2 \log_{28} 7) \cdot \log_7 2 - \log_2 7 = \left(\frac{1}{\log_7 2} + 4 \log_7 2 + 4\right) \left(\frac{1}{\log_7 2} - \frac{2}{2 \log_7 2 + 1}\right) \cdot \log_7 2 - \frac{1}{\log_2 7} = \left(\frac{1}{a} + 4a + 4\right) \left(\frac{1}{a} - \frac{2}{2a+1}\right) \cdot a - \frac{1}{a} = \frac{(2a+1)^2}{a} \cdot \frac{1}{(2a+1)a} \cdot a - \frac{1}{a} = 2$$

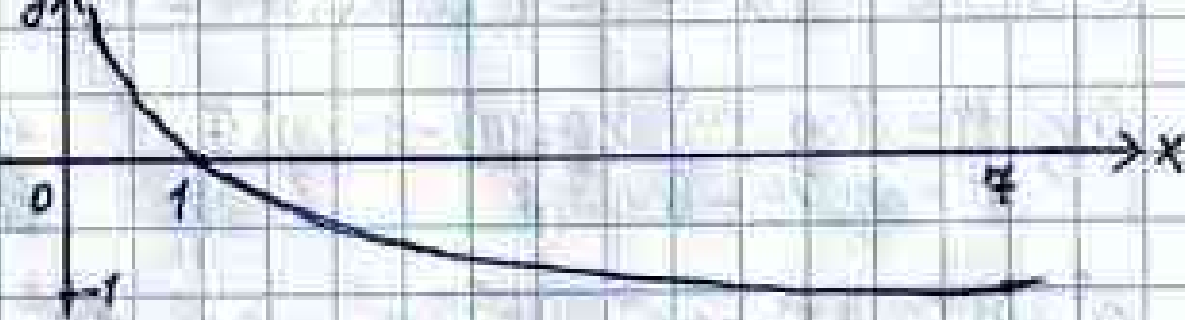
$$3) (\log_8 3 + \log_3 1296 + 4)(\log_8 3 - \log_{108} 9) \cdot \log_8 6 - \log_8 3 = \left(\frac{1}{\log_8 6} + 4 \log_8 6 + 4\right) \left(\frac{1}{\log_8 6} - \frac{2}{2 \log_8 6 + 1}\right) \cdot \log_8 6 - \frac{1}{\log_8 3} = \left(\frac{1}{a} + 4a + 4\right) \left(\frac{1}{a} - \frac{2}{2a+1}\right) \cdot a - \frac{1}{a} = \frac{(2a+1)^2}{a} \cdot \frac{1}{(2a+1)a} \cdot a - \frac{1}{a} = 2$$

~256

1)  $f(x) = \log_5 x$



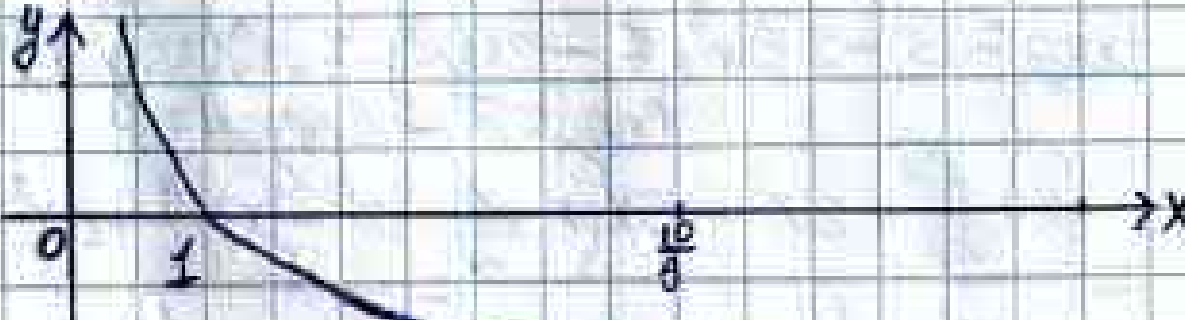
2)  $f(x) = \log_{\frac{1}{7}} x$



3)  $f(x) = \log_{12,5} x$



4)  $\log_{0,9} x$



~257

1)  $f(x) = \log_8 x$   
 $8 > 1 \Rightarrow \varphi^{-a}$  өспөү

2)  $f(x) = \log_{0,1} x$   
 $0,1 < 1 \Rightarrow \varphi^{-a}$  кемүү

3)  $f(x) = \log_{\frac{1}{2}} x$   
 $\frac{1}{2} < 1 \Rightarrow \varphi^{-a}$  кемүү

4)  $f(x) = \log x$   
 $10 > 1 \Rightarrow \varphi^{-a}$  өспөү

~260

1)  $f(x) = \log_2(x+1)$   
 $x+1 > 0$   
 $x > -1$

2)  $f(x) = \log_{0,2}(x-8)$   
 $x-8 > 0$   
 $x > 8$

3)  $f(x) = \log_{\frac{1}{3}}(3x+4)$   
 $3x+4 > 0$   
 $x > -\frac{4}{3}$

4)  $f(x) = \log_5(2x-1)$   
 $2x-1 > 0$   
 $x > 0,5$

~261

1)  $f(x) = \log_{\frac{1}{2}}(2-x)$   
 $2-x > 0$   
 $x < 2$

2)  $f(x) = \log_{2,5}(5-2x)$   
 $5-2x > 0$   
 $x < 2,5$

3)  $f(x) = \log_5(11-4x)$   
 $11-4x > 0$   
 $x < \frac{11}{4}$

4)  $f(x) = \log_7(6-5x)$   
 $6-5x > 0$   
 $x < 1,2$

~262

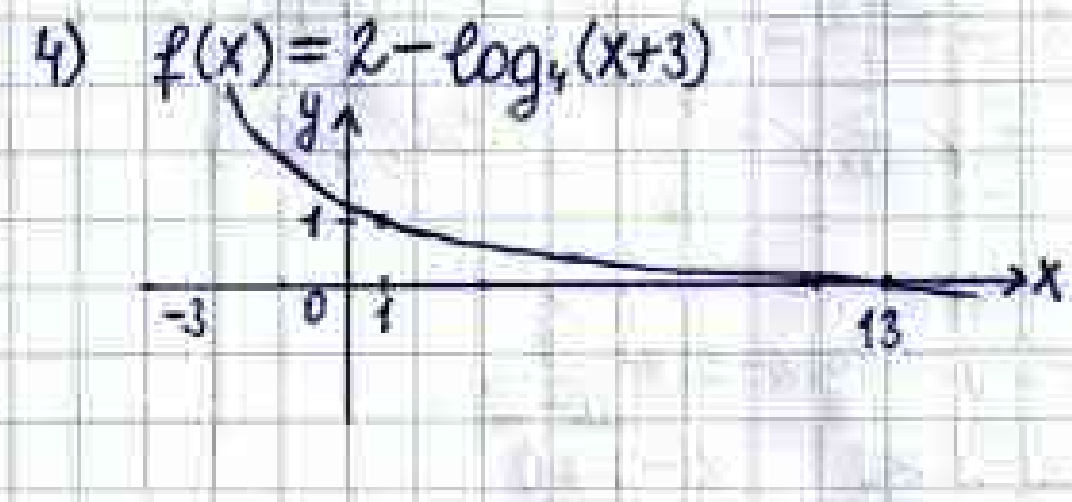
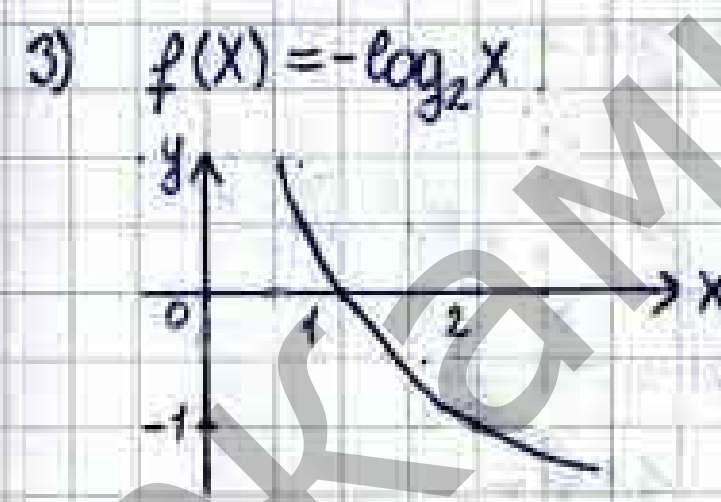
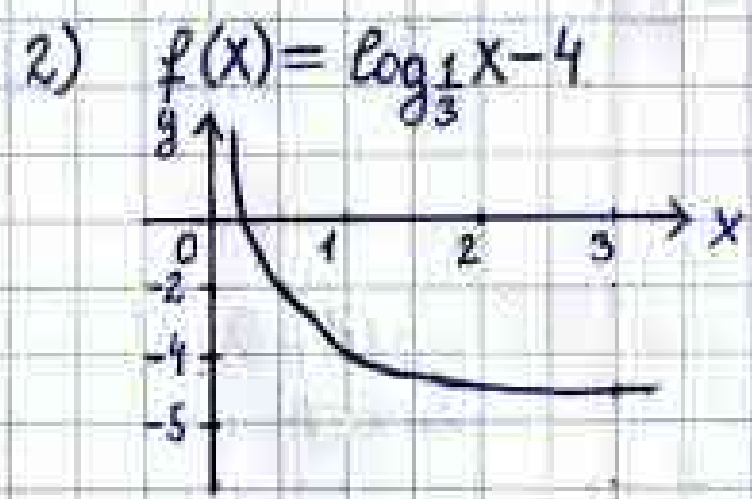
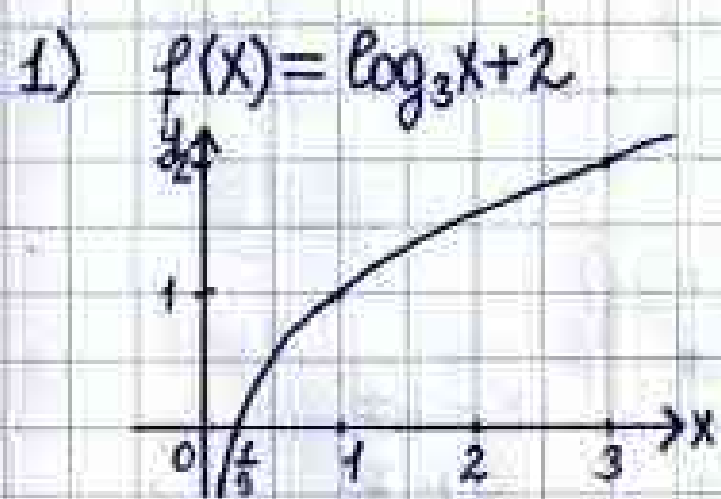
1)  $f(x) = \log(3x-1) + \log(x^2+x+1)$   
 $\begin{cases} 3x-1 > 0 \\ x^2+x+1 > 0 \end{cases}$   
 $\begin{cases} x > \frac{1}{3} \\ x > \frac{1}{3} \end{cases}$   
 $D = 1-4 < 0$  XER  
MC:  $(\frac{1}{3}; +\infty)$

2)  $f(x) = \log(x-5) + \log(x^2+x+2)$   
 $\begin{cases} x-5 > 0 \\ x^2+x+2 > 0 \end{cases}$   
 $\begin{cases} x > 5 \\ x \in R \end{cases}$   
 $D = 1-8 < 0$   
MC:  $(5; +\infty)$

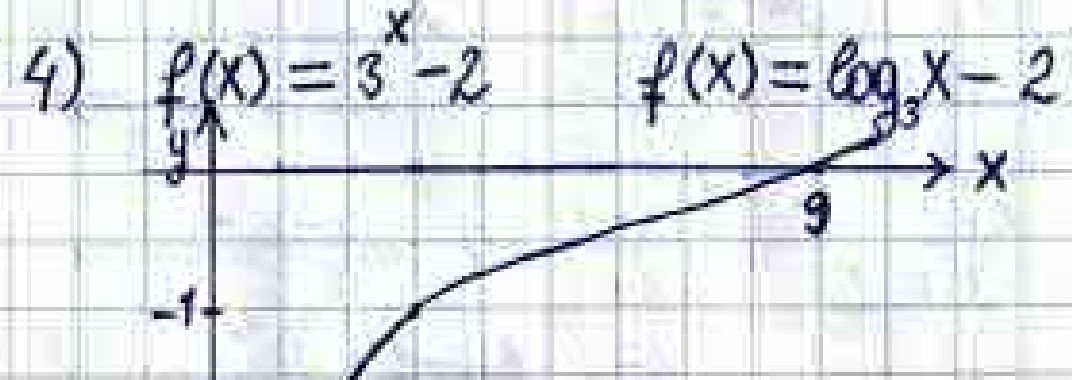
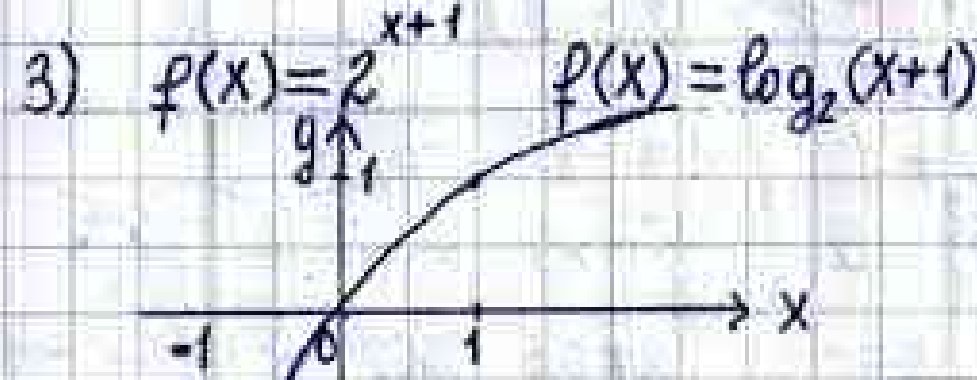
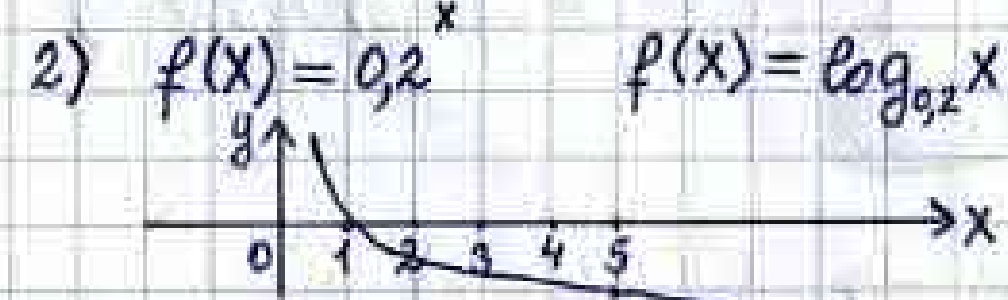
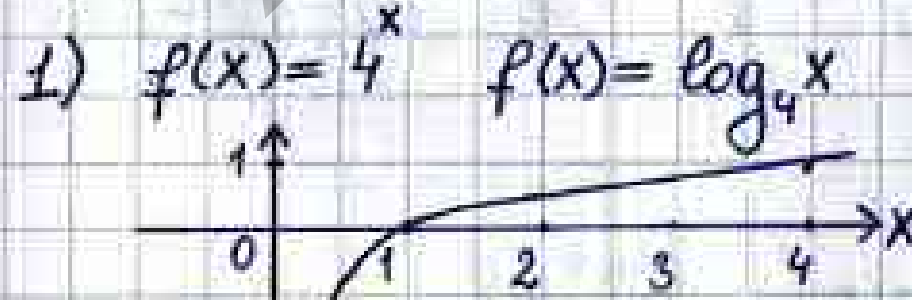
3)  $f(x) = \log_3(x-1) + \log_2(x+5)$   
 $\begin{cases} x-1 > 0 \\ x+5 > 0 \end{cases}$   
 $\begin{cases} x > 1 \\ x > -5 \end{cases}$   
MC:  $(1; +\infty)$

4)  $f(x) = \log_7(3-x) - \log_{0,3}(x+2)$   
 $\begin{cases} 3-x > 0 \\ x+2 > 0 \end{cases}$   
 $\begin{cases} x < 3 \\ x > -2 \end{cases}$   
MC:  $(-2; 3)$

~263



~264





1)  $f(x) = \log_2(x^2-1) + \sqrt{x}$   
 $\begin{cases} \sqrt{x} \geq 0 \\ x^2-1 > 0 \end{cases}$   
 $x_1 = -1, x_2 = 1$   
 MC:  $(1; +\infty)$

2)  $f(x) = \log_{0,8}(1-x^4) - \sqrt{x}$   
 $\begin{cases} 1-x^4 > 0 \\ x \geq 0 \end{cases}$   
 $x_1 = -1, x_2 = 1$   
 MC:  $[0; 1)$

3)  $f(x) = \sqrt{x+2} - \log_{0,1}(6-2x)$   
 $\begin{cases} x+2 \geq 0 \\ 6-2x > 0 \end{cases}$   
 $x \geq -2, x < 3$   
 MC:  $[-2; 3)$

4)  $f(x) = \sqrt{3-x} + \log_5(9+4x)$   
 $\begin{cases} 3-x \geq 0 \\ 9+4x > 0 \end{cases}$   
 $x \leq 3, x > 2,25$   
 MC:  $(2,25; 3]$

1)  $f(x) = \log_3(x(x-3)) - \log_3(x+4)$   
 $\begin{cases} x+4 > 0 \\ x(x-3) > 0 \end{cases}$   
 $x > -4, x_1 = 0, x_2 = 3$   
 MC:  $(-4; 0) \cup (3; +\infty)$

2)  $f(x) = \ln(3+5x) - \ln(4-9x^2)$   
 $\begin{cases} 3+5x > 0 \\ 4-9x^2 > 0 \end{cases}$   
 $x > -3/5, x = \pm 2/3$   
 MC:  $(-3/5; 2/3)$

3)  $f(x) = \log_{0,5}(x^2+x) + \sqrt{2-x}$   
 $\begin{cases} 2-x \geq 0 \\ x^2+x > 0 \end{cases}$   
 $x \leq 2, x_1 = 0, x_2 = -1$   
 MC:  $(-\infty; -1) \cup (0; 2]$

4)  $f(x) = \sqrt{1-x} + \ln(9-x^2)$   
 $\begin{cases} 1-x \geq 0 \\ 9-x^2 > 0 \end{cases}$   
 $x \leq 1, x = \pm 3$   
 MC:  $(-3; 1]$

1)  $f(x) = \frac{\lg(3+2x-x^2)}{2-\sqrt{x}}$   
 $\begin{cases} 3+2x-x^2 > 0 \\ 2-\sqrt{x} \neq 0 \end{cases}$   
 $x^2-2x-3 < 0, x \neq 4$   
 $D = 4 + 12 = 16$   
 $x_1 = \frac{2-4}{2} = -1, x_2 = \frac{2+4}{2} = 3$   
 MC:  $(-1; 3)$

2)  $f(x) = \frac{\ln(x^2+5x)}{x-7}$   
 $\begin{cases} x^2+5x > 0 \\ x-7 \neq 0 \end{cases}$   
 $x_1 = 0, x_2 = -5, x \neq 7$   
 MC:  $(-\infty; -5) \cup (0; 7) \cup (7; +\infty)$


3)  $f(x) = \lg|x-3| + \frac{1}{\sqrt{x-2}}$   
 $\begin{cases} x-2 > 0 \\ x-3 \neq 0 \end{cases}$   
 $x > 2, x \neq 3$   
 MC:  $(2; 3) \cup (3; +\infty)$

4)  $f(x) = 10 \lg|x+4| - \frac{3}{\sqrt{8-x}}$   
 $\begin{cases} 8-x > 0 \\ x+4 \neq 0 \end{cases}$   
 $x < 8, x \neq -4$   
 MC:  $(-\infty; -4) \cup (-4; 8)$

1)  $f(x) = \log_{x-2} \left( \frac{2x}{x+1} - 1 \right)$   
 $\begin{cases} x-2 > 0 \\ \frac{2x}{x+1} - 1 > 0 \end{cases}$   
 $x > 2, \frac{2x-x-1}{x+1} > 0$   
 $x \neq 3, \frac{x-1}{x+1} > 0$   
 $x_1 = 1, x_2 = -1$   
 MC:  $(2; 3) \cup (3; +\infty)$

2)  $f(x) = \log_{x+5} \left( \frac{3x+2}{2x-1} \right)$   
 $\begin{cases} x+5 > 0 \\ \frac{3x+2}{2x-1} > 0 \end{cases}$   
 $x > -5, \frac{3x+2}{2x-1} > 0$   
 $x_1 = -5/3, x_2 = 0,5$   
 MC:  $(-5; 4) \cup (4; +\infty)$

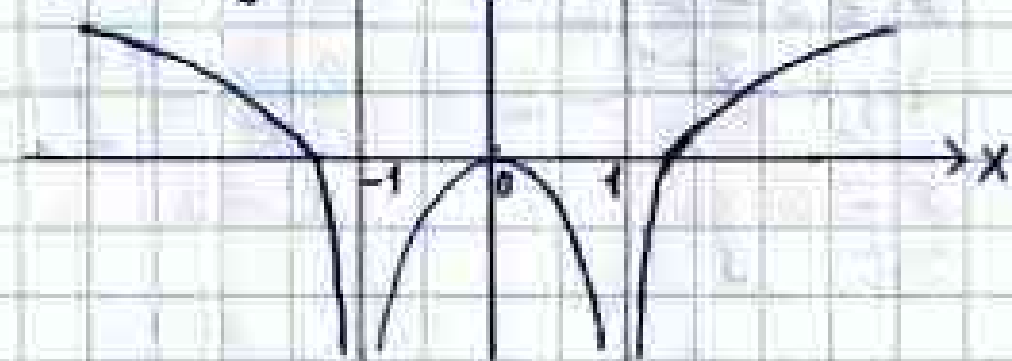
$$3) f(x) = \log_{x-1} \left( \frac{x}{9-x^2} \right)$$

$$\begin{cases} x-1 > 0 \\ x-1 \neq 1 \\ \frac{x}{9-x^2} > 0 \end{cases} \quad \begin{cases} x > 0 \\ x \neq 2 \\ \frac{x}{9-x^2} > 0 \end{cases}$$


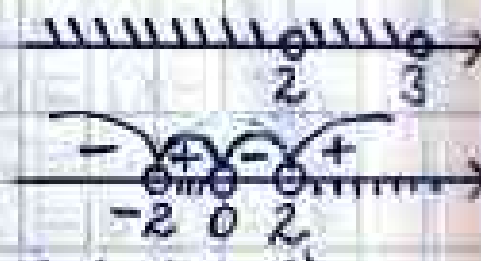
$x=0 \quad x \neq \pm 3 \quad \text{MC: } (0; 2) \cup (2; 3)$

~270

$$1) f(x) = \lg|x^2-1|$$

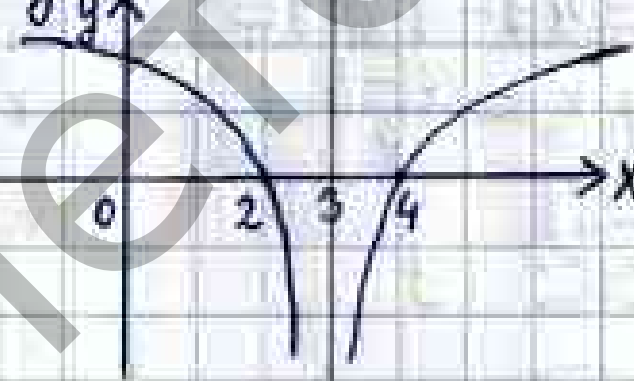


$$4) f(x) = \log_{3-x} \frac{x^2-4}{x}$$

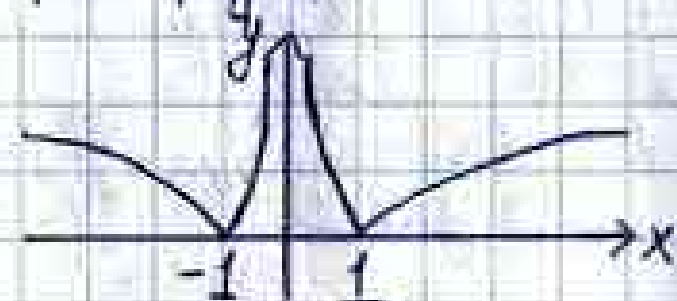
$$\begin{cases} 3-x > 0 \\ 3-x \neq 1 \\ \frac{x^2-4}{x} > 0 \end{cases} \quad \begin{cases} x < 3 \\ x \neq 2 \\ \frac{x^2-4}{x} > 0 \end{cases}$$


$x = \pm 2 \quad x \neq 0 \quad \text{MC: } (-2; 0)$

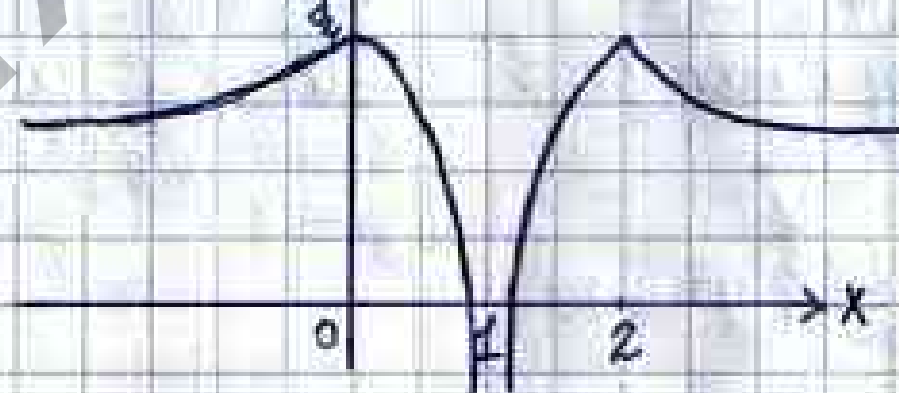
$$2) f(x) = \lg|x-3|$$



$$3) f(x) = |\lg|x||$$



$$4) f(x) = 4 - |\log_3|x-1||$$



~271

$$1) \log_3(2x-1) = 2$$

$$2x-1 = 3^2$$

$$2x = 9 + 1$$

$$x = 5$$

$$2) \ln(3x-5) = 0$$

$$3x-5 = e^0$$

$$3x = 1 + 5$$

$$x = 2$$

$$3) \log_4(4-x) = 1$$

$$4-x = 4$$

$$-x = 4 - 4$$

$$x = -3$$

$$4) \lg(2x-1) = \lg 3$$

$$\log_2(2x-1) = 1$$

$$2x-1 = 3$$

$$x = 2$$

~272

$$1) \lg(3-x) = \lg(x+2)$$

$$3-x = x+2$$

$$2x = 1$$

$$x = 0,5$$

$$2) \lg x + \lg(x-1) = \lg 2$$

$$\lg(x^2-x) = \lg 2$$

$$x^2-x-2 = 0$$

$$D = 1+8 = 9$$

$$x_1 = \frac{1-3}{2} = -1$$

$$x_2 = \frac{1+3}{2} = 2$$

$$3) \ln(6-x) + \ln x = \ln 5$$

$$\ln(6x-x^2) = \ln 5$$

$$x^2-6x+5 = 0$$

$$D = 36-20 = 16$$

$$x_1 = \frac{6-4}{2} = 1$$

$$x_2 = \frac{6+4}{2} = 5$$

$$4) \log_2(4-x) = \log_2(1-2x)$$

$$4-x = 1-2x$$

$$x = -3$$

~273

$$1) \lg(5-x) + \lg x = \lg 4$$

$$\lg(5x-x^2) = \lg 4$$

$$x^2-5x+4 = 0$$

$$D = 25-16 = 9$$

$$x_1 = \frac{5-3}{2} = 1$$

$$5-x > 0$$

$$x < 5$$

$$x_2 = \frac{5+3}{2} = 4$$

$$2) \ln(6-x) + \ln x = \ln 5$$

$$\ln(6x-x^2) = \ln 5$$

$$x^2-6x+5 = 0$$

$$D = 36-20 = 16$$

$$x_1 = \frac{6-4}{2} = 1$$

$$x_2 = \frac{6+4}{2} = 5$$

$$3) \lg(x+1) + \lg(x-1) = \lg 3$$

$$\lg(x^2-1) = \lg 3$$

$$x^2-1 = 3$$

$$x^2 = 4$$

$$x \neq -2 \quad x = 2$$

$$4) \lg x + \lg(x-3) = 1$$

$$\lg(x(x-3)) = 1$$

$$x(x-3) = 0$$

$$x_1 \neq 0 \quad x-3 = 0$$

$$x_2 \neq 3$$

274

1)  $\log(x^2 - x) = 1 - \log 5$   
 $\log(x^2 - x) = \log 2$   
 $x^2 - x - 2 = 0$   
 $\Delta = 1 + 8 = 9$   
 $x_1 = \frac{1-3}{2} = -1$   $x_2 = \frac{1+3}{2} = 2$   
 MC: 2

2)  $\log_6(2x^2 - x) = 1 - \log_6 2$   
 $\log_6(2x^2 - x) = \log_6 3$   
 $2x^2 - x - 3 = 0$   
 $\Delta = 1 + 4 \cdot 6 = 25$   
 $x_1 = \frac{1-5}{2} = -2$   $x_2 = \frac{1+5}{2} = 3$   
 MC: 3

3)  $2 \log_3^2 x - 7 \log_3 x + 3 = 0$   
 $\Delta = 49 - 4 \cdot 2 \cdot 3 = 25$   
 $\log_3 x_1 = \frac{7-5}{4} = \frac{1}{2}$   $\log_3 x_2 = \frac{7+5}{4} = 3$   
 $x_1 = \sqrt{3}$   $x_2 = 9$   
 MC: 9

4)  $\log^2 x - 3 \log_3 x + 2 = 0$   
 $\Delta = 9 - 8 = 1$   
 $\log_3 x = \frac{3-1}{2} = 1$   $\log_3 x = \frac{3+1}{2} = 2$   
 $x = 3$   $x = 9$   
 MC: 9

275

1)  $\begin{cases} x^2 + y^2 = 80 \\ \log_2 x + \log_2 y = 5 \end{cases}$   
 $\begin{cases} x^2 + y^2 = 80 \\ \log_2 xy = \log_2 32 \end{cases}$   
 $\begin{cases} x = 32 \\ y = 4 \end{cases}$  H/e  $\begin{cases} x_2 = 4 \\ y_2 = 8 \end{cases}$   
 $\frac{1024}{y^2} + y^2 = 80$   
 $y^4 - 80y^2 + 1024 = 0$   
 $\Delta = 6400 - 4 \cdot 1024 = 2304$   
 $y_1^2 = \frac{80-48}{2} = 16$   $y_2^2 = \frac{80+48}{2} = 64$   
 $y_1 = 4$   $y \neq -4$   $y_2 = 8$   $y \neq -8$

2)  $\begin{cases} \log x + \log y = \log 2 \\ x^2 + y^2 = 5 \end{cases}$   
 $\begin{cases} xy = 2 \\ x^2 + y^2 = 5 \end{cases}$   $x > 0$   $y > 0$   
 $\begin{cases} x = \frac{2}{y} \\ \frac{4}{y^2} + y^2 = 5 \end{cases}$   
 $\begin{cases} x_1 = 2 \\ y_1 = 1 \end{cases}$  H/e  $\begin{cases} x_2 = 1 \\ y_2 = 2 \end{cases}$   
 $y^4 - 5y^2 + 4 = 0$   
 $\Delta = 25 - 16 = 9$   
 $y_1^2 = \frac{5-3}{2} = 1$   $y_2^2 = \frac{5+3}{2} = 4$   
 $y_1 = 1$   $y_2 = 2$

276

1)  $\begin{cases} \log_2(x+y) = 3 \\ \log_{15} x = 1 - \log_{15} y \end{cases}$   
 $\begin{cases} \log_2(x+y) = \log_2 8 \\ \log_{15}(xy) = 1 \end{cases}$   
 $\begin{cases} x+y = 8 \\ xy = 15 \end{cases}$   
 $\begin{cases} x = 8-y \\ (8-y)y = 15 \end{cases}$   
 $y^2 - 8y + 15 = 0$   
 $\Delta = 64 - 60 = 4$   
 $y_1 = \frac{8-2}{2} = 3$   $x_1 = 8-3 = 5$   
 $y_2 = \frac{8+2}{2} = 5$   $x_2 = 8-5 = 3$

2)  $\begin{cases} \log_3(xy) = 2 + \log_3 2 \\ \log_3(x+y) = 2 \end{cases}$   
 $\begin{cases} \log_3(\frac{xy}{2}) = \log_3 9 \\ \log_3(x+y) = \log_3 9 \end{cases}$   
 $\begin{cases} xy = 18 \\ x+y = 9 \end{cases}$   
 $\begin{cases} x = 9-y \\ (9-y)y = 18 \end{cases}$   
 $y^2 - 9y + 18 = 0$   
 $\Delta = 81 - 72 = 9$   
 $y_1 = \frac{9-3}{2} = 3$   $x_1 = 9-3 = 6$   
 $y_2 = \frac{9+3}{2} = 6$   $x_2 = 9-6 = 3$

277

1)  $\begin{cases} \log_2(3x-y) = 5 \\ \log_3(x^2 - y^2) - \log_3(x-y) = 0,5 \end{cases}$   
 $\begin{cases} 3x-y = 5 \\ \log_3(x+y) = 0 \end{cases}$   
 $\begin{cases} y = 3x-5 \\ x = 2 \end{cases}$   
 $\begin{cases} x+3x-5 = 3 \\ y = 1 \end{cases}$

2)  $\begin{cases} \log_3(x-y) = 1 \\ \log_3(2x-1) + \log_3 y = 1 \end{cases}$   
 $\begin{cases} x-y = 3 \\ \log_3(2xy-y) = \log_3 3 \end{cases}$   
 $\begin{cases} x = 1+y \\ 2y(1+y) - y = 3 \end{cases}$   
 $\begin{cases} x = 2 \\ y = 1 \end{cases}$   
 $2y + 2y^2 - y - 3 = 0$   
 $2y^2 + y - 3 = 0$   
 $\Delta = 1 + 24 = 25$   
 $y_1 = \frac{-1-5}{4} = -\frac{3}{2}$   $y_2 = \frac{-1+5}{4} = 1$

278

1)  $\log_3 \sqrt{2x+1} = 1$   $\log_3 \sqrt{2x+1} = \log_3 3$   
 $2x+1 = 9$   $x = 4$

2)  $\log_{\frac{1}{2}} \sqrt[3]{2x-2} = -2$   $\log_{\frac{1}{2}} \sqrt[3]{2x-2} = \log_{\frac{1}{2}} 4$   
 $2x-2 = 64$   $x = 33$

$$3) \log_{\frac{3}{5}} \frac{2x+3}{x-2} = 1$$

$$\log_{\frac{3}{5}} \frac{2x+3}{x-2} = \log_{\frac{3}{5}} \frac{3}{5}$$

$$10x+15 = 3x-6$$

$$7x = -21$$

$$x = -3$$

~279

$$1) \log \sqrt{3x+1} + \log \sqrt{x+4} = \log 12 \quad x > -4$$

$$\log \sqrt{3x^2+13x+4} = \log 12$$

$$3x^2+13x-140=0$$

$$D = 169 + 4 \cdot 3 \cdot 140 = 1849$$

$$x_1 = \frac{-13-43}{6} = -\frac{28}{3} \quad x_2 = \frac{-13+43}{6} = 5$$

$$4) \log_{\sqrt{3}} \frac{1}{3x-5} = 0$$

$$\log_{\sqrt{3}} \frac{1}{3x-5} = \log_{\sqrt{3}} 1$$

$$3x-5 = 1$$

$$3x = 6$$

$$x = 2$$

$$2) \log(x-2) - \log \sqrt{x-4} = \log 3 \quad x > 4$$

$$\log \frac{x-2}{\sqrt{x-4}} = \log 3$$

$$(x-2)^2 = (3\sqrt{x-4})^2$$

$$x^2-4x+4 = 9x-36$$

$$x^2-13x+40=0$$

$$D = 169-160 = 9$$

$$x_1 = \frac{13-3}{2} = 5 \quad x_2 = \frac{13+3}{2} = 8$$

$$3) (x^2-4) \log_3(1-x^2-3x) = 0 \quad -2,6 < x < 0,38$$

$$\log_3(1-x^2-3x) = 0$$

$$x^2+3x-1 = -1$$

$$x(x+3) = 0$$

$$x_1 = 0 \quad x_2 = -3$$

$$x^2-4=0$$

$$x^2=4$$

$$x_2 = 2 \quad x_3 = -2$$

$$4) (x^2-x-2) \log_2(x^2-4x+4) = 0 \quad x \neq 2$$

$$x^2-x-2=0$$

$$D = 1+8 = 9$$

$$x_1 = \frac{1-3}{2} = -1$$

$$x_2 = \frac{1+3}{2} = 2$$

$$x^2-4x+4=1$$

$$x^2-4x+3=0$$

$$D = 16-12 = 4$$

$$x_2 = \frac{4-2}{2} = 1$$

$$x_3 = 3$$

~280

$$1) \log x + \log x^2 + \log x^3 = 6 \quad x > 0$$

$$\log(x \cdot x^2 \cdot x^3) = \log 10^6$$

$$x^6 = 10^6$$

$$x = 10$$

$$2) \frac{\log x}{1-\log x} = 3 \quad x > 0$$

$$\frac{\log x}{1-\log x} = 3$$

$$\log 10 - \log x = 3$$

$$\log_{10} x = 3$$

$$x = \frac{10^3}{x^3} \Rightarrow x = \sqrt[4]{1000}$$

$$3) \log_2 \log_2 \log_2 x = 0$$

$$\log_2 \log_2 x = 1$$

$$\log_2 x = 2$$

$$x = 4$$

$$4) \begin{matrix} x + \log 2 \\ 10^{\log 10^x + \log 2} \\ 10^{\log 10^x \cdot 2} \end{matrix} = 20$$

$$10^{\log 10^x + \log 2} = 20$$

$$10^{\log 10^x \cdot 2} = 20 \Rightarrow 10^x = 10 \Rightarrow x = 1$$

~281

$$1) \log_3(5^{2x} - 2 \cdot 5^x) = 2 \log_3 15$$

$$\log_3(5^{2x} - 2 \cdot 5^x) = \log_3 225$$

$$5^{2x} - 2 \cdot 5^x - 15 = 0$$

$$D = 4 + 4 \cdot 15 = 64$$

$$5^x = \frac{2+8}{2} = 5 \quad 5^x \neq \frac{2-8}{2} = -3$$

$$x = 1$$

$$2) \log_2(2^{2(x+1)} + 2^{4x}) = 2 \log_4 5$$

$$\log_2(2^{2(x+1)} + 2^{4x}) = \log_2 5$$

$$4 \cdot 2^{2x} + 2^{4x} = 5$$

$$2^{4x} + 4 \cdot 2^{2x} - 5 = 0$$

$$D = 16 + 4 \cdot 5 = 36$$

$$2^{2x} = \frac{-4+6}{2} = 2$$

$$2x = 1$$

$$x = 0,5$$

$$2^{2x} \neq \frac{-4-6}{2} = -5$$

3)  $\log_3(3^x - 8) = 2 - x$   
 $\log_3(3^x - 8) = \log_3 \frac{1}{3^x}$   
 $3^{2x} - 8 \cdot 3^x - 9 = 0$   
 $D = 64 + 36 = 100$   
 $3^x = \frac{8+10}{2} = 9$       $3^x = \frac{8-10}{2} = -1$   
 $x = 2$

~282

1)  $\begin{cases} \log_3(y-x) = 1 \\ 3^{2x+1} \cdot 2^y = 24 \end{cases}$       $\begin{cases} y = x+3 \\ 3^{x+1} \cdot 2^{x+3} = 24 \end{cases}$   
 $\begin{cases} y = 3 \\ x = 0 \end{cases}$       $\begin{cases} 24 \cdot 6^x = 24 \\ 6^x = 6^0 \\ x = 0 \end{cases}$

4)  $\log_7(6+7^{-x}) = 1+x$   
 $\log_7(6+7^{-x}) = \log_7(7 \cdot 7^x)$   
 $6 + \frac{1}{7^x} = 7 \cdot 7^x$   
 $7 \cdot 7^{2x} - 6 \cdot 7^x - 1 = 0$   
 $D = 36 + 28 = 64$   
 $3^x = 1 \Rightarrow x = 0$

2)  $\begin{cases} \log_2(x-y) = 1 \\ 2^{x+2} \cdot 3^{y+1} = 72 \end{cases}$       $\begin{cases} x = y+2 \\ 2^{y+2} \cdot 3^{y+1} = 72 \end{cases}$   
 $\begin{cases} x = 3 \\ y = 1 \end{cases}$       $\begin{cases} 4 \cdot 3 \cdot 2^y \cdot 3^y = 72 \\ 6^y = 6 \\ y = 1 \end{cases}$

~283

1)  $\begin{cases} 3^x \cdot 2^y = 576 \\ \log_{\sqrt{2}}(y-x) = 4 \end{cases}$       $\begin{cases} \log_{\sqrt{2}}(y-x) = \log_{\sqrt{2}} 4 \\ 3^x \cdot 2^y = 576 \end{cases}$   
 $\begin{cases} y = 4+x \\ 2^4 \cdot 2^x \cdot 3^x = 576 \\ 6^x = 36 \\ x = 2 \end{cases}$       $\begin{cases} y = 6 \\ x = 2 \end{cases}$

2)  $\begin{cases} 3^x \cdot 2^y = 972 \\ \log_{\sqrt{3}}(x-y) = 2 \end{cases}$       $\begin{cases} \log_{\sqrt{3}}(x-y) = \log_{\sqrt{3}} 3 \\ 3^x \cdot 2^y = 972 \end{cases}$   
 $\begin{cases} x = y+3 \\ 3^3 \cdot 3^y \cdot 2^y = 972 \\ 6^y = 36 \\ y = 2 \end{cases}$       $\begin{cases} x = 5 \\ y = 2 \end{cases}$

~284

1)  $\begin{cases} 10^{2-\lg(x-y)} = 25 \\ \lg(x-y) + \lg(x+y) = 1 + 2\lg 2 \end{cases}$       $\begin{cases} 10^{\lg \frac{100}{x-y}} = 25 \\ \lg(x^2 - y^2) = \lg 40 \end{cases}$   
 $\begin{cases} x-y = 25 \\ x^2 - y^2 = 40 \end{cases}$       $\begin{cases} x = 4+y \\ (4+y)^2 - y^2 = 40 \\ 16 + 8y + y^2 - y^2 = 40 \\ 8y = 24 \\ y = 3 \end{cases}$

2)  $\begin{cases} 10^{1+\lg(x+y)} = 50 \\ \lg(x-y) + \lg(x+y) = 2 - \lg 5 \end{cases}$       $\begin{cases} 10^{\lg(10(x+y))} = 50 \\ \lg(x^2 - y^2) = \lg 20 \end{cases}$   
 $\begin{cases} 10(x+y) = 50 \\ x^2 - y^2 = 20 \end{cases}$       $\begin{cases} x = 5-y \\ (5-y)^2 - y^2 = 20 \\ 25 - 10y + y^2 - y^2 = 20 \\ 10y = 5 \\ y = 0,5 \end{cases}$   
 $\begin{cases} x = 4,5 \\ y = 0,5 \end{cases}$

~285

1)  $\begin{cases} \log_3 x - 2 = 27 \\ x \log_5 x - 2 = x \log_x 24 \end{cases}$   
 $\log_3 x - 2 = \frac{3}{\log_3 x}$   
 $\log_3^2 x - 2 \log_3 x - 3 = 0$   
 $D = 4 + 4 \cdot 3 = 16$   
 $\log_3 x = \frac{2-4}{2} = -1$       $\log_3 x = \frac{2+4}{2} = 3$   
 $x_1 = 3^{-1}$       $\log_3 x = 3$   
 $x_1 = \frac{1}{3}$       $x_2 = 3^3$   
 $x_2 = 27$

2)  $\begin{cases} \log_2 x - 3 = 16 \\ x \log_2 x - 3 = x \log_x 16 \end{cases}$   
 $\log_2 x - 3 = \frac{4}{\log_2 x}$   
 $\log_2^2 x - 3 \log_2 x - 4 = 0$   
 $D = 9 + 4 \cdot 4 = 25$   
 $\log_2 x = \frac{3-5}{2} = -1$       $\log_2 x = \frac{3+5}{2} = 4$   
 $x_1 = 2^{-1}$       $\log_2 x = 4$   
 $x_1 = \frac{1}{2}$       $x_2 = 2^4$   
 $x_2 = 16$

3)  $\begin{cases} 3 - \log_3 x = 9 \\ x^{3-\log_3 x} = x \log_x 9 \end{cases}$   
 $3 - \log_3 x = \log_x 3^2$   
 $3 - \log_3 x = \frac{\log_3 x}{2}$   
 $3 \log_3 x - \log_3^2 x = 2$   
 $\log_3^2 x - 3 \log_3 x + 2 = 0$   
 $D = 9 - 4 \cdot 2 = 1$   
 $\log_3 x = 1$       $\log_3 x = 2$   
 $x_1 = 3$       $x_2 = 9$

4)  $\begin{cases} \log_5 x + 2 = 12,5 \\ x \log_5 x + 2 = x \log_x 12,5 \end{cases}$   
 $\log_5 x + 2 = \log_x 5^3$   
 $\log_5 x + 2 = \frac{3}{\log_5 x}$   
 $\log_5^2 x + 2 \log_5 x - 3 = 0$   
 $D = 4 + 4 \cdot 3 = 16$   
 $\log_5 x = \frac{-2-4}{2} = -3$       $\log_5 x = \frac{-2+4}{2} = 1$   
 $x_1 = \frac{1}{125}$       $x = 5$

~286

$$1) \log_{2x+3} \frac{1}{4} + 2 = 0$$

$$\frac{\log_4 (2x+3)}{-1} + 2 = 0$$

$$2 \log_4 (2x+3) - 1 = \log_4 1$$

$$\log_4 \frac{(2x+3)^2}{4} = \log_4 1$$

$$\frac{(2x+3)^2}{4} = 1$$

$$2x+3 = 2$$

$$x = -\frac{1}{2}$$

$$2) \log_{\frac{2x-1}{x+2}} 3 - 1 = 0$$

$$\log_3 \frac{2x-1}{x+2} - 1 = 0$$

$$\log_3 3 - \log_3 \frac{2x-1}{x+2} = 0$$

$$\log_3 \frac{3x+6}{2x-1} = \log_3 1$$

$$3x+6 = 2x-1$$

$$x = -7$$

$$3) \log_{\sqrt{6-x}} 3 - 2 = 0$$

$$\frac{\log_3 \sqrt{6-x}}{1} - 2 = 0$$

$$\log_3 3 - \log_3 (6-x) = 0$$

$$\log_3 \frac{3}{6-x} = \log_3 1$$

$$\frac{3}{6-x} = 1$$

$$6-x = 3$$

$$x = 3$$

$$4) \log_{\frac{1}{\sqrt{x+2}}} 5 + 2 = 0$$

$$\frac{\log_5 1}{\sqrt{x+2}} + 2 = 0$$

$$\log_5 5 + \log_5 \frac{1}{x+2} = 0$$

$$\log_5 \frac{5}{x+2} = \log_5 1$$

$$\frac{5}{x+2} = 1$$

$$x = 3$$

~287

$$1) \frac{4}{2} \log_2 (2x-2) = 2 \frac{1}{3}$$

$$\frac{1}{3} \log_2 (2x-2) = 2 \frac{1}{3}$$

$$(2x-2)^{\frac{1}{3}} = 2^{\frac{4}{3}}$$

$$2x = 16 + 2$$

$$x = 9$$

$$2) \log_x \sqrt{5} + \log_x (5x) - \frac{9}{4} = (\log_x \sqrt{5})^2$$

$$\log_x \sqrt{5} + \log_x \sqrt{5^2} + \log_x x - \frac{9}{4} = (\log_x \sqrt{5})^2$$

$$\log_x \sqrt{5} + 2 \log_x \sqrt{5} + 1 - \frac{9}{4} - (\log_x \sqrt{5})^2 = 0$$

$$3(\log_x \sqrt{5})^2 - 3 \log_x \sqrt{5} + \frac{5}{4} = 0$$

$$9 - 4 \cdot \frac{5}{4} = 4$$

$$\log_x \sqrt{5} = \frac{3-2}{2} = \frac{1}{2} \quad \log_x \sqrt{5} = \frac{3+2}{2} = \frac{5}{2}$$

$$x = 5 \quad x = \sqrt[5]{5}$$

$$3) \log_{x+4} (x^4 + x^2 + 2x) \cdot \log_{x+1} (x+4) = 2$$

$$\log_{x+4} (x+4) \cdot \log_{x+1} (x^4 + x^2 + 2x) = \log_{x+1} (x+4)^2$$

$$x^4 + x^2 + 2x = (x+1)^2$$

$$x^4 + x^2 + 2x = x^2 + 2x + 1$$

$$x^4 = 1$$

$$x \neq -1 \quad x = 1$$

$$4) (x+1)^{\log_3 (x-2)} + 2(x-2)^{\log_3 (x+1)} = 3x^2 + 6x + 3$$

$$(x+1)^{\log_3 (x-2)} + 2(x+1)^{\log_3 (x-2)} = 3(x+1)^2$$

$$3(x+1)^{\log_3 (x-2)} = 3(x+1)^2$$

$$\log_3 (x-2) = 2$$

$$x-2 = 3^2$$

$$x = 11$$

~288

$$1) \begin{cases} \log_2 (x+y) + \log_2 (7-y) = 1 + \log_2 5 \\ 2 \log_2 (x-y) = 4 \end{cases} \quad \begin{cases} \log_2 (7x+7y-xy-y^2) = \log_2 8 \cdot 5 \\ 2 \log_2 (x-y) = 2 \log_2 4 \end{cases}$$

$$\begin{cases} 7x+7y-xy-y^2 = 40 \\ \log_2 (x-y) = \log_2 4 \end{cases} \quad \begin{cases} x = 4+y \\ 7(4+y)+7y-xy-y^2 = 40 \\ 28+7y+7y-4y-y^2-y^2 = 40 \\ y^2-5y+6=0 \\ y_1=2 \quad y_2=3 \end{cases} \quad \begin{cases} x_1=6 \\ y_1=2 \end{cases} \quad \text{H/e} \quad \begin{cases} x_2=4 \\ y_2=3 \end{cases}$$

$$2) \begin{cases} \log_3 (3y-x+24) = 27 \\ \log_2 (2x-2y) - \log_2 (5-y^2) = 1 \end{cases} \quad \begin{cases} 3y-x+24 = 27 \\ \log_2 \frac{2(x-y)}{5-y^2} = \log_2 2 \end{cases} \quad \begin{cases} x = 3y-3 \\ 3y-3-y = 5-y^2 \\ y^2+2y-8=0 \\ y_1=2 \quad y_2=-4 \end{cases} \quad \begin{cases} x = 3 \\ y = 2 \end{cases}$$

~289

$$1) \begin{cases} \log_2^2 y + \log_2 y - 2 \log_2^2 x = 0 \\ 9x^2 y - xy^2 = 64 \\ x = 2 \\ y = 2 \end{cases}$$

$$2) \begin{cases} 2 \log_3^2 x + \log_3 x \cdot \log_3 y - \log_3^2 y = 0 & x > 0 \\ xy^2 + \frac{x^2}{y} = 28 & y > 0 \\ \log_3 x = a & 2a^2 + \log_3 y a - \log_3^2 y = 0 \\ \Delta = \log_3^2 y + 8 \log_3^2 y = 9 \log_3^2 y \\ a_1 = \frac{-\log_3 y - 3 \log_3 y}{4} = -\log_3 y \neq \log_3 x \neq \phi \\ a_2 = \frac{-\log_3 y + 3 \log_3 y}{4} = \frac{\log_3 y}{2} \\ \log_3 x = \frac{1}{2} \log_3 y \Rightarrow \frac{\log_3 x^2}{y} = \log_3 y \\ x \cdot x^2 + \frac{x^2}{x^2} = 28 \Rightarrow x^3 = 27 \Rightarrow x = 3 \\ y = 3^2 = 9 \\ \text{MC: } (3, 9) \end{cases}$$

~290

$$1) \begin{cases} \log_2 x + \log_4 y = 4 \\ \log_4 x + \log_2 y = 5 \\ x = \frac{16}{\sqrt{y}} \\ y \cdot \sqrt{\frac{16}{y}} = 32 \end{cases} \quad \begin{cases} \log_2 x + \frac{1}{2} \log_2 y = 4 \\ \frac{1}{2} \log_2 x + \log_2 y = 5 \\ x = \frac{16}{\sqrt{y}} \\ y \cdot \frac{4}{y^{\frac{1}{2}}} = 32 \end{cases} \quad \begin{cases} \log_2(x\sqrt{y}) = \log_2 16 \\ \log_2(4\sqrt{x}) = \log_2 32 \\ x = \frac{16}{\sqrt{y}} \\ y = 16 \end{cases} \quad \begin{cases} x\sqrt{y} = 16 \\ y\sqrt{x} = 32 \\ x = 4 \\ y = 16 \end{cases}$$

$$2) \begin{cases} \log_3 x + \log_9 y = 5 \\ 2 \log_3 x - \log_3 y = -1 \\ y = 3x \\ x\sqrt{3x} = 243 \end{cases} \quad \begin{cases} \log_3 x + \frac{1}{2} \log_3 y = 5 \\ \log_3 x - \log_3 y = -1 \\ y = 3x \\ 3x^3 = 59049 \\ x = 19683 \end{cases} \quad \begin{cases} \log_3(x\sqrt{y}) = \log_3 243 \\ \log_3 \frac{x}{y} = \log_3 \frac{27}{3} \\ y = 3 \cdot 27 \\ x = 27 \end{cases} \quad \begin{cases} x\sqrt{y} = 243 \\ \frac{x}{y} = \frac{1}{3} \\ y = 81 \\ x = 27 \end{cases}$$

~291

$$1) \begin{cases} \log_5(3+8x) > 0 \\ 3+8x > 0 \\ 3+8x > 1 \\ x > -\frac{1}{4} \end{cases} \quad \begin{cases} \log_5(3+8x) > \log_5 1 \\ x > -\frac{1}{8} \\ x > -\frac{1}{4} \end{cases}$$

MC:  $(-\frac{1}{4}, +\infty)$

$$2) \begin{cases} \log_3(7-x) > -2 \\ 7-x > 0 \\ 7-x < 9 \\ x < 7 \\ x > -2 \end{cases} \quad \begin{cases} \log_3(7-x) > \log_3 9 \\ x < 7 \\ x > -2 \end{cases}$$

MC:  $(-2, 7)$

$$3) \begin{cases} \log_2(x-3) \leq 3 \\ x-3 > 0 \\ x-3 \leq 8 \\ x > 3 \\ x \leq 11 \end{cases} \quad \begin{cases} \log_2(x-3) \leq \log_2 8 \\ x > 3 \\ x \leq 11 \end{cases}$$

MC:  $(3, 11]$

$$4) \begin{cases} \lg(4x-1) < 1 \\ 4x-1 > 0 \\ 4x-1 \leq 10 \\ x > 0,25 \\ x \leq 2,75 \end{cases} \quad \begin{cases} \lg(4x-1) \leq \lg 10 \\ x > 0,25 \\ x \leq 2,75 \end{cases}$$

MC:  $(0,25; 2,75]$

~292

$$1) \begin{cases} \log_2(2x+5) > \log_2(x-7) \\ 2x+5 > 0 \\ x-7 > 0 \\ 2x+5 > x-7 \\ x > -2,5 \\ x > 7 \\ x > -12 \end{cases}$$

MC:  $(7, +\infty)$

$$2) \begin{cases} \log_5(3x-2) > \log_5(x+6) \\ 3x-2 > 0 \\ x+6 > 0 \\ 3x-2 > x+6 \\ x > \frac{2}{3} \\ x > -6 \\ x > 4 \end{cases}$$

MC:  $(4, +\infty)$



x > 0  
y > 0  
x < y  
φ

3)  $\log_3(3x-1) > \log_3(2x+3)$   
 $\begin{cases} 3x-1 > 0 \\ 2x+3 > 0 \\ 3x-1 > 2x+3 \end{cases} \Rightarrow \begin{cases} x > \frac{1}{3} \\ x > -\frac{3}{2} \\ x > 4 \end{cases}$   
 $\text{MC: } (4; +\infty)$

~293

1)  $\log_2(2x-1) > \log_2(x+1)$   
 $\begin{cases} 2x-1 > 0 \\ x+1 > 0 \\ 2x-1 > x+1 \end{cases} \Rightarrow \begin{cases} x > 0,5 \\ x > -1 \\ x > 2 \end{cases} \text{MC: } (2; +\infty)$

4)  $\log_{\frac{1}{3}}(4x-3) \geq \log_{\frac{1}{3}}(x+3)$   
 $\begin{cases} 4x-3 > 0 \\ x+3 > 0 \\ 4x-3 \leq x+3 \end{cases} \Rightarrow \begin{cases} x > \frac{3}{4} \\ x > -3 \\ x \leq 2 \end{cases}$   
 $\text{MC: } (\frac{3}{4}; 2]$

3/4  
3

2)  $\log_5(3x+1) > \log_5(x-2)$   
 $\begin{cases} 3x+1 > 0 \\ x-2 > 0 \\ 3x+1 > x-2 \end{cases} \Rightarrow \begin{cases} x > -\frac{1}{3} \\ x > 2 \\ x > -\frac{3}{2} \end{cases} \text{MC: } (2; +\infty)$

3)  $\log_{\frac{1}{4}}(12-x) \geq -2 \quad \log_{\frac{1}{4}}(12-x) \geq \log_{\frac{1}{4}} 49$   
 $\begin{cases} 12-x > 0 \\ 12-x \leq 49 \end{cases} \Rightarrow \begin{cases} x < 12 \\ x \geq -37 \end{cases} \text{MC: } [-37; 12]$

~294

1)  $\log_2^2 x + \log_2 x - 2 \leq 0 \quad x > 0$   
 $\text{D} = 1 + 8 = 9$   
 $\log_2 x = \frac{-1 \pm 3}{2} = -2 \quad \log_2 x = \frac{-1 \pm 3}{2} = 1$   
 $x_1 = \frac{1}{4} \quad x_2 = 2$   
 $\text{MC: } [\frac{1}{4}; 2]$

2)  $\log_{0,2}^2 x - 5 \log_{0,2} x + 6 < 0 \quad x > 0$   
 $\log_{0,2}^2 x - 5 \log_{0,2} x + 6 < 0$   
 $\text{D} = 25 - 24 = 1$   
 $\log_{0,2} x = \frac{5 \pm 1}{2} = 2 \quad \log_{0,2} x = \frac{5 \pm 1}{2} = 3$   
 $x_1 = 0,04 \quad x_2 = 0,008$   
 $\text{MC: } (0,008; 0,04)$

2/3

3)  $\log_{0,1}^2 x + 3 \log_{0,1} x > 4 \quad x > 0$   
 $\log_{0,1}^2 x + 3 \log_{0,1} x - 4 > 0$   
 $\text{D} = 9 + 4 \cdot 4 = 25$   
 $\log_{0,1} x = \frac{-3 \pm 5}{2} = -4 \quad \log_{0,1} x = \frac{-3 \pm 5}{2} = 1$   
 $x_1 = 10000 \quad x_2 = 0,1$   
 $\text{MC: } (0; 0,1) \cup (10000; +\infty)$

4)  $2 - \log^2 x \geq \log x \quad x > 0$   
 $\log^2 x + \log x - 2 \leq 0$   
 $\text{D} = 1 + 2 \cdot 4 = 9$   
 $\log x = \frac{-1 \pm 3}{2} = -2 \quad \log x = \frac{-1 \pm 3}{2} = 1$   
 $x_1 = 0,01 \quad x_2 = 10$   
 $\text{MC: } [0,01; 10]$

2/3

10

~295

1)  $\log_{0,5}(x-2) > 1$  onga  $x-2 < 0,5$  w

2)  $\log_{0,2}(x-2) > \log_{0,2} 3$  onga  $x-2 < 3$  w

75]

~295

1)  $f(x) = \sqrt{\log_{\frac{1}{2}} \frac{2x}{x-1}}$   
 $\log_{\frac{1}{2}} \frac{2x}{x-1} \geq 0 \quad \log_{\frac{1}{2}} \frac{2x}{x-1} \geq \log_{\frac{1}{2}} 1$   
 $\begin{cases} \frac{2x}{x-1} > 0 \\ \frac{2x}{x-1} \leq 1 \end{cases} \Rightarrow \begin{cases} \frac{2x}{x-1} > 0 \\ 2x=0 \quad x-1 \neq 0 \\ x=0 \quad x \neq 1 \end{cases} \Rightarrow \begin{cases} \frac{2x}{x-1} \leq 1 \\ 2x-x+1=0 \\ x=-1 \end{cases}$   
 $\text{MC: } [-1; 0)$

2)  $f(x) = \sqrt{\log_{0,3} \frac{x-1}{x+5}}$   
 $\log_{0,3} \frac{x-1}{x+5} \geq 0 \quad \log_{0,3} \frac{x-1}{x+5} \geq \log_{0,3} 1$   
 $\begin{cases} \frac{x-1}{x+5} > 0 \\ \frac{x-1}{x+5} \leq 1 \end{cases} \Rightarrow \begin{cases} \frac{x-1}{x+5} > 0 \\ x-1=0 \quad x+5 \neq 0 \\ x=1 \quad x \neq -5 \end{cases} \Rightarrow \begin{cases} \frac{x-1}{x+5} \leq 1 \\ x-1=0 \quad x+5 \neq 0 \\ x=1 \quad x \neq -5 \end{cases}$

∞)



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1)  $\log_2(x^2+2x+2) < 1$

$$\begin{cases} x^2+2x+2 > 0 \\ x^2+2x+2 < 10 \\ x^2+2x-8 < 0 \end{cases}$$

$x^2+2x+2 > 0$   
 $\Delta = 4 - 4 \cdot 2 = -5 < 0$   
 $x \in \mathbb{R}$

$x^2+2x-8 < 0$   
 $\Delta = 4 + 32 = 36$   
 $x_1 = \frac{-2-6}{2} = -4$   
 $x_2 = \frac{-2+6}{2} = 2$

MC:  $(-4; 2)$

2)  $\log_{\frac{1}{2}}(x^2-x-2) > -2$

$$\log_{\frac{1}{2}}(x^2-x-2) > \log_{\frac{1}{2}}4$$

$$\begin{cases} x^2-x-2 > 0 \\ x^2-x-2 < 4 \\ x^2-x-6 < 0 \end{cases}$$

$x^2-x-2 > 0$   
 $\Delta = 1 + 4 \cdot 2 = 9$   
 $x_1 = -1$   
 $x_2 = 2$

$x^2-x-6 < 0$   
 $\Delta = 1 + 24 = 25$   
 $x_1 = -2$   
 $x_2 = 3$

MC:  $(-2; -1) \cup (2; 3)$

3)  $\log_2(x^2+3x-1) < -1$

$$\begin{cases} x^2+3x-1 > 0 \\ x^2+3x-1 > 3 \\ x^2+3x-4 = 0 \end{cases}$$

$x^2+3x-1 = 0$   
 $\Delta = 9 + 4 = 13$   
 $x = \frac{-3 \pm \sqrt{13}}{2}$

$x^2+3x-4 = 0$   
 $\Delta = 9 + 16 = 25$   
 $x_1 = \frac{-3-5}{2} = -4$   
 $x_2 = \frac{-3+5}{2} = 1$

MC:  $(-\infty; -4) \cup (1; +\infty)$

4)  $\log_2(x^2+10) < 4$

$$\log_2(x^2+10) < \log_2 16$$

$$\begin{cases} x^2+10 > 0 \\ x^2+10 < 16 \\ x^2 < 6 \end{cases}$$

$x^2 < 6$   
 $x_1 = \sqrt{6}$   
 $x_2 = -\sqrt{6}$

MC:  $(-\sqrt{6}; \sqrt{6})$

~298

1)  $\frac{2}{2} \log_3 \frac{x-1}{3x+3} \leq \frac{1}{2} \log_3 \frac{1}{9}$

$$\log_3 \frac{x-1}{3x+3} \leq \log_3 \frac{1}{9}$$

$$\begin{cases} \frac{x-1}{3x+3} > 0 \\ \frac{x-1}{3x+3} \leq \frac{1}{9} \end{cases}$$

$\frac{x-1}{3x+3} > 0$   
 $\frac{x-1}{3x+3} = 0$   
 $x = 1$   
 $x \neq -1$

$\frac{x-1}{3x+3} \leq \frac{1}{9}$   
 $6x - 12 = 0$   
 $x = 2$   
 $x \neq -1$

MC:  $(1; 2]$

2)  $\frac{3}{3} \log_2 \frac{x-1}{x+1} < \frac{1}{3} \log_2 \frac{1}{4}$

$$\log_2 \frac{x-1}{x+1} < \log_2 \frac{1}{4}$$

$$\begin{cases} \frac{x-1}{x+1} > 0 \\ \frac{x-1}{x+1} < \frac{1}{4} \end{cases}$$

$\frac{x-1}{x+1} > 0$   
 $\frac{x-1}{x+1} = 0$   
 $x = 1$   
 $x \neq -1$

$\frac{x-1}{x+1} < \frac{1}{4}$   
 $x = 1$   
 $x \neq -1$   
 $x = \frac{5}{3}$

MC:  $(1; \frac{5}{3})$

3)  $(5x+1) \log(4-x) \leq 0$

$$\begin{cases} 5x+1 = 0 \\ \log(4-x) = 0 \end{cases}$$

$5x+1 = 0$   
 $x = -0,2$

$\log(4-x) = 0$   
 $4-x = 1$   
 $x = 3$

MC:  $(-\infty; -0,2] \cup [3; 4)$

4)  $(3-x) \log(2x-1) \geq 0$

$$\begin{cases} 3-x = 0 \\ 2x-1 > 0 \\ \log(2x-1) = 0 \end{cases}$$

$3-x = 0$   
 $x = 3$

$2x-1 > 0$   
 $x > 0,5$

$\log(2x-1) = 0$   
 $2x-1 = 1$   
 $x = 1$

MC:  $(0,5; 3]$

~299

1)  $\log_{\frac{1}{6}}(\log_2 \sqrt{6-x}) > 0$

$$\begin{cases} \log_2 \sqrt{6-x} > 0 \\ \log_2 \sqrt{6-x} < 1 \end{cases}$$

$\log_2 \sqrt{6-x} > 0$   
 $\sqrt{6-x} > 1$   
 $6-x > 1$   
 $x < 5$

$\log_2 \sqrt{6-x} < 1$   
 $\sqrt{6-x} < 2$   
 $6-x < 4$   
 $x > 2$

MC:  $(2; 5)$

2)  $\log_{\frac{1}{2}} \log_3 \frac{x+1}{x-1} \geq 0$

$$\begin{cases} \log_3 \frac{x+1}{x-1} > 0 \\ \log_3 \frac{x+1}{x-1} \leq 1 \end{cases}$$

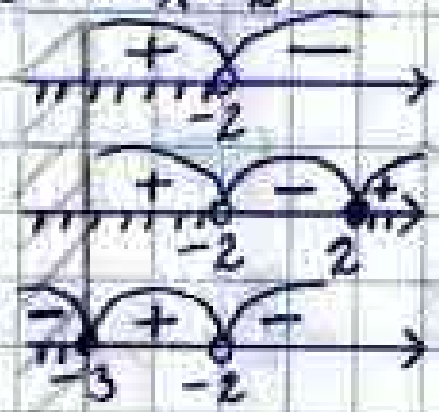
$\log_3 \frac{x+1}{x-1} > 0$   
 $\frac{x+1}{x-1} > 1$   
 $x+1 > x-1$   
 $2 > -2$   
 $x > -1$

$\log_3 \frac{x+1}{x-1} \leq 1$   
 $\frac{x+1}{x-1} \leq 3$   
 $x+1 \leq 3(x-1)$   
 $x+1 \leq 3x-3$   
 $4 \leq 2x$   
 $x \geq 2$

MC:  $(2; +\infty)$

3)  $\log_{0,5} \log_5 \frac{x-2}{x+2} \geq \log_{0,5} 1$

$$\begin{cases} \log_5 \frac{x-2}{x+2} > 0 \\ \log_5 \frac{x-2}{x+2} \leq 1 \end{cases} \quad \begin{cases} \frac{x-2}{x+2} > 1 \\ \frac{x-2}{x+2} > 0 \\ \frac{x-2}{x+2} \leq 5 \end{cases}$$



$\mathcal{M}: (-\infty; -3]$

~300

4)  $\log_{2,5} (\log_3 (9^x - 6)) \geq 0$

$$\begin{cases} \log_3 (9^x - 6) > 0 \\ \log_3 (9^x - 6) \geq 1 \end{cases} \quad \begin{cases} 9^x - 6 > 1 \\ 9^x - 6 > 0 \\ 9^x - 6 \geq 3 \end{cases}$$

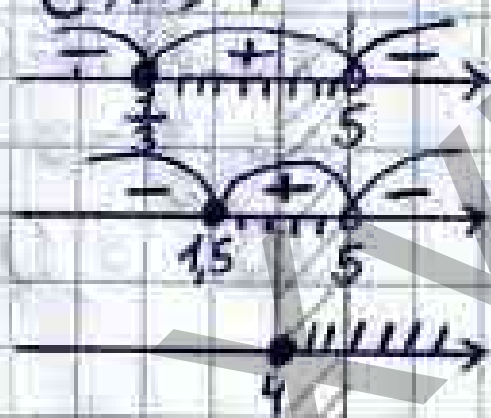
$$\begin{cases} 9^x > 7 \\ 9^x > 6 \\ 9^x \geq 9 \Rightarrow x \geq 1 \end{cases}$$

$\mathcal{M}: [1; +\infty)$

1)  $f(x) = \sqrt{\log_{2,1} \frac{3x-1}{5-x}} + \sqrt{x-4}$

$$\begin{cases} \log_{2,1} \frac{3x-1}{5-x} \geq 0 \\ x-4 \geq 0 \end{cases} \quad \begin{cases} \frac{3x-1}{5-x} > 0 \\ \frac{3x-1}{5-x} \geq 1 \\ x \geq 4 \end{cases}$$

$$\frac{3x-1=0}{5-x} \quad \frac{3x-1=1}{5-x} \quad \begin{cases} x = \frac{1}{3} \quad x \neq 5 \\ x = 1,5 \end{cases}$$



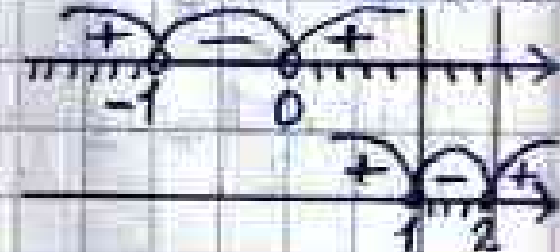
$\mathcal{M}: [4; 5)$

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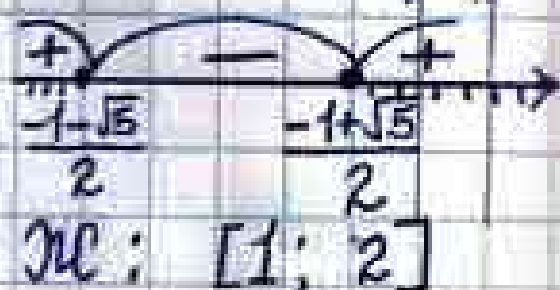
2)  $f(x) = \sqrt{-x^2 + 3x - 2} + \sqrt{\log_6 (x+x^2)}$

$$\begin{cases} \log_6 (x+x^2) \geq 0 \\ -x^2 + 3x - 2 \geq 0 \\ x+x^2 = 0 \end{cases} \quad \begin{cases} x+x^2 > 0 \\ x+x^2 \geq 1 \\ x^2 - 3x + 2 \leq 0 \end{cases}$$

$$\begin{cases} x_1 = 0 \quad x_2 = -1 \\ x^2 + x - 1 = 0 \\ \mathcal{D} = 1+4 = 5 \\ x = \frac{-1 \pm \sqrt{5}}{2} \end{cases}$$



$$\begin{cases} x^2 - 3x + 2 = 0 \\ \mathcal{D} = 9 - 8 = 1 \\ x_1 = 1 \quad x_2 = 2 \end{cases}$$



$\mathcal{M}: [1; 2]$

1)  $\log_{\frac{1}{16}} (x-1) + \log_{\frac{1}{16}} (x-2) > -2$  onga  $\begin{cases} (x-1)(x-2) > 0 \\ (x-1)(x-2) < 5 \end{cases}$  ✓

2)  $3^{3x} - 4 \cdot 3^x \leq 0$  onga  $0 \leq x \leq \log_3 4$

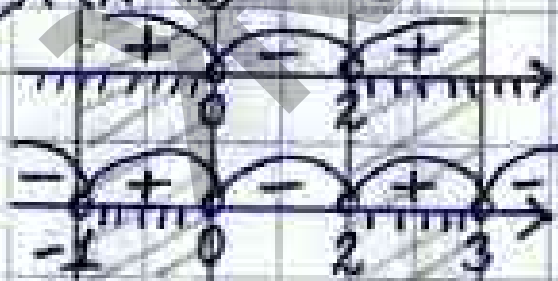
~302

1)  $\log_{0,1} (x-2) - \lg x > \log_{0,1} 3$

$$\lg (x-2)^{-1} - \lg x > \lg 3^{-1}$$

$$\lg \frac{1}{(x-2)x} > \lg \frac{1}{3}$$

$$\begin{cases} \frac{1}{x(x-2)} > 0 \\ \frac{1}{x(x-2)} > \frac{1}{3} \end{cases} \quad \begin{cases} \frac{1}{x(x-2)} = 0 \\ x \neq 0 \quad x \neq 2 \end{cases} \quad \begin{cases} \frac{1}{x(x-2)} = \frac{1}{3} \\ x^2 - 2x - 3 = 0 \\ \mathcal{D} = 4 + 12 = 16 \\ x_1 = -1 \\ x_2 = 3 \end{cases}$$



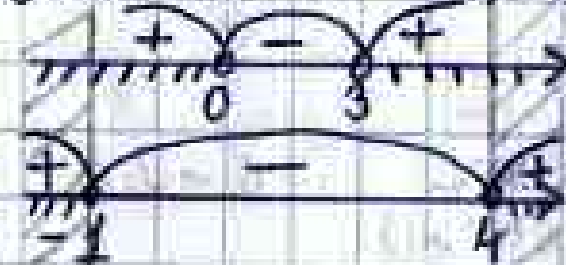
$\mathcal{M}: -0,5 \text{ и } 2, 5$

2)  $\log_{0,5} x - \log_2 (x-3) < \log_{0,5} 4$

$$\log_{0,5} x + \log_{0,5} (x-3) < \log_{0,5} 4$$

$$\log_{0,5} (x(x-3)) < \log_{0,5} 4$$

$$\begin{cases} x(x-3) > 0 \\ x(x-3) > 4 \end{cases} \quad \begin{cases} x(x-3) = 0 \\ x_1 = 0 \quad x_2 = 3 \end{cases} \quad \begin{cases} x^2 - 3x - 4 = 0 \\ \mathcal{D} = 9 + 16 = 25 \\ x_1 = -1 \\ x_2 = 4 \end{cases}$$



$\mathcal{M}: -2; 5$

3)  $\log_{0,2} x - \log_5 (x-2) < \log_{0,2} 3$

$$\log_{0,2} (x(x-2)) < \log_{0,2} 3$$

$$\begin{cases} x(x-2) > 0 \\ x(x-2) > 3 \end{cases} \quad \begin{cases} x(x-2) = 0 \\ x_1 = 0 \quad x_2 = 2 \end{cases}$$

$$\begin{cases} x^2 - 2x - 3 = 0 \\ \mathcal{D} = 4 + 12 = 16 \\ x_1 = -1 \quad x_2 = 3 \end{cases}$$



$\mathcal{M}: -2; 4$

4)  $\lg x - \log_{0,1} (x-1) > \log_{0,1} 0,5$

$$\lg (x(x-1)) > \lg 2$$

$$\begin{cases} x(x-1) > 0 \\ x(x-1) > 2 \end{cases} \quad \begin{cases} x(x-1) = 0 \\ x_1 = 0 \quad x_2 = 1 \end{cases}$$

$$\begin{cases} x^2 - x - 2 = 0 \\ \mathcal{D} = 1 + 8 = 9 \\ x_1 = -1 \quad x_2 = 2 \end{cases}$$



$\mathcal{M}: -2; 3$

1)  $(\log_2 X - 4)(5X^2 + X - 6) \geq 0$   
 $\log_2 X - 4 = 0 \quad X > 0$   
 $\log_2 X = 4$   
 $\log_2 X = \log_2 16$   
 $X = 16$

$5X^2 + X - 6 = 0$   
 $D = 1 + 120 = 121$   
 $X_1 = \frac{-1-11}{10} = -1.2$   
 $X_2 = \frac{-1+11}{10} = 1$

MC:  $(0; 1] \cup [16; +\infty)$

2)  $(\log_3 X + 3)(X^2 + 2X - 8) \geq 0$   
 $\log_3 X + 3 = 0 \quad X > 0$   
 $\log_3 X = -3$   
 $X = \frac{1}{27}$

$X^2 + 2X - 8 = 0$   
 $D = 4 + 32 = 36$   
 $X_1 = \frac{-2-6}{2} = -4$   
 $X_2 = \frac{-2+6}{2} = 2$

MC:  $(0; \frac{1}{27}] \cup [2; +\infty)$

3)  $\frac{2-X}{(X+4)\log_{0.5}(2X^2+6X+5)} \geq 0$   
 $\log_{0.5} 2X^2+6X+5 \neq 0$   
 $2X^2+6X+5 \neq 0$   
 $X^2+3X+2.5 \neq 0$   
 $D = 9 - 10 = -1$   
 $X_1 = \frac{-3-1}{2} = -2$   
 $X_2 = \frac{-3+1}{2} = -1$

$X+4 \neq 0 \quad X \neq -4$   
 $2-X=0 \quad X=2$

MC:  $(-4; -2) \cup (-1; 2]$

4)  $\log_4(3 - \frac{1}{X-1}) + \log_4 \frac{1}{X} \geq 0$   
 $\log_4(\frac{3X-4}{X(X-1)}) \geq \log_4 1$   
 $\frac{3X-4}{X(X-1)} > 0$   
 $\frac{3X-4}{X(X-1)} \geq 1$   
 $3X-4 = X(X-1)$   
 $X^2 - 4X + 4 = 0$   
 $D = 16 - 16 = 0$   
 $X = \frac{4}{2} = 2$

$\frac{3X-4}{X(X-1)} = 0$   
 $X \neq 0 \quad X \neq 1 \quad 3X-4=0$   
 $X = \frac{4}{3}$

MC:  $(0; 1) \cup (2; +\infty)$

1)  $\log_{1-X}(2X+3) \geq 1$   
 $\log_{1-X}(2X+3) \geq \log_{1-X} 1-X$   
 $\begin{cases} 2X+3 > 0 \\ 1-X > 0 \\ 2X+3 \geq 1-X \end{cases}$   
 $\begin{cases} X > -1.5 \\ X < 1 \\ X \geq -2/3 \end{cases}$   
 MC:  $[-2/3; 1)$

2)  $\log_{X-1}(X-8) \leq 1$   
 $\log_{X-1}(X-8) \leq \log_{X-1}(X-1)$   
 $\begin{cases} X-8 > 0 \\ X-1 > 0 \\ X-8 \leq X-1 \end{cases}$   
 $\begin{cases} X \geq 8 \\ X > 1 \\ X \leq 7 \end{cases}$   
 MC:  $(8; +\infty)$

3)  $2 \log_{2X} \sqrt{X+1} < 0$   
 $\log_{2X} \sqrt{X+1} < \log_{2X} 1$   
 $\begin{cases} X+1 > 0 \\ X+1 < 1 \\ 2X > 0 \\ 2X \neq 1 \end{cases}$   
 $\begin{cases} X > -1 \\ X < 0 \\ X > 0 \\ X \neq 0.5 \end{cases}$   
 MC:  $(-1; 0)$

4)  $\log_{3X}(2.5X+1) \geq 0$   
 $\begin{cases} 2.5X+1 > 0 \\ 2.5X+1 \geq 1 \\ 3X > 0 \\ 3X \neq 1 \end{cases}$   
 $\begin{cases} X > -0.4 \\ X \geq 0 \\ X > 0 \\ X \neq 1/3 \end{cases}$   
 MC:  $(-0.4; 0)$

1)  $8^{\log_2 X} - 2X > X-2 \quad X > 0$   
 $2^{\log_2 X^3} - 2X^2 - X + 2 > 0$   
 $X^3 - 2X^2 - X + 2 > 0$   
 Тонкер схемасы бойынша  
 такшаларга тектейміз  
 $(X-1)(X+1)(X-2) > 0$   
 $X=1 \quad X \neq -1 \quad X=2$

MC:  $(0; 1) \cup (2; +\infty)$

2)  $X^{\frac{1}{10}} \cdot \lg X < 1$   
 $\lg X \neq 0$   
 $X \neq 1$

$X^{\frac{1}{10}} \cdot \lg X < 1$   
 $\lg X < \frac{1}{10}$   
 $\lg X < \lg \sqrt[10]{10}$

$\begin{cases} X > 0 \\ X < \sqrt[10]{10} \\ X \neq 1 \end{cases}$

MC:  $(0; 1) \cup (1; \sqrt[10]{10})$

$$3) \begin{aligned} x^3 &> 2^{15 \log_2 \sqrt{2}} \cdot 3^{\log_2 3} \\ x^3 &> 2^{15 \cdot \frac{1}{2} \log_2 2} \cdot 3^{\log_2 3} \\ x^3 &> 2^{\frac{15}{2}} \cdot 3^{\log_2 3} \\ x^3 &> 2^{\log_2 3^{10}} \cdot \sqrt{2} \\ x^3 &> 3^{10} \cdot \sqrt{2} \\ x &> \sqrt[3]{3^{10} \cdot \sqrt{2}} \end{aligned}$$

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$$4) \begin{aligned} x^{-64 \log_5^2 x + 5 \log_5 x} &\leq \left(\frac{1}{5}\right)^{2 + \log_5 8} \quad x \geq 0 \\ -64 \log_5^2 x + 5 \log_5 x &\leq \log_5 \left(\frac{1}{5}\right)^{2 + \log_5 8} \\ \log_5 x \cdot (-64 \log_5 x + 5) &\leq 0 \\ 64 \log_5^2 x - 20 \log_5 x + 7 &\geq 0 \\ \left(\log_5^2 x - \frac{1}{4}\right) \left(\log_5^2 x - \frac{1}{16}\right) &\geq 0 \end{aligned}$$

MC:  $\left[\frac{1}{\sqrt{5}}; \sqrt{5}\right] \cup \left[\frac{1}{5}; 5\right]$

$$1) f(x) = \lg(4-x^2) + \sqrt{\frac{1+\lg^2 x}{\lg x^2}} - 1$$

$$\begin{cases} 4-x^2 > 0 \\ \frac{1+\lg^2 x}{\lg x^2} - 1 \geq 0 \\ 1+\lg^2 x - \lg x^2 \geq 0 \\ \lg^2 x - 2\lg x + 1 \geq 0 \\ (\lg x - 1)^2 \geq 0 \\ \lg x - 1 = 0 \\ \lg x = 1 \\ x = 10 \end{cases} \Rightarrow \begin{cases} 4-x^2 > 0 \\ 4-x^2 = 0 \\ x^2 = 4 \\ x_1 = 2 \\ x_2 = -2 \end{cases}$$

MC: (0; 2)

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$$2) f(x) = \sqrt{\log_3 \log_3(x-3)} + \sqrt{x^2 - 25}$$

$$\begin{cases} \log_3 \log_3(x-3) \geq 0 \\ x^2 - 25 \geq 0 \end{cases} \Rightarrow \begin{cases} \log_3(x-3) > 0 \\ \log_3(x-3) < 0 \\ x-3 > 0 \quad x > 3 \\ x-3 > 1 \quad x > 4 \\ x-3 \leq 3 \quad x \leq 6 \end{cases}$$

MC: [5; 6]

$$1) f(x) = \frac{15+x^2}{\sqrt{\log_4(5x-x^2)} - 1}$$

$$\begin{cases} \log_4(5x-x^2) - 1 > 0 \\ \log_4(5x-x^2) > \log_4 \frac{1}{4} \end{cases}$$

$$\begin{cases} 5x-x^2 > 0 \\ 5x-x^2 < \frac{1}{4} \end{cases} \Rightarrow \begin{cases} 5x-x^2 = 0 \\ x_1 = 0, x_2 = 5 \end{cases}$$

$$\begin{cases} x^2 - 5x + 0,25 > 0 \\ \Delta = 25 - 4 \cdot 0,25 = 24 \\ x_1 = \frac{5-2\sqrt{6}}{2} \\ x_2 = \frac{5+2\sqrt{6}}{2} \end{cases}$$

MC:  $\left(0; \frac{5-2\sqrt{6}}{2}\right) \cup \left(\frac{5+2\sqrt{6}}{2}; 5\right)$

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$$2) f(x) = \frac{\sqrt{17-15x-2x^2}}{\log_x(x+3)}$$

$$\begin{cases} \log_x(x+3) > 0 \\ 17-15x-2x^2 \geq 0 \end{cases} \Rightarrow \begin{cases} x+3 > 0 \\ x > 0 \quad x \neq 1 \\ x+3 > 1 \\ x > -3 \\ x > 0 \quad x \neq 1 \\ x > -2 \end{cases}$$

$$\begin{cases} 2x^2 + 15x - 17 \leq 0 \\ \Delta = 225 + 136 = 361 \\ x_1 = \frac{-15-19}{4} = -8,5 \\ x_2 = \frac{-15+19}{4} = 1 \end{cases}$$

MC: (0; 1)

$$1) f(x) = 3^{x^2-7x} \quad f'(x) = 3^{x^2-7x} \cdot (2x-7) \ln 3$$

$$3) f(x) = 0,8^{1-x^2} \quad f'(x) = 0,8^{1-x^2} \cdot (-3x^2) \ln 0,8$$

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$$2) f(x) = 2^{x+3x^2} \quad f'(x) = 2^{x+3x^2} \cdot (1+6x) \ln 2$$

$$4) f(x) = \left(\frac{1}{7}\right)^{4-x} = -\left(\frac{1}{7}\right)^{4-x} \cdot \ln\left(\frac{1}{7}\right)$$

$$1) f(x) = 7 + x - 5 \ln x$$

$$f'(1) = 1 - 5 \cdot \frac{1}{1} = 1 - 5 = -4$$

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$$2) f(x) = 4 + \frac{1}{8} \ln 2x$$

$$f'(3) = \frac{1}{8} \cdot \frac{2}{2x} = \frac{1}{8} \cdot \frac{1}{3} = \frac{1}{24}$$

$$1) f(x) = \log_{0,5}(2+x)$$

$$f'(1) = \frac{1}{(2+x) \ln 0,5} = \frac{1}{3 \ln 0,5}$$

$$2) \log_3(5+x) = f(x)$$

$$f'(4) = \frac{1}{(5+x) \ln 3} = \frac{1}{9 \ln 3}$$

$$3) f(x) = 0,2^{x-3}$$

$$f'(4) = 0,2^{x-3} \ln 0,2 = 0,2 \ln 0,2$$

$$4) f(x) = 2,5^{x-1}$$


$$f'(2) = 2,5^{x-1} \ln 2,5 = 2,5 \ln 2,5$$


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1)  $f(x) = x \ln x, x_0 = 0,5$   
 $f(0,5) = 0,5 \ln 0,5$   
 $f'(x) = \ln x + x \cdot \frac{1}{x} = \ln 0,5 + 1$   
 $y = 0,5 \ln 0,5 + (\ln 0,5 + 1)(x - 0,5)$

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1)  $f(x) = 2 \ln x + x^{-2}, x > 0$   
 $f'(x) = \frac{2}{x} - \frac{2}{x^3} = 0, x \neq 0$   
 $2x^2 - 2 = 0$   
 $x = 1, x = -1$   
  
 MC:  $(0; 1)$  kengigi  
 $(1; +\infty)$  o'cegi


3)  $f(x) = x^3 \cdot e^{-3x}$   
 $f'(x) = 3x^2 \cdot e^{-3x} + x^3 \cdot (-3e^{-3x}) = 0$   
 $3e^{-3x}(x^2 - x^3) = 0$   
 $x^2(1-x) = 0$   
 $x = 0, x = 1$   
  
 MC:  $(-\infty; 0)$  MC/e  
 $(1; +\infty)$  kengigi  
 $(0; 1)$  o'cegi

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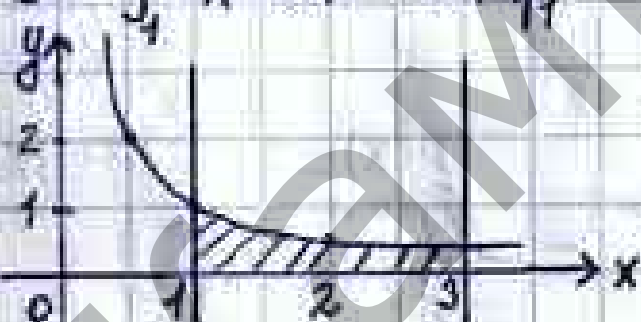
1)  $f(x) = x \ln x, (0; \frac{1}{e})$   
 $f'(x) = \ln x + 1 = 0, x > 0$   
 $\ln x = -1$   
 $x = \frac{1}{e}$   
  
 MC:  $(0; \frac{1}{e})$  kengigi  
 $(\frac{1}{e}; +\infty)$  MC

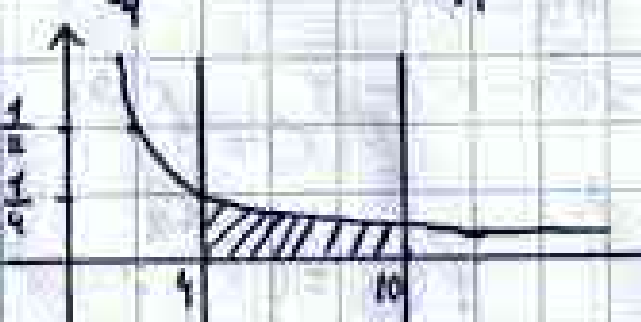
2)  $f(x) = \ln(x^2 + 2x), x_0 = 2$   
 $f(2) = \ln 8$   
 $f'(x) = \frac{2x+2}{x^2+2x} = \frac{2 \cdot 2 + 2}{4+4} = \frac{6}{8} = \frac{3}{4}$   
 $y = \ln 8 + \frac{3}{4}(x-2) = \ln 8 + \frac{3x}{4} - 1,5$

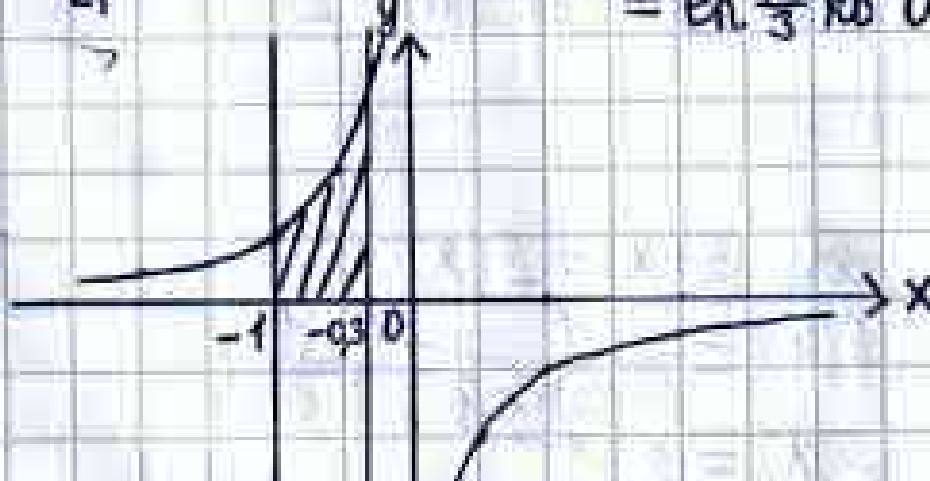
2)  $f(x) = x^2 \cdot e^x$   
 $f'(x) = 2x \cdot e^x + x^2 \cdot e^x = 0$   
 $e^x(2x + x^2) = 0$   
 $2x + x^2 = 0$   
 $x = 0, x = -2$   
  
 MC:  $(-\infty; -2]$  MC/e  
 $(0; +\infty)$  o'cegi,  $[-2; 0]$  MC


4)  $f(x) = x^3 - 3 \ln(2x), x > 0$   
 $f'(x) = 3x^2 - \frac{3}{x} = 0, x \neq 0$   
 $3x^3 - 3 = 0$   
 $x = 1$   
  
 MC:  $(0; 1)$  kengigi,  $(1; +\infty)$  o'cegi

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1)  $y = \frac{1}{x}, y = 0, x = 1, x = 3$   
 $S = \int_1^3 \frac{1}{x} dx = \ln|x| \Big|_1^3 = \ln 3 - \ln 1 = \ln 3$   
  
 MC:  $\ln 3$  kb 5

2)  $y = \frac{1}{x}, y = 0, x = 4, x = 10$   
 $S = \int_4^{10} \frac{1}{x} dx = \ln|x| \Big|_4^{10} = \ln 10 + \ln 4$   
  
 MC:  $\ln 10 + \ln 4$

3)  $y = \frac{1}{x}, y = 0, x = -0,3, x = -1$   
 $S = \int_{-1}^{-0,3} \frac{1}{x} dx = -\ln|x| \Big|_{-1}^{-0,3} = -\ln 0,3 + \ln 1 = \ln \frac{10}{3}$   
  
 MC:  $\ln \frac{10}{3}$  kb 5

4)  $y = -\frac{1}{x}, y = 0, x = -3, x = -2$   
 $S = \int_{-3}^{-2} (-\frac{1}{x}) dx = -\ln|x| \Big|_{-3}^{-2} = -\ln 2 + \ln 3 = \ln \frac{3}{2}$   
  
 MC:  $\ln 0,5$  kb 5

~315

1)  $f(x) = \frac{5^x}{x^2+1}$   
 $f'(1) = \frac{(5^x)'(x^2+1) - (x^2+1)'(5^x)}{(x^2+1)^2} = \frac{5 \ln 5 \cdot (x^2+1) - 2x \cdot 5^x}{(x^2+1)^2}$   
 $= \frac{10 \ln 5 - 10}{4} = 2,5(\ln 5 - 1)$

2)  $f(x) = \frac{\ln x}{x^3}$   
 $f'(e) = \frac{(\ln x)'x^3 - (x^3)'(\ln x)}{x^6} = \frac{1 - 3 \ln x}{x^4} = \frac{1 - 3 \ln e}{e^4} = -\frac{2}{e^4}$

3)  $f(x) = e^{-x^2}$   
 $f'\left(\frac{1}{\sqrt{2}}\right) = -2x e^{-x^2} = -\frac{2}{\sqrt{2}} \cdot e^{-\frac{1}{2}} = -\frac{2}{\sqrt{2} \cdot \sqrt{e}} = -\sqrt{\frac{2}{e}}$

~316

1)  $f(x) = \frac{2}{2x-1}$        $f(x) = 2 \cdot \frac{1}{2} \ln|2x-1| + C = \ln|2x-1| + C$

2)  $f(x) = e^{3x+2}$        $F(x) = \frac{1}{3} e^{3x+2} + C$

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1)  $f(x) = \frac{x^2}{0,5^{1-2x}}$   
 $f'(1) = \frac{2x \cdot 0,5^{1-2x} - (-2) \cdot 0,5^{1-2x} \cdot \ln 0,5 \cdot x^2}{0,5^{(1-2x)^2}} = \frac{2 \cdot 0,5^{1-2} + 2 \cdot 0,5^{1-2} \cdot \ln 0,5 \cdot 1}{0,5^{(1-2)^2}} = 1 + \ln 0,5$

2)  $f(x) = \frac{3^{1-2x}}{x^{-4}}$   
 $f'(2) = \frac{-2 \cdot 3^{1-2x} \cdot \ln 3 \cdot x^{-4} + 4x^{-5} \cdot 3^{1-2x}}{x^{-8}} = \frac{-2 \cdot 3^{1-4} \cdot \ln 3 \cdot 4 \cdot 2^{-5} \cdot 3 + 4 \cdot 2^{-5} \cdot 3^{1-4}}{2^{-8}} = \frac{1}{432} - \frac{\ln 3}{108}$

3)  $f(x) = \ln(1,5-x) - e^{x-1}$        $f(1) = \frac{-1}{1,5-1} - e^{1-1} = \frac{-1}{0,5} - 1 = -2 - 1 = -3$

4)  $f(x) = \ln(2-3x) + x$        $f\left(\frac{1}{3}\right) = \frac{-3}{2-1} + 1 = -3 + 1 = -2$

~318

1)  $f(x) = x \cdot e^{2x-1}$   
 $f'(x) = e^{2x-1} + 2 \cdot e^{2x-1} \cdot x = 0$   
 $e^{2x-1}(1+2x) = 0$   
 $1+2x = 0$   
 $x = -0,5$   
 MC:  $(-\infty; -0,5]$  kresnugi,  $[-0,5; +\infty)$  oseg

2)  $f(x) = \log_5(1-3x)$   
 $f'(x) = \frac{-3}{(1-3x) \ln 5} = 0$   
 $1-3x \neq 0$   
 $x = \frac{1}{3}$   
 MC:  $(-\infty; \frac{1}{3})$  kresnugi,  $(\frac{1}{3}; +\infty)$  oseg

~319

1)  $f(x) = \ln(2x) + x^{-1}$        $x_0 = 0,5$   
 $f(0,5) = \ln(2 \cdot 0,5) + 0,5^{-1} = 2$   
 $f'(0,5) = \frac{1}{x} - \frac{1}{x^2} = \frac{1}{0,5} - \frac{1}{0,25} = -2$   
 $y = 2 - 2(x-0,5) = 3 - 2x$

2)  $f(x) = e^{1+2x} - 4x^3$        $x_0 = -0,5$   
 $f(-0,5) = e^{1-1} - 4 \cdot 0,125 = 1,5$   
 $f'(-0,5) = 2e^{1+2x} - 12x^2 = 2 - 3 = -1$   
 $y = 1,5 - 1(x+0,5) = -x$

3)  $f(x) = \ln(-0,5x) - x^2$        $x_0 = -2$   
 $f(-2) = \ln 1 - 4 = -4$   
 $f'(-2) = \frac{1}{x} - 2x = \frac{1}{-2} + 4 = 3 \frac{1}{2} = 3,5$   
 $y = -4 + 3,5(x+2) = 3,5x + 3$

4)  $f(x) = e^{1-2x} - x^{-2}$        $x_0 = 0,5$   
 $f(0,5) = e^{1-1} - 4 = -3$   
 $f'(0,5) = -2 \cdot e^{1-2x} + 2x^{-3} = -2e^{1-1} + \frac{2}{0,125} = 14$   
 $y = -3 + 14(x-0,5) = 14x - 10$  59

~320

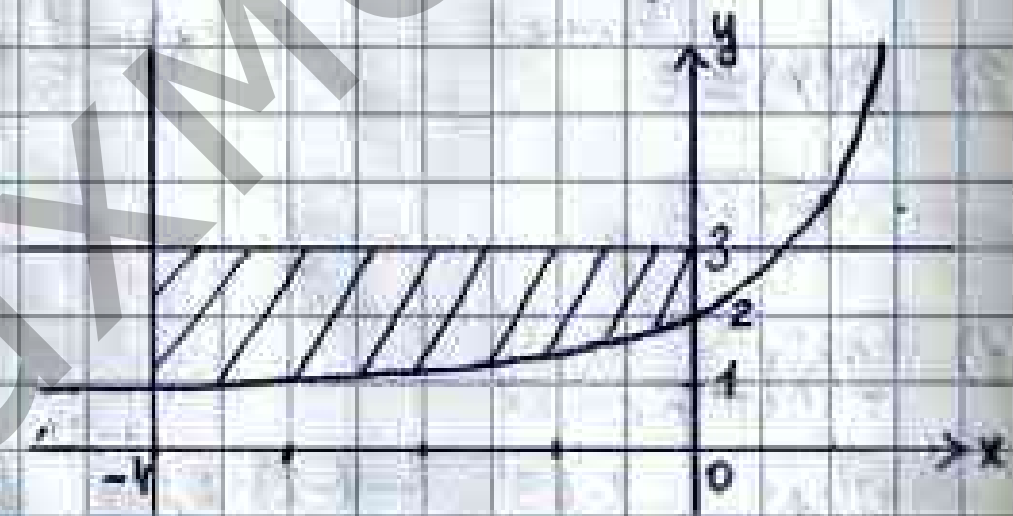
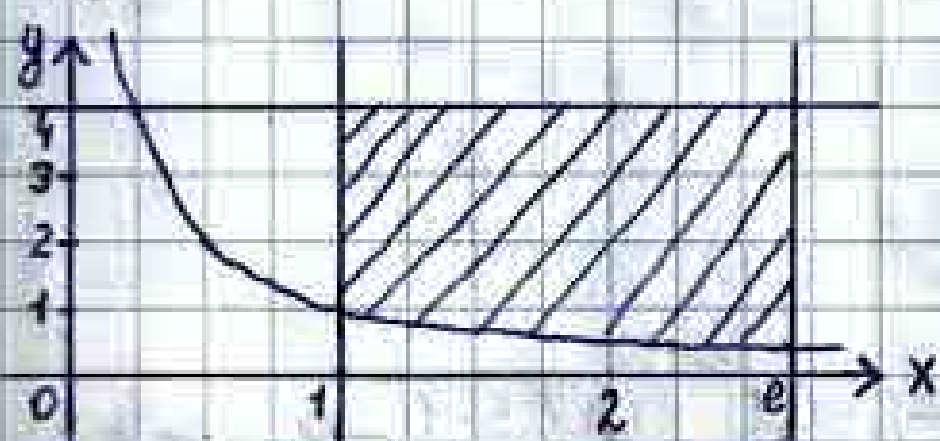
1)  $y = x + \ln(-x) \quad [-4; -0,5]$   
 $y' = 1 + \frac{1}{x} = 0 \quad x \neq 0$   
 $x + 1 = 0$   
 $x = -1$   
 $y(-1) = -1 + \ln(1) = -1$  ең үлкен  
 $y(-4) = -4 + \ln 4 = -2,6$  ең кіші  
 $y(-0,5) = -0,5 + \ln 0,5 = -1,2$

2)  $y = x + e^{-x} \quad [-\ln 4; \ln 2]$   
 $y' = 1 - e^{-x} = 0$   
 $e^{-x} = 1 \Rightarrow x = 0$   
 $y(0) = 0 + e^0 = 1$   
 $y(-\ln 4) = -\ln 4 + e^{\ln 4} = 4$  ең үлкен  
 $y(\ln 2) = \ln 2 + e^{-\ln 2} = 1,2$

~321

1)  $y = \frac{1}{x}, \quad y = 4, \quad x = 1, \quad x = e$   
 $S = \int_1^e (4 - \frac{1}{x}) dx = 4x - \ln|x| \Big|_1^e =$   
 $= 4e - \ln e - 4 + \ln 1 = 4e - 5$  кв. бірлік

2)  $y = 1 + e^x, \quad x = 0, \quad x = 4, \quad y = 3$   
 $S = \int_0^4 (3 - 1 - e^x) dx = 2x - e^x \Big|_0^4 =$   
 $= e^0 - (-8 - e^{-4}) = 7 + \frac{1}{e^4}$



~322

1)  $f(x) = e^{-1-x} + \ln(e^x)$   
 $f'(x) = -e^{-1-x} + \frac{1}{x} = 0$   
 $f(-1) = 0 = 0$

2)  $f(x) = e^{1+2x} \ln(-x)$   
 $f'(-0,5) = 2 \cdot e^{1+2x} \cdot \frac{1}{x} = 2 \cdot e^{1-2 \cdot 0,5} \cdot \frac{1}{0,5} = 4$   
 $f''(-0,5) = 4 > 0$

~324

1)  $g(x) = e^{3x-6}$   
 $g(2) = e^{3 \cdot 2 - 6} = 1, \quad x_0 = 2$   
 $g'(2) = 3 \cdot e^{3x-6} = 3 \cdot e^{3 \cdot 2 - 6} = 3$   
 $y = 1 + 3(x - 2) = 3x - 5$

2)  $f(x) = x^{-2} \cdot \ln(4x+3), \quad x_0 = -0,5$   
 $f(-0,5) = 4 \cdot \ln(4 \cdot (-0,5) + 3) = 0$   
 $f'(-0,5) = -2x^{-3} \ln(4x+3) + x^{-2} \cdot \frac{4}{4x+3} =$   
 $= -2 \cdot (-0,5)^{-3} \ln(-2+3) + 4 \cdot \frac{4}{-2+3} = 16$   
 $y = 16(x + 0,5) = 16x + 8$

3)  $f(x) = x^{-3} \cdot \ln(2x-3), \quad x_0 = 2$   
 $f(2) = \frac{1}{8} \cdot \ln(2 \cdot 2 - 3) = \frac{1}{8} \ln 1 = 0$   
 $f'(2) = \frac{-3}{x^4} \cdot \ln(2x-3) + \frac{1}{x^3} \cdot \frac{2}{2x-3} =$   
 $= \frac{-3}{16} \cdot \ln 1 - \frac{1}{8} \cdot \frac{2}{2 \cdot 2 - 3} = -0,25$   
 $y = -0,25(x - 2) = -0,25x + 0,5$

4)  $f(x) = x^{-2} \cdot e^{1+2x}, \quad x_0 = -0,5$   
 $f(-0,5) = (-0,5)^{-2} \cdot e^{1+2 \cdot (-0,5)} = 4$   
 $f'(-0,5) = \frac{-2}{x^3} \cdot e^{1+2x} + \frac{1}{x^2} \cdot 2e^{1+2x} =$   
 $= \frac{-2}{(-0,5)^3} \cdot e^{1-1} + \frac{1}{(-0,5)^2} \cdot 2 \cdot e^{1+2 \cdot (-0,5)} = 6$   
 $y = 4 + 16(x + 0,5) = 16x + 12$

~326

$f(x) = \frac{\ln^2 x}{x} \quad [2; e^2]$   
 $\ln x \left( \frac{2}{x} - 1 \right) = 0$   
 $\ln x = 0 \quad \frac{2}{x} - 1 = 0$   
 $x = 1 \text{ и } 2 - x = 0$   
 $x = 2$

$f'(x) = \frac{2}{x} \ln x - \ln x = 0 \quad x \neq 0$   
 $f(1) = \frac{\ln^2 1}{1} = 0$  ең кіші  
 $f(2) = \frac{\ln^2 2}{2} \approx \frac{0,49}{2} = 0,245$  ең кіші  
 $f(e^2) = \frac{\ln^2 e^2}{e^2} = \frac{4}{e^2}$  ең үлкен

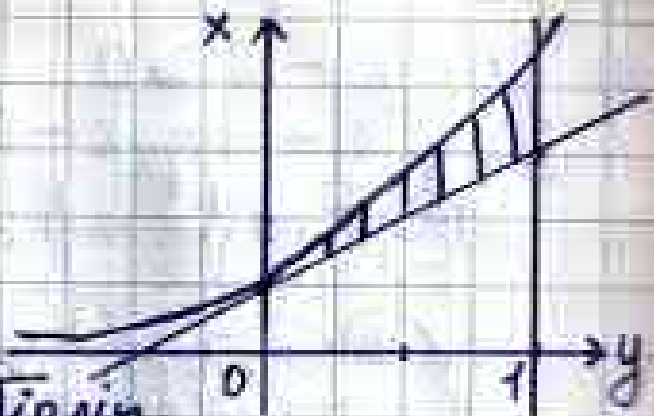
~327

$$y = e^{2x} \quad (0; 1) \quad x=1$$

$$y(0) = e^{2 \cdot 0} = 1 \quad y'(0) = 2 \cdot e^{2 \cdot 0} = 2$$

$$y = 1 + 2(x-0) = 2x + 1$$

$$S = \int_0^1 (e^{2x} - (2x+1)) dx = \left. \frac{1}{2} e^{2x} - x^2 - x \right|_0^1 = \frac{e^2}{2} - 1 - 1 - \frac{e^0}{2} = \frac{e^2}{2} - 2,5 \text{ кв. единиц}$$



Өзінгі тапсырма!

1)  $y = \frac{x+2}{343-49^x}$   $343 - 49^x \neq 0 \quad 49^x \neq 343 \quad 7^{2x} = 7^3 \quad x \neq 1,5$   
 (A)  $(-\infty; 1,5) \cup (1,5; +\infty)$

2)  $\log_{1,2} \left[ \frac{25}{36} \cdot \left(\frac{6}{5}\right)^{3,2} \right] = \log_{1,2} \left[ \left(\frac{6}{5}\right)^{-2} \cdot \left(\frac{6}{5}\right)^{3,2} \right] = \log_{1,2} [1,2^{-2+3,2}] = \log_{1,2} 1,2^{1,2} = 1,2$  (B)

3)  $\left(\frac{1}{625}\right)^{-\log_5 a} - 49^{1+\log_7 a} = 5^{4\log_5 a} - 49^{\log_7 a + \log_7 a} = 5^{\log_5 a^4} - 7^{\log_7 49a^2} = a^4 - 49a^2$  (C)

4)  $y = \log_{3,4} (-2x^2 + 3x - 1)$   
 $(-2x^2 + 3x - 1) > 0$   
 $2x^2 - 3x + 1 < 0$   
 $D = 9 - 4 \cdot 2 = 1$   
 $x_1 = \frac{3-1}{2 \cdot 2} = 0,5 \quad x_2 = \frac{3+1}{2 \cdot 2} = 1$   
 MC: (0,5; 1)

5)  $\left(\frac{1}{2}\right)^{-4}; 1; 4^{-\sqrt{3}}; 8 \Rightarrow 4^{-\sqrt{3}}; 1; 8; \left(\frac{1}{2}\right)^{-4}$  (D)

6)  $\log_{22} 9 = a$  немесе  $\log_{22} 46 = b$   $\log_{81} 414 = ?$   
 $\log_{81} 414 = \frac{\log_{22} 9 + \log_{22} 46}{\log_{22} 9^3} = \frac{a+b}{2a}$  (B)

7)  $11^{x-1} - 11^{x+2} + 1330 = 0$   
 $\frac{11^x}{11} - 11^x \cdot 1331 = -1330$   
 $-1330 \cdot 11^x = 11 \cdot (-1330)$   
 $11^x = 11$   
 $x = 1$  (D)

8)  $0,25^{2+0,5x^2} > 32^x$   
 $\frac{1}{2}^{-4-x^2} > 2^{5x}$   
 $x^2 + 5x + 4 < 0$   
 $D = 25 - 16 = 9$   
 $x_1 = \frac{-5-3}{2} = -4 \quad x_2 = \frac{-5+3}{2} = -1$   
 MC: (-4; -1) және -2 (B)

9)  $y = \log_6 (x^2 + 6x) - 3$   
 $x^2 + 6x > 0$   
 $x^2 + 6x < 216$   
 $\log_6 (x^2 + 6x) - 3 < 0$   
 $x^2 + 6x - 216 = 0$   
 $D = 36 + 4 \cdot 216 = 900$   
 $x_1 = \frac{-6-30}{2} = -18 \quad x_2 = \frac{-6+30}{2} = 12$   
 $\log_6 (x^2 + 6x) < \log_6 216$   
 $x^2 + 6x < 216$   
 $x(x+6) = 0$   
 $x_1 = 0 \quad x_2 = -6$   
 MC: (-18; -6) \cup (0; 12)

10)  $\begin{cases} 3^y = 27^x \\ \log_2 (y - x^2) = 1 \\ y - x^2 = 2 \end{cases} \begin{cases} y = 3x \\ 3x - x^2 = 2 \\ x^2 - 3x + 2 = 0 \\ x_1 = 1 \quad x_2 = 2 \end{cases} \begin{cases} y = 3 \\ x = 1 \end{cases}$  Н/е  $\begin{cases} y = 6 \\ x = 2 \end{cases}$  (D)



$$\begin{cases} x^2 + x - 6 \geq 0 \\ \log_4^2 x - \log_4 x - 6 < 0 \end{cases}$$



$$x \in [2; 64) \quad \text{B}$$

$$x^2 + x - 6 = 0$$

$$D = 1 + 24 = 25$$

$$x_1 = \frac{-1-5}{2} = -3$$

$$x_2 = \frac{-1+5}{2} = 2$$

$$\log_4^2 x - \log_4 x - 6 = 0$$

$$D = 1 + 24 = 25$$

$$\log_4 x = \frac{1-5}{2} = -2$$

$$x = 4^{-2} \\ x = \frac{1}{16}$$

$$\log_4 x = \frac{1+5}{2} = 3$$

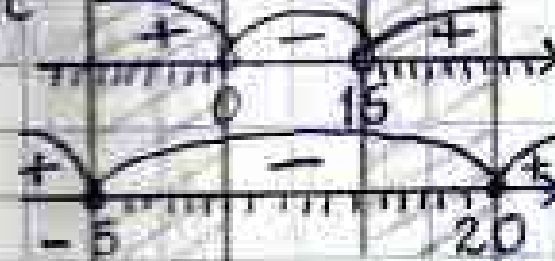
$$x = 4^3 \\ x = 64$$

$$12) \frac{1}{125} \leq 5^{-x+5} < 3125 \Rightarrow 5^{-3} \leq 5^{-x+5} < 5^5 \quad -3 \leq -x+5 < 5 \quad 8 > x > 0 \quad \text{C}$$

$$13) y = \log_7(\cos 2x) \quad y'(\frac{\pi}{8}) \Rightarrow y'(\frac{\pi}{8}) = \frac{-2 \sin 2x}{\cos 2x \ln 7} = \frac{-2 \tan 2x}{\ln 7} = \frac{-2 \tan(2 \cdot \frac{\pi}{8})}{\ln 7} = \frac{-2}{\ln 7} \quad \text{A}$$

$$14) \log(x^2 - 15x) \leq 2$$

$$\begin{cases} x^2 - 15x > 0 \\ x^2 - 15x \leq 100 \end{cases}$$



$$x^2 - 15x = 0$$

$$x(x - 15) = 0$$

$$x_1 = 0 \quad x - 15 = 0$$

$$x_2 = 15$$

$$x^2 - 15x - 100 = 0$$

$$D = 225 + 4 \cdot 100 = 625$$

$$x_1 = \frac{15-25}{2} = -5$$

$$x_2 = \frac{15+25}{2} = 20$$

$$x \in [-5; 0) \cup (15; 20] \quad \text{A}$$

$$15) y = x e^{-3x} + 4 \quad x = -1 \\ y(-1) = -e^3 + 4 \quad y'(-1) = e^{-3x} - 3e^{-3x} \cdot x = e^3 + 3e^3 = 4e^3 \\ y = -e^3 + 4 + 4e^3(x+1) = -e^3 + 4 + 4e^3x + 4e^3 = 4e^3x + 3e^3 + 4 \quad \text{A}$$

$$16) y = x^3 - 3 \ln x \quad x > 0 \\ y' = 3x^2 - \frac{3}{x} = 0 \Rightarrow 3x^3 - 3 = 0 \Rightarrow x = 1 \quad \text{nc: } (0; 1) \text{ kenuji} \\ (1; +\infty) \text{ o'cegi} \quad \text{B}$$

$$17) y = 0,5x^2 - 6x + 2 \ln x^4 \quad x > 0 \\ y' = x - 6 + \frac{8x^3}{x^4} = 0 \\ x^2 - 6x + 8 = 0 \\ D = 36 - 32 = 4 \\ x_1 = \frac{6-2}{2} = 2 \quad x_2 = \frac{6+2}{2} = 4$$



$$x_{\max} = 2 \\ x_{\min} = 4 \quad \text{C}$$

$$18) y = x^2 e^x \quad [-1; 2] \\ y' = 2xe^x + x^2 e^x = 0 \\ e^x x(2+x) = 0 \\ x_1 = 0 \quad x_2 = -2 \notin [-1; 2] \\ y(-1) = e^{-1} = \frac{1}{e} \\ y(0) = 0 \cdot e^0 = 0 \\ y(2) = 4 \cdot e^2 \quad \text{en kishi} \\ \text{en y'ken} \quad \text{D}$$

$$19) \int_1^2 (3^x - \frac{3}{x}) dx = \left[ \frac{3^x}{\ln 3} - 3 \ln x \right]_1^2 = \frac{9}{\ln 3} - 3 \ln 2 - \frac{3}{\ln 3} + 3 \ln 1 = \frac{6}{\ln 3} - 3 \ln 2 \quad \text{C}$$

$$20) y = 0,5^x \quad y = 0, \quad x = 1, \quad x = 2 \\ S = \int_1^2 0,5^x dx = \left[ \frac{0,5^x}{\ln 0,5} \right]_1^2 = \frac{0,5}{\ln 0,5} - \frac{0,5}{\ln 0,5} = \frac{0,25 + 0,5}{\ln 2} = \frac{1}{4 \ln 2} \quad \text{C}$$



~328

$\frac{+5}{2} = 3$

1)  $x^2 = 1$  яки  $(x+2)x^2 = x+2$   
 $x = \pm 1$   $(x+2)(x^2-1) = 0$   
 $x = -2$   $x = \pm 1$

я: екінші теньдеу бірінші теньдеудің салдары болады

2)  $x^3 = x$   $x^3 + \frac{1}{x} = x + \frac{1}{x}$   
 $x = \pm 1$   $x = \pm 1$

я: бірінші теньдеу екінші теньдеудің салдары болады

~329

С

2 А  
Ент

1)  $x^2 - 3 + \frac{1}{x+1} = 2x + \frac{1}{x+1}$   
 $x^2 - 3 = 2x$   
 $x^2 - 2x - 3 = 0$   
 $D = 4 + 12 = 16$   
 $x_1 = \frac{2-4}{2} = -1$   $x_2 = \frac{2+4}{2} = 3$

2)  $\sqrt{(x-2)(x+3)} = 6$   
 $x^2 + 3x - 2x - 6 = 36$   
 $x^2 + x - 42 = 0$   
 $D = 1 + 168 = 169$   
 $x_1 = \frac{-1-13}{2} = -7$   $x_2 = \frac{-1+13}{2} = 6$

~330

1)  $(x^2-10)^2 + 2(x-10) + 1 = 0$   $(x^2-10)=t$   
 $t^2 + 2t + 1 = 0$   
 $D = 4 - 4 = 0$   
 $t = \frac{-2}{2} = -1$   $(x^2-10) = -1$   
 $x = \pm 3$

2)  $(x^2-8)^2 + 4(x^2-8) - 5 = 0$   $(x^2-8)=t$   
 $t^2 + 4t - 5 = 0$   
 $D = 16 + 20 = 36$   
 $t_1 = \frac{-4-6}{2} = -5$   $(x^2-8) = -5$   
 $x_1 = +\sqrt{3}$   
 $t_2 = \frac{-4+6}{2} = 1$   $(x^2-8) = 1$   
 $x_2 = \pm 3$

~331

1)  $\frac{x+56}{9x^2-16} + \frac{1}{2(3x-4)} = \frac{18}{x(3x+4)}$   
 $\frac{x+56}{9x^2-16} + \frac{1}{2(3x-4)} - \frac{18}{x(3x+4)} = 0$   
 $2x^2 + 142x - 3x^2 - 4x - 108x + 144 = 0$   
 $-x^2 + 144 = 0$   
 $x = \pm 12$

2)  $\sqrt{x^2+10x+25} - \sqrt{x^2-8x+16} = 5$   
 $\sqrt{(x+5)^2} - \sqrt{(x-4)^2} = 5$   
 $\sqrt{(x+5)^2} - \sqrt{(4-x)^2} = 5$   
 $x+5-4+x=5$   
 $2x=4$   
 $x=2$

~332

1)  $\cos^2 x + 3 \cos x = 0$   $[0^\circ; 90^\circ]$   
 $\cos x (\cos^2 x + 3) = 0$   
 $\cos x = 0$   
 $x = \frac{\pi}{2} + \pi n$   $n=0$   $x = \frac{\pi}{2}$

2)  $\operatorname{tg}^2 x = \operatorname{tg} x$   $[0^\circ; 45^\circ]$   
 $\operatorname{tg} x = 1$   
 $x = \frac{\pi}{4} + \pi n$   $n \in \mathbb{Z}$   
 $n=0$   $x = \frac{\pi}{4}$

~333

1)  $7^{2x} - 6 \cdot 7^x + 5 = 0$   
 $D = 36 - 4 \cdot 5 = 16$   
 $7^x = \frac{6-4}{2} = 1$   $7^x = \frac{6+4}{2} = 5$   
 $x_1 = 0$   $x = \log_7 5$

2)  $3^{x+1} + \frac{18}{3^x} = 29$   
 $3 \cdot 3^{2x} - 29 \cdot 3^x + 18 = 0$   
 $D = 841 - 4 \cdot 3 \cdot 18 = 625$   
 $3^x = \frac{29-25}{2 \cdot 3} = \frac{2}{3}$   $3^x = \frac{29+25}{2 \cdot 3} = 9$   
 $x = \log_3 \frac{2}{3}$   $x = 2$

~334

$$1) \begin{cases} x^2 - y = 14 \\ 3x + y = 4 \end{cases} \quad \begin{cases} y = x^2 - 14 \\ x^2 + 3x - 18 = 0 \\ D = 9 + 4 \cdot 18 = 81 \\ x_1 = \frac{-3+9}{2} = 3 \\ x_2 = \frac{-3-9}{2} = -6 \end{cases}$$

$$\begin{cases} x = 3 \\ y = -5 \end{cases} \text{ и/е } \begin{cases} x = -6 \\ y = 22 \end{cases}$$

$$2) \begin{cases} 2^x - 2^y = 24 \\ x + y = 8 \end{cases} \quad \begin{cases} x = 8 - y \\ 2^{8-y} - 2^y = 24 \\ 256 - 2^y - 24 = 0 \\ 2^y + 24 \cdot 2^y - 256 = 0 \\ D = 576 + 4 \cdot 256 = 1600 \\ 2^y = \frac{-24 \pm 40}{2} = 8 \\ y = 3 \end{cases}$$

$$\begin{cases} x = 5 \\ y = 3 \end{cases}$$

~335

$$1) \begin{cases} \cos^2 x = 0 \\ 1 + \cos 2x = 0 \\ \cos 2x = -1 \\ 2x = \pi + 2\pi n \\ x = \frac{\pi}{2} + \pi n \quad n \in \mathbb{Z} \end{cases}$$

$$\begin{cases} x^2 = \frac{\pi}{2} \\ x = \pm \sqrt{\frac{\pi}{2}} \end{cases}$$

п: болмайды

$$2) \begin{cases} 2 \log_a x = 0 \\ \log_a x^2 = 0 \\ x^2 = a^0 \\ x = \pm 1 \end{cases} \quad \begin{cases} \log_a x^2 = 0 \\ x^2 = a^0 \\ x^2 = 1 \\ x = \pm 1 \end{cases}$$

п: бірінші теңдеу екінші теңдеудің салдары болады

~336

$$1) \begin{cases} (\sqrt{x^2+1})^2 = (x+1)^2 \\ x^2+1 = x^2+2x+1 \\ 2x = 0 \\ x = 0 \end{cases}$$

$$2) \begin{cases} \sin x = \frac{\sqrt{2}}{2} \\ x = (-1)^n \frac{\pi}{4} + \pi n \quad n \in \mathbb{Z} \end{cases} \quad \begin{cases} \operatorname{tg} x = 1 \\ x = \frac{\pi}{4} + \pi k \end{cases}$$

п: бірінші теңдеу екінші теңдеудің салдары болады

~337

$$1) \frac{1}{x^2+2x-3} + \frac{18}{x^2+2x+2} - \frac{18}{x^2+2x+1} = 0$$

$$\frac{1}{t-4} + \frac{18}{t+1} - \frac{18}{t} = 0$$

$$t^2 + t + 18t^2 - 72t - 18(t^2 - 3t - 4) = 0$$

$$19t^2 - 71t - 18t^2 + 54t + 72 = 0$$

$$t^2 - 17t + 72 = 0$$

$$D = 289 - 288 = 1$$

$$t_1 = \frac{17-1}{2} = 8 \quad t_2 = \frac{17+1}{2} = 9$$

$$\begin{cases} x^2 + 2x + 1 = t \\ (x+1)^2 = t \end{cases}$$

$$\begin{cases} (x+1)^2 = 8 \\ x+1 = \pm 2\sqrt{2} \\ x = \pm 2\sqrt{2} - 1 \end{cases} \quad \begin{cases} (x+1)^2 = 9 \\ x+1 = 3 & x+1 = -3 \\ x = 2 & x = -4 \end{cases}$$

(~338)

1)  $\cos^2(\pi-x) + 8\cos(\pi+x) = 0, [90^\circ; 270^\circ]$   
 $\cos^2 x - 8\cos x = 0$   
 $\cos x(\cos x - 1) = 0$   
 $\cos x = 0 \quad \cos x = 1$   
 $x = \frac{\pi}{2} + \pi n, \quad x = 2\pi n \quad n \in \mathbb{Z}$   
 $n_1 = 0 \quad x = \frac{\pi}{2}; \quad n_2 = 1 \quad x = \frac{3\pi}{2}$   
 MC:  $\frac{\pi}{2}, \frac{3\pi}{2}$

2)  $\cos 2x - 2\sin^2 x = -3 \quad [0^\circ; 180^\circ]$   
 $1 - 2\sin^2 x - 2\sin^2 x = -3$   
 $4\sin^2 x = 4$   
 $1 - \cos 2x = 1$   
 $\cos 2x = -1$   
 $x = \frac{\pi}{2} + \pi n \quad n \in \mathbb{Z}$   
 $n = 0 \quad x = \frac{\pi}{2}$  MC:  $\frac{\pi}{2}$  н/е  $90^\circ$

(~339)

1)  $25^{\sqrt{x}} - 124 \cdot 5^{\sqrt{x}} = 125 \quad 5^{\sqrt{x}} = t$   
 $t^2 - 124t - 125 = 0$   
 $D = 15376 + 4 \cdot 125 = 15876$   
 $t_1 = \frac{124 - 126}{2} = -1$   
 $t_2 = \frac{124 + 126}{2} = 125$   
 $5^{\sqrt{x}} = 125$   
 $5^{\sqrt{x}} = 5^3$   
 $x = 9$

2)  $\log_x \sqrt{5} - 1,25 = (\log_x \sqrt{5})^2 \quad \log_x \sqrt{5} = t$   
 $t^2 - 3t + 1,25 = 0$   
 $D = 9 - 4 \cdot 1,25 = 4$   
 $t_1 = \frac{3-2}{2} = \frac{1}{2}$   
 $t_2 = \frac{3+2}{2} = \frac{5}{2}$   
 $\log_x \sqrt{5} = 0,5$   
 $x_1 = 5$   
 $\log_x \sqrt{5} = 2,5$   
 $x_2 = 5$

(~340)

1)  $\begin{cases} x^2 - y^2 = 35 \\ x^2 y - xy^2 = 30 \end{cases}$   
 $(x-y)(x+y) = 35$   
 $xy(x-y) = 30$   
 $6(x+y) = 7xy$   
 $7xy - 6x - 6y = 0$   
 $x(7y-6) - 6y = 0$   
 $x = \frac{6y}{7y-6}$

$\begin{cases} (7y-6)^2 - y^2 = 35 \\ -(y^2+35)(7y-6)^2 + 36y^2 = 0 \\ -49y^4 + 84y^3 - 175y^2 + 252y - 1260 + 36y^2 = 0 \\ 7y^4 - 12y^3 + 245y^2 - 420y + 980 = 0 \\ 7y^4 - 7y^3 - 5y^3 + 5y^2 + 240y^2 - 240y - 180y + 180 = 0 \\ 7y^3(y-1) - 5y^3(y-1) + 240y(y-1) - 180(y-1) = 0 \\ (y-1)(7y^3 - 5y^3 + 240y - 180) = 0 \\ y-1=0 \Rightarrow y=1 \text{ generic} \end{cases}$

$x = \frac{6 \cdot 1}{7 \cdot 1 - 6} = 6$   
 $\begin{cases} x = 6 \\ y = 1 \end{cases}$

(~341)

1)  $\begin{cases} \sqrt{3y+x} - \sqrt{6y-x} = x \\ \sqrt{3y+x} + \sqrt{6y-x} = 3y \end{cases}$   
 $\sqrt{3y+x} = a$   
 $\sqrt{6y-x} = b$   
 $\begin{cases} 3y+x = a^2 \\ 6y-x = b^2 \end{cases} \Rightarrow \begin{cases} y = \frac{a^2+b^2}{9} \\ x = \frac{2a^2-b^2}{3} \end{cases}$   
 $\begin{cases} 2a-b = \frac{2a^2-b^2}{3} \\ a+b = \frac{a^2+b^2}{3} \end{cases}$   
 $3a = \frac{2a^2-b^2+a^2+b^2}{3}$   
 $a(3-a) = 0$   
 $\begin{cases} a_1 = 0 \\ b_1 = 0 \\ b_2 = 3 \end{cases} \quad \begin{cases} a_2 = 3 \\ b_2 = 0 \\ b_2 = 3 \end{cases}$   
 $\begin{cases} \sqrt{3y+x} = 0 \Rightarrow x = 0 \\ \sqrt{6y-x} = 0 \Rightarrow y = 0 \end{cases}$   
 $\begin{cases} \sqrt{3y+x} = 0 \Rightarrow x = -1 \\ \sqrt{6y-x} = 3 \Rightarrow y = \frac{1}{3} \end{cases}$   
 $\begin{cases} \sqrt{3y+x} = 3 \Rightarrow x = 3 \\ \sqrt{6y-x} = 3 \Rightarrow y = 2 \end{cases}$

2)  $\begin{cases} 2^{\sqrt{x}+\sqrt{y}} = 512 \\ \lg \sqrt{xy} = \frac{1}{2} + \lg 20 \end{cases}$   
 $\sqrt{x} + \sqrt{y} = 9$   
 $\sqrt{xy} = 20\sqrt{10}$   
 $\begin{cases} \sqrt{x} = a \\ \sqrt{y} = b \end{cases}$   
 $\begin{cases} ab = 20\sqrt{10} \\ a+b = 9 \end{cases}$  внем меор  
 $a = 1 \quad b = 9 \quad c = 20\sqrt{10}$   
 $x^2 - 9x + 20\sqrt{10} = 0$   
 $D = 81 - 80\sqrt{10} < 0$   
 MC:  $\emptyset$

~342

1)  $2 \log_a f(x) = C$  яки  $\log_a f^2(x) = C$   
 $f(x) = a^{\frac{C}{2}}$  яки  $f(x) = a^{\frac{C}{2}}$   
 я: мандес тендүүлөр

2)  $\frac{1}{2} \log_a f(x) = C$  яки  $\log_a \sqrt{f(x)} = C$   
 $f(x) = a^{2C}$  яки  $f(x) = a^{2C}$   
 я: мандес емес тендүүлөр

3)  $3 \log_a f(x) = C$  яки  $\log_a f^3(x) = C$   
 $f(x) = a^{\frac{C}{3}}$  яки  $f(x) = a^{\frac{C}{3}}$   
 я: мандес тендүүлөр

4)  $\log_a f^2(x) = C$  яки  $2 \log_a |f(x)| = C$   
 $f(x) = a^{\frac{C}{2}}$  яки  $f(x) = a^{\frac{C}{2}}$   
 я: мандес тендүүлөр

~243

1)  $\begin{cases} x^2 + y^2 = 5 \\ \log_x^4 + \log_y^4 = \log 2 \end{cases} \begin{cases} x = \frac{2}{y} \\ \frac{4}{y^2} + y^2 = 5 \end{cases} \begin{cases} x > 0 \\ y > 0 \end{cases}$

$4 + y^4 = 5y^2$   
 $t^2 - 5t + 4 = 0$   
 $D = 25 - 4 \cdot 4 = 9$

$t_1 = \frac{5-3}{2} = 1$

$t_2 = \frac{5+3}{2} = 4$

$y^2 = t$

$y_1^2 = 1$

$y_1 = \pm 1 \Rightarrow x = \pm 2$

$y_2^2 = 4$

$y_2 = \pm 2 \Rightarrow x = \pm 1$

$x > 0$

$y > 0$

$x \neq -2$

$x \neq -1$

$x = 2$

$y = 1$

$x = 1$

$y = 2$

~344

1)  $x^4 + 4x^3 - 18x^2 - 12x + 9 = 0$

Торкер схемасы бойынша квадратка жіктөлөү:

$(x+1)(x-3)(x^2+6x-3) = 0$

$x_1 = -1, x_2 = 3$

$x^2 + 6x - 3 = 0$

$D = 36 + 12 = 48$

$x = \frac{-6 \pm 4\sqrt{3}}{2}$

$= -3 \pm 2\sqrt{3}$

2)  $32x^4 - 48x^3 - 10x^2 + 21x + 5 = 0$

Торкер схемасы бойынша квадратка жіктөлөү:

$(x-1)(x+0,5)(32x^2-32x-10) = 0$

$x_1 = 1, x_2 = -0,5$

$32x^2 - 32x - 10 = 0$

$D = 1024 + 4 \cdot 320 = 2304$

$x_3 = \frac{32-48}{32 \cdot 2} = -\frac{1}{4}, x_4 = \frac{32+48}{32 \cdot 2} = \frac{5}{4}$

~345

1)  $\sqrt{1 + \cos x} = \sin x$

$1 + \cos x = \sin^2 x$

$1 + \cos x - 1 + \cos^2 x = 0$

$\cos x (\cos x + 1) = 0$

$\cos x = 0$

$\cos x + 1 = 0$

$x = \frac{\pi}{2} + \pi n$

$x = \pi + 2\pi n, n \in \mathbb{Z}$

2)  $2 \sin 2x + 2 \sin x = 0$

$4 \sin x \cos x + 2 \sin x = 0$

$2 \sin x (2 \cos x + 1) = 0$

$2 \sin x = 0$

$2 \cos x + 1 = 0$

$x = \pi n$

$x = \pm \frac{2\pi}{3} + 2\pi n, n \in \mathbb{Z}$

~346

1)  $\log_3(4 + 15 \cdot 2^x + 27) = 2 \log_3(4 \cdot 2^x - 3)$

$4 + 15 \cdot 2^x + 27 = (4 \cdot 2^x - 3)^2$

$16 \cdot 2^{2x} - 24 \cdot 2^x + 9 - 2^{2x} - 15 \cdot 2^x - 27 = 0$

$15 \cdot 2^{2x} - 39 \cdot 2^x - 18 = 0$

$5 \cdot 2^{2x} - 13 \cdot 2^x - 6 = 0$

$D = 169 + 4 \cdot 5 \cdot 6 = 289$

$2^x \neq \frac{13-17}{10} \neq -\frac{2}{5}$

$2^x = \frac{13+17}{10} = 3$

$x = \log_2 3$

2)  $\log_{\sqrt{10}} \sqrt{x^3} \cdot \log(100x) = 3 \log x$

$-\log_{\sqrt{10}} x^{\frac{3}{2}} (\log 100 + \log x) - 3 \log x = 0$

$-\frac{3}{2} \log x (2 + \log x) - 3 \log x = 0$

$-\frac{3}{2} \log x - \frac{3}{2} \log^2 x - 3 \log x = 0$

$-\log x (\frac{3}{2} \log x + 4 \frac{3}{2}) = 0$

$-\log x = 0$

$\frac{3}{2} \log x + 3 = 0$

$\log x = \log 1$

$\log x = -\frac{2}{3} \cdot 3$

$x = 1$

$x = 10^{-2}$

$x = 0,1$

(~346)

$$1) \log_3(4^x + 15 \cdot 2^x + 27) = 2 \log_3(4 \cdot 2^x - 3)$$

$$4^x + 15 \cdot 2^x + 27 = (4 \cdot 2^x - 3)^2$$

$$2^{2x} + 15 \cdot 2^x + 27 = 16 \cdot 2^{2x} - 24 \cdot 2^x + 9$$

$$15 \cdot 2^{2x} - 39 \cdot 2^x - 18 = 0 \quad 2^x = t$$

$$5t^2 - 13t - 6 = 0$$

$$D = 169 + 4 \cdot 5 \cdot 6 = 289$$

$$t_1 = \frac{13 - 17}{10} = -0,4 \quad 2^x \neq -0,4$$

$$t_2 = \frac{13 + 17}{10} = 3 \quad 2^x = 3$$

$$x = \log_2 3$$

$$2) \log_{100} \sqrt{x^3} \lg(100x) = 3 \lg x$$

$$-\lg x^{\frac{3}{2}} \cdot (2 + \lg x) - 3 \lg x = 0$$

$$-\frac{3}{2} \lg x - \frac{3}{2} \lg^2 x - 3 \lg x = 0$$

$$-\lg x \left( \frac{3}{2} \lg x + 6 \right) = 0$$

$$-\lg x = 0 \quad \frac{3}{2} \lg x + 6 = 0$$

$$\lg x = \lg 1 \quad x = 1$$

$$x = 0,001$$

(~347)

$$1) \begin{cases} 2 \log_4^2 x - \log_4 x \cdot \log_4 y - 6 \log_4^2 y = 0 \\ \log_2^2 x - 2 = -\log_{0,5} y \end{cases}$$

$$\begin{cases} x = 4 \cdot 4 = 16 \\ y = 4 \end{cases} \text{ H/e } \begin{cases} x = 4 \sqrt[5]{16} \\ y = \sqrt[5]{16} \end{cases}$$

$$\log_2 x - 2 = \log_2 y$$

$$\log_2 x - \log_2 y - \log_2 4 = 0$$

$$\log_2 \frac{x}{4y} = \log_2 1$$

$$\frac{x}{4y} = 1$$

$$x = 4y$$

$$2 \log_4^2(4y) - \log_4(4y) \cdot \log_4 y - 6 \log_4^2 y = 0 \quad 6 \log_4^2 y = 0$$

$$2(\log_4 4 + \log_4 y)(\log_4 4 + \log_4 y) - (\log_4 4 + \log_4 y) \log_4 y - 6 \log_4^2 y = 0$$

$$2(1 + 2 \log_4 y + \log_4^2 y) - (1 + \log_4 y) \log_4 y - 6 \log_4^2 y = 0$$

$$2 + 4 \log_4 y + 2 \log_4^2 y - \log_4 y - \log_4^2 y - 6 \log_4^2 y = 0$$

$$5 \log_4^2 y - 3 \log_4 y - 2 = 0$$

$$D = 9 + 4 \cdot 5 \cdot 2 = 49$$

$$\log_4 y = \frac{3+7}{10} = 1 \Rightarrow y = 4; \quad \log_4 y = \frac{3-7}{10} = -\frac{2}{5}$$

$$2) \begin{cases} x \cdot 3^y + 3^{y+1} = 3^{y+1} + x \cdot 3^x \\ y - (x+2)^{0,5} = 0 \end{cases}$$

$$x \cdot 3^y + 3 \cdot 3^y - 3 \cdot 3^y - x \cdot 3^x = 0$$

$$x(3^y - 3^x) - 3(3^y - 3^x) = 0$$

$$(3^y - 3^x)(x - 3) = 0$$

$$3^y - 3^x = 0 \quad x - 3 = 0$$

$$3^y = 3^x \quad x_3 = 3$$

$$x = y$$

$$\begin{cases} x_1 = -1 \\ y_1 = -1 \end{cases}; \quad \begin{cases} x_2 = 2 \\ y_2 = 2 \end{cases}; \text{ H/e } \quad \begin{cases} x_3 = 3 \\ y_3 = \sqrt{5} \end{cases}$$

$$y = \sqrt{x+2} \quad \text{H/e} \quad y = \sqrt{x+2}$$

$$y^2 = x + 2$$

$$9 = 1 + 8 = 9$$

$$y_1 = \frac{1-3}{2} = -1 \Rightarrow x_1 = -1$$

$$y_2 = \frac{1+3}{2} = 2 \Rightarrow x_2 = 2$$

(~348)

$$1) \begin{cases} (x+y) 3^{y-x} = \frac{5}{27} \\ 3 \log_5(x+y) = x-y \\ \log_5(x+y) = \log_5 5^{x-y} \end{cases}$$

$$x = 5^{\frac{x-y}{3}} - y, \text{ H/e } x = \sqrt[3]{5}^{x-y} - y$$

$$\begin{cases} x = 4 \\ y = 1 \end{cases}$$

$$\left( \frac{\sqrt[3]{5}}{3} \right)^{x-y} - \frac{y+y}{3} \cdot 3^{y-x} = \frac{5}{27}$$

$$\left( \frac{\sqrt[3]{5}}{3} \right)^{x-y} = \left( \frac{\sqrt[3]{5}}{3} \right)^3$$

$$x - y = 3$$

$$y = x - 3 \Rightarrow x = \sqrt[3]{5}^{x-x+3} - x + 3$$

$$x = 4 \quad y = 4 - 3 = 1$$

~349

1)  $47 - x(3x+4) < 2(17-2x) - 62$   
 $47 - 3x^2 - 4x < 34 - 4x - 62$   
 $-3x^2 < -45$   
 $x^2 > 15$   
 $x_1 = -5 \quad x_2 = 5 \quad \text{MC: } (-\infty; -5) \cup (5; +\infty)$

2)  $\frac{3}{8}x^2 - 11 + \frac{2}{12}44 - 2x^2 \leq \frac{24}{10}$   
 $9x^2 - 33 + 148 - 4x^2 - 240 \leq 0$   
 $5x^2 \leq 125$   
 $x^2 \leq 25$   
 $x_1 = -5 \quad x_2 = 5 \quad \text{MC: } [-5; 5]$

3)  $\frac{x-1}{x+3} \geq 2$      $x-1=2x+6$      $x+3 \neq 0$   
 $x = -7$      $x \neq -3$   
 $\text{MC: } [-7; -3)$

4)  $\frac{x^2+5x+4}{x^2-5x+6} > 0$      $x^2+5x+4=0$      $x^2-5x+6=0$   
 $D=25-16=9$      $D=1$   
 $x_1 = \frac{-5-3}{2} = -4$      $x_1 = \frac{5-1}{2} = 2$   
 $x_2 = \frac{-5+3}{2} = -1$      $x_2 = \frac{5+1}{2} = 3$   
 $\text{MC: } (-\infty; -4) \cup (-1; 2) \cup (3; +\infty)$

~350

1)  $\sqrt{x^2+x+1} < 1$   
 $x^2+x+1 < 1$   
 $x(x+1) = 0$   
 $x_1 = 0 \quad x_2 = -1$   
 $\text{MC: } (-1; 0)$

2)  $\sqrt{x-5} < 3$      $x-5 \geq 0$   
 $x-5 < 81$      $x \geq 5$   
 $x < 86$   
 $\text{MC: } [5; 86]$

~351

1)  $4 - 3 \cdot 2^x - 1 > 0$   
 $4 \cdot 2^{-x} - 3 \cdot 2^x - 1 > 0$   
 $D = 9 + 16 = 25$   
 $2^x \pm \frac{3-5}{8} \pm \frac{1}{2}$      $2^x = \frac{3+5}{8} = 1$   
 $x = 0$   
 $\text{MC: } (0; +\infty)$

2)  $\log_{\sqrt{2}}(x^2-3x) \leq 4$      $\log_{\sqrt{2}}(x^2-3x) < \log_{\sqrt{2}} 4$   
 $x^2-3x > 0$      $x^2-3x-4=0$      $x^2-3x=0$   
 $D=9+16=25$      $D=1$   
 $x_1 = -1$      $x_1 = 0$   
 $x_2 = 4$      $x_2 = 3$   
 $\text{MC: } [-1; 0) \cup (3; 4]$

~352

1)  $\frac{(x+3)^2}{5} + 1 - \frac{(3x-1)^2}{5} < \frac{x(2x-3)}{2}$   
 $\frac{x^2+6x+9+5-9x^2+6x-1}{5} < \frac{2x^2-3x}{2}$   
 $\frac{-8x^2+12x+13}{5} < \frac{2x^2-3x}{2}$   
 $-16x^2+24x+26 < 10x^2-15x$   
 $26x^2-39x-26 > 0$   
 $D = 1521 + 2704 = 4225$   
 $x_1 = \frac{39-65}{2 \cdot 26} = -0,5$   
 $x_2 = \frac{39+65}{2 \cdot 26} = 2$   
 $\text{MC: } (-\infty; -0,5) \cup (2; +\infty)$

2)  $x-7 + \frac{(x-6)^2}{3} \geq \frac{(x+4)^2}{2} - \frac{(x+2)(x+6)}{4}$   
 $\frac{3x-21+x^2-12x+36}{3} \geq \frac{2x^2+16x+32-x^2-8x-12}{4}$   
 $\frac{x^2-9x+15}{3} \geq \frac{x^2+8x+20}{4}$   
 $4x^2-36x+60 \geq 3x^2+24x+60$   
 $x^2-60x \geq 0$   
 $x(x-60) = 0$   
 $x_1 = 0 \quad x_2 = 60$   
 $\text{MC: } (-\infty; 0] \cup [60; +\infty)$

~353

1)  $\frac{\sqrt{x-2}-2}{x} < 0$   
 $\sqrt{x-2}-2=0$      $\sqrt{x-2} \geq 0$      $x \neq 0$   
 $x-2=4$      $x \geq 2$   
 $x=6$   
 $\text{MC: } [2; 6)$

2)  $\frac{\sqrt{2x-1}-1}{x} < 0$   
 $\sqrt{2x-1}-1=0$      $2x-1 \geq 0$      $x \neq 0$   
 $2x-1=1$      $x \geq 0,5$   
 $x=1$   
 $\text{MC: } [0,5; 1)$

~354

1)  $\frac{0,2^x - 0,008}{x^2 - 10x + 25} \leq 0$   
 $0,2^x - 0,008 = 0$   
 $0,2^x = 0,2^3$   
 $x = 3$

$x^2 - 10x + 25 \neq 0$   
 $D = 100 - 100 = 0$   
 $x \neq 5$



$MC: [3; 5) \cup (5; +\infty)$

2)  $\frac{x^2 + 6x + 9}{2^x - 4} > 0$   
 $x^2 + 6x + 9 = 0$   
 $D = 36 - 36 = 0$   
 $x = -3$

$2^x - 4 \neq 0$   
 $2^x \neq 2^2$   
 $x \neq 2$

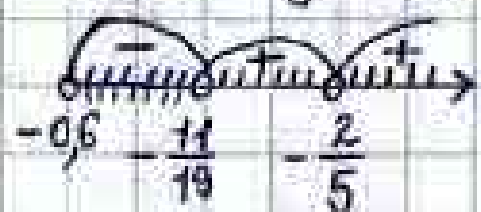


$MC: (-2; +\infty)$

~355

1)  $\frac{\log_{0,5}(5x+3)}{11+19x} < 0$   
 $\log_{0,5}(5x+3) = 0$   
 $5x+3=1$   
 $x = -\frac{2}{5} = -0,4$

$x > -0,6$   
 $11+19x \neq 0$   
 $x \neq -\frac{11}{19}$



$MC: (0,6; -\frac{11}{19})$

2)  $\frac{13x+16}{\log_{0,8}(4x+5)} \geq 0$   
 $13x+16=0$   
 $x = -\frac{16}{13}$

$x > -1,25$   
 $\log_{0,8}(4x+5) \neq 0$   
 $4^x + 5 \neq 1$   
 $x \neq -1$



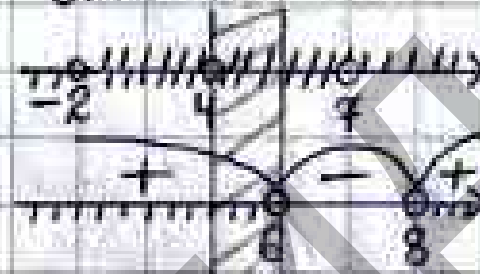
$MC: [-\frac{16}{13}; -1)$

~356

1)  $\begin{cases} 2 \geq \frac{1}{4} \\ 1 + \log_3(x-4) < \log_3(x+2) - 1 \\ \log_3(3x-12) < \log_3 \frac{x+2}{3} \end{cases}$

$\begin{cases} 3x-12 > 0 \\ \frac{x+2}{3} > 0 \\ 3x-12 < x+2 \end{cases}$

$\begin{cases} x > 4 \\ x > -2 \\ x < 7 \end{cases}$



$MC: (4; 6)$

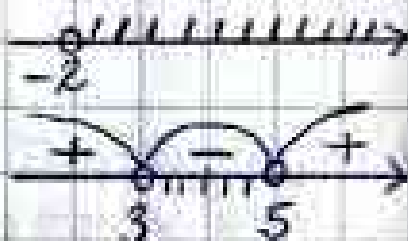
$x^2 - 14x + 48 = 0$   
 $D = 196 - 192 = 4$   
 $x = \frac{14-2}{2} = 6$   
 $x = \frac{14+2}{2} = 8$

2)  $\begin{cases} (0,7)^{x^2-8x+13} < \frac{100}{49} \\ \log_{0,5}(x+16) < \log_{0,5}(x+2) + 1 \\ \log_{0,5}(x+16) < \log_{0,5}(0,5x+1) \end{cases}$

$\begin{cases} x+16 > 0 \\ 0,5x+1 > 0 \\ x+16 < 0,5x+1 \end{cases}$

$(0,7)^{x^2-8x+13} < \frac{100}{49}$

$x^2 - 8x + 13 = 0$   
 $D = 64 - 4 \cdot 15 = 4$   
 $x_1 = \frac{8-2}{2} = 3$   
 $x_2 = \frac{8+2}{2} = 5$



$MC: (3; 5)$

~357

$y = \log_{0,5} \frac{3x^2 - 11x - 4}{5x^2 - 7x + 2}$   
 $3x^2 - 11x - 4 = 0$   
 $D = 121 + 4 \cdot 4 \cdot 3 = 169$   
 $x_1 = \frac{11-13}{6} = -\frac{1}{3}$   
 $x_2 = \frac{11+13}{6} = 4$

$\frac{3x^2 - 11x - 4}{5x^2 - 7x + 2} < 0$

$5x^2 - 7x + 2 \neq 0$   
 $D = 49 - 4 \cdot 2 \cdot 5 = 9$   
 $x_1 = \frac{7-3}{10} = \frac{2}{5}$   
 $x_2 = \frac{7+3}{10} = 1$



$MC: (-\frac{1}{3}; \frac{2}{5}) \cup (1; 4)$

~358

1)  $\frac{2x^2 - 9x + 4}{3x^2} \geq -1$   
 $2x^2 - 9x + 4 + 3x^2 = 0$   
 $5x^2 - 9x + 4 = 0$   
 $D = 81 - 80 = 1$   
 $x = \frac{9-1}{10} = 0,8$   
 $x = \frac{9+1}{10} = 1$

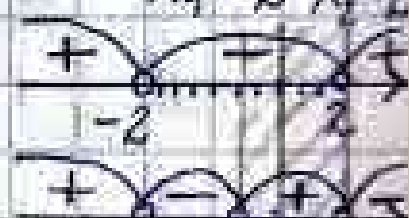
$3x^2 \neq 0$   
 $x \neq 0$



$MC: (-\infty; 0) \cup (0; 0,8] \cup [1; +\infty)$

2)  $\frac{\sqrt{4-x^2} \cdot (3-5x)^2}{(x+3)(x-0,5)(2x+3)} > 0$   
 $\sqrt{4-x^2} \cdot (3-5x)^2 = 0$   
 $4-x^2 = 0$   
 $x = \pm 2$   
 $(x+3)(x-0,5)(2x+3) \neq 0$   
 $x \neq -3$   
 $x \neq 0,5$   
 $x \neq -\frac{3}{2}$

$x^2 - 4 < 0$   
 $x_1 = 2$   
 $x_2 = -2$



$MC: (\frac{3}{2}; 0,5) \cup (0,5; 2)$



~381

1)  $x - a = 1$   
 $x = a + 1$

2)  $5x = a$   
 $x = \frac{a}{5}$

3)  $\frac{x}{2} = a$   
 $x = 2a$

4)  $x^3 = a$   
 $x = \sqrt[3]{a}$

~382

1)  $ax = 10$   
 $x = \frac{10}{a}$

2)  $\sqrt{x} = a$   
 $x = \sqrt{a}$

3)  $|x| = a$   
 $x = \pm a$

4)  $|x+3| = a$   
 $x = -a - 3$   
 $x = a - 3$

~383

1)  $6(ax-1) - a = 2(a+x) - 7$   
 $6ax - 6 - a - 2a - 2x + 7 = 0$   
 $2x(3a-1) - (3a-1) = 0$   
 $(3a-1)(2x-1) = 0$   
 $a = \frac{1}{3} \quad x = \frac{1}{2}$

2)  $0,5(5x-1) = 4,5 - 2a(x-2)$   
 $2,5x - 0,5 - 4,5 + 2ax - 4a = 0$   
 $5x - 10 + 4ax - 8a = 0$   
 $x(4a+5) - 2(4a+5) = 0$   
 $(x-2)(4a+5) = 0$   
 $x = 2 \quad a = -1,25$

~384


1)  $2(a-2x) = ax+3$   
 $2a - 4x - ax = 3$   
 $-x(a+4) = -2a+3$   
 $x = \frac{2a+3}{a+4}$   
MC:  $a \neq -4$

2)  $a^2x = a(x+2) - 2$   
 $a^2x - ax = 2a - 2$   
 $x(a^2 - a) = 2a - 2$   
 $x = \frac{2a-2}{a^2-a} \Rightarrow \begin{matrix} a^2-a \neq 0 \\ a \neq 0 \quad a \neq 1 \end{matrix}$   
MC:  $a \neq 0 \quad a \neq 1$

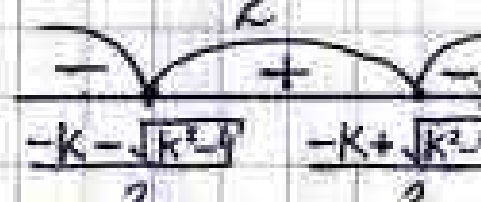
~385


1)  $ax - 4 = 3x \Rightarrow ax - 3x = 4 \Rightarrow x = \frac{4}{a-3} \quad x = 8$   
 $\frac{4}{a-3} = 8 \Rightarrow 4 = 8a - 24 \Rightarrow 8a = 28 \Rightarrow a = \frac{7}{2}$

~386

1)  $ax^2 < 9$   
 $ax^2 = 9$   
 $x = \pm \frac{3}{\sqrt{a}}$   
  
MC:  $(-\frac{3}{\sqrt{a}}; \frac{3}{\sqrt{a}})$

2)  $ax^2 > -1$   
 $ax^2 = -1$   
 $\emptyset$

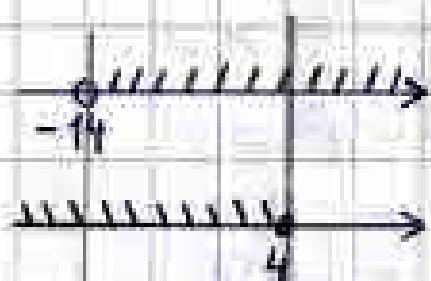
3)  $x^2 + kx + 1 \geq 0$   
 $D = k^2 - 4$   
 $x = \frac{-k \pm \sqrt{k^2 - 4}}{2}$   
  
MC:  $(\frac{-k - \sqrt{k^2 - 4}}{2}; \frac{-k + \sqrt{k^2 - 4}}{2}]$

4)  $(n+5)x \leq n^2 - 5$   
 $x \leq n - 5$   
  
MC:  $(-\infty; n-5]$

~387

$|x^2 - 6x - 5| = a$   
 $x^2 - 6x - 5 = -a$   
 $x^2 - 6x + (a-5) = 0$   
 $D = 36 - 4(a-5) \geq 0$   
 $-4a \geq -16$   
 $a \leq 4$

$x^2 - 6x - 5 = a$   
 $x^2 - 6x - (a+5) = 0$   
 $D = 36 + 4(a+5) > 0$   
 $4a > -56$   
 $a > -14$



MC:  $(-14; 4]$  и/е  $[-14; 4)$

~389

1)  $4x^2 - 15x + 4a^3 = 0$

$D = 225 - 64a^3$

$x_1 = \frac{15 + \sqrt{225 - 64a^3}}{8}$

$x_2 = \frac{15 - \sqrt{225 - 64a^3}}{8}$

$\sqrt{225 - 64a^3} = t$

$\frac{15+t}{8} = \left(\frac{15+t}{8}\right)^2 \Rightarrow$

$\frac{15+t}{8} = \frac{225 - 30t + t^2}{64} \Rightarrow$

$120 + 8t = t^2 - 30t + 225$

$t^2 - 38t + 105 = 0$

$D = 1444 - 420 = 1024$

$t_1 = \frac{38 - 32}{2} = 3$

$t_2 = \frac{38 + 32}{2} = 35$

$\sqrt{225 - 64a^3} = 3$

$\sqrt{225 - 64a^3} = 35$

$225 - 64a^3 = 9$

$225 - 64a^3 = 1225$

$a^3 = \frac{216}{64} \Rightarrow a = \frac{3}{2}$

$a^3 = \frac{-1000}{64} \Rightarrow a = \frac{5}{2}$  ж.к.  $\frac{3}{2}; \frac{5}{2}$

2)  $x^2 - ax + a - 1 = 0$

$D = a^2 - 4(a-1) = a^2 - 4a + 4 = (a-2)^2$

$x_1 = \frac{a - (a-2)}{2} = 1$

$x_2 = \frac{a + (a-2)}{2} = a-1$

$(a-1)^2 + 1 = 17$

$a^2 - 2a - 15 = 0$

$D = 4 + 4 \cdot 15 = 64$

$a_1 = \frac{2 - 8}{2} = -3$

$a_2 = \frac{2 + 8}{2} = 5$

~390

1)  $\begin{cases} x + ay = 1 \\ x - 3ay = 2a + 3 \end{cases}$

$4ay = 2a - 2$

$y = \frac{-2(a+1)}{4a} \Rightarrow y = \frac{a+1}{-2a}$

ж.к.  $a=0$  шешімі болмайды

2)  $\begin{cases} x + ay = 1 \\ ax + y = 2a \end{cases} \begin{cases} x = 1 - ay \\ a(1 - ay) + y = 2a \end{cases}$

$a - a^2y + y = 2a$

$y(1 - a^2) = a$

$y = \frac{a}{1 - a^2} \quad 1 - a^2 \neq 0$

ж.к.  $0; \pm 1$  шешімі болмайды

~391

1)  $\begin{cases} 3x + ay = 3 \\ ax + 3y = 3 \end{cases}$

$3x - ax + ay - 3y = 0$

$x(3-a) - (3-a)y = 0$

$(3-a)(x-y) = 0$

$3-a=0$

$a=3$

ж.к. 3

2)  $\begin{cases} ax - (a+1)y = 6 \\ 7ax - 28y = 6(a+4) \end{cases} \quad x = \frac{ay + y + 6}{a}$

$7a \cdot \frac{ay + y + 6}{a} - 28y = 6a + 24$

$7ay + 7y + 42 - 28y = 6a + 24$

$7ay - 21y - 6a + 18 = 0$

$7y(a-3) - 6(a-3) = 0$

$(a-3)(7y-6) = 0$

$a-3=0$

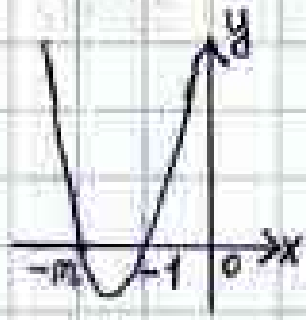
$a=3$  ж.к. 3

~392

1)  $x^2 + (m+1)x + m \leq 0$

$D = (m+1)^2 - 4m = (m-1)^2$

$x_1 = \frac{-(m+1) - (m-1)}{2} = -m$



$x_2 = \frac{-(m+1) + (m-1)}{2} = -1$

ж.к.  $-m-1$   
ж.к.  $-1; -m$

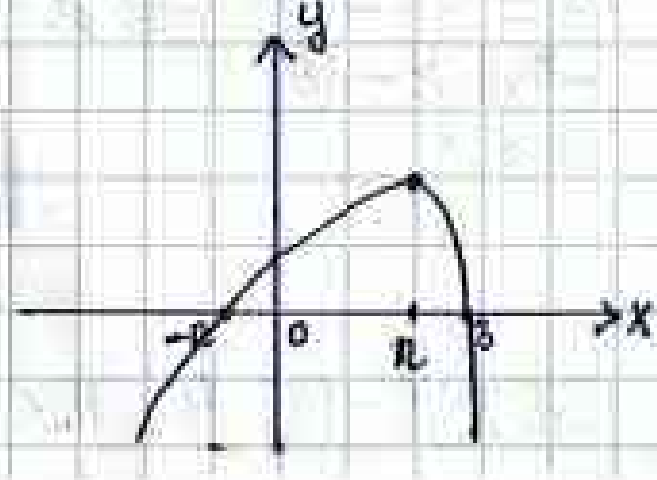
2)  $\frac{x^2 - n^2}{x-3} > 0$

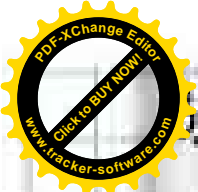
$x^2 - n^2 = 0$

$x = \pm n$

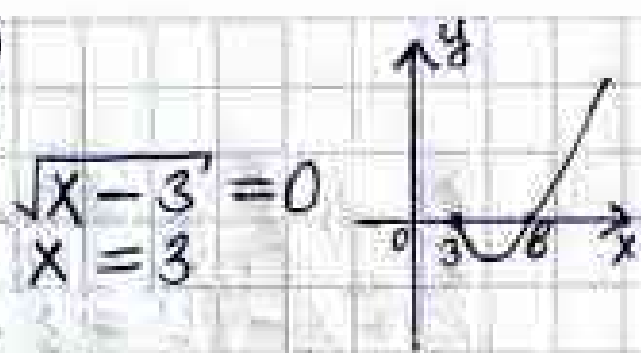
$x-3 \neq 0$

$x \neq -3$





$$\begin{aligned} (x-b)\sqrt{x-3} &< 0 \\ (x-b)\sqrt{x-3} &= 0 \\ x-b &= 0 \\ x &= b \\ \text{mc: } (3; b) \end{aligned}$$



$$4) \log_7(x-5) \leq \log_7(a-x)$$

$$\begin{cases} x-5 > 0 \\ a-x > 0 \\ x-5 \leq a-x \end{cases} \Rightarrow \begin{cases} x > 5 \\ x < a \\ x \leq \frac{a+5}{2} \end{cases}$$

~393

$$(k-2)x^2 - 2(k+3)x + 4k = 0$$

$$D = 4(k+3)^2 - 4 \cdot 4k(k-2) = 4(k^2 + 6k + 9) - 16k(k-2) = 4(-3k^2 + 14k + 9)$$

$$x = \frac{2(k+3) \pm 2\sqrt{-3k^2 + 14k + 9}}{2(k-2)} = \frac{(k+3) \pm \sqrt{-3k^2 + 14k + 9}}{k-2}$$

$$x_1 > 3 \quad x_2 < 2$$

$$\frac{k+3 + \sqrt{-3k^2 + 14k + 9}}{k-2} > 3$$

$$\frac{k+3 - \sqrt{-3k^2 + 14k + 9}}{k-2} < 2$$

$$k+3 + \sqrt{-3k^2 + 14k + 9} > 3k-6$$

$$k+3 - \sqrt{-3k^2 + 14k + 9} < 2k-4$$

$$(\sqrt{-3k^2 + 14k + 9})^2 > (2k-9)^2$$

$$(\sqrt{-3k^2 + 14k + 9})^2 > (7-k)^2$$

$$-3k^2 + 14k + 9 > 4k^2 - 36k + 81$$

$$-3k^2 + 14k + 9 > 49 - 14k + k^2$$

$$7k^2 - 50k + 72 < 0$$

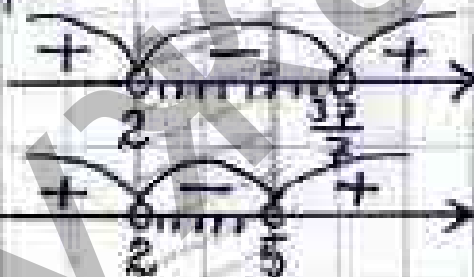
$$k^2 - 7k + 10 < 0$$

$$D = 2500 - 4 \cdot 7 \cdot 72 = 484$$

$$D = 49 - 4 \cdot 10 = 9$$

$$k_1 = \frac{50-22}{14} = 2 \quad k_2 = \frac{50+22}{14} = \frac{37}{7}$$

$$k_1 = \frac{7-3}{2} = 2 \quad k_2 = \frac{7+3}{2} = 5$$



mc: (2; 5)

~394

$$x^2 - (2k+1)x + k^2 + 2 = 0$$

$$D = (2k+1)^2 - 4(k^2+2) = 4k^2 + 4k + 1 - 4k^2 - 8 = 4k - 7$$

$$x = \frac{(2k+1) \pm \sqrt{4k-7}}{2}$$

$$x_1 = 2x_2$$

$$\frac{2k+1 + \sqrt{4k-7}}{2} = 2 \cdot \frac{2k+1 - \sqrt{4k-7}}{2}$$

$$2k+1 + \sqrt{4k-7} = 4k+2 - 2\sqrt{4k-7}$$

$$(2k+1)^2 = (3\sqrt{4k-7})^2$$

$$4k^2 + 4k + 1 = 9(4k-7)$$

$$k^2 - 8k + 16 = 0$$

$$D = 64 - 4 \cdot 16 = 0$$

$$k = \frac{8 \pm 0}{2} = 4$$

mc: 4

~395

$$3x^3 + ax^2 + bx + 18 = 0 \quad x_1 = 1 + \sqrt{3}$$

$$3(1+\sqrt{3})^3 + a(1+\sqrt{3})^2 + b(1+\sqrt{3}) + 18 = 3(1+3\sqrt{3}+3 \cdot 3+3\sqrt{3}) + a(1+2\sqrt{3}+3) + b(1+\sqrt{3}) + 18 = 30 + 18\sqrt{3} + 4a + a \cdot 2\sqrt{3} + b + b\sqrt{3} + 18 = 4a + b + (2a+b)\sqrt{3} + 48 + 18\sqrt{3} = 0$$

$$\begin{cases} 4a + b = -48 \\ (2a+b)\sqrt{3} = -18\sqrt{3} \end{cases}$$

$$\begin{cases} 4a + b = -48 \\ 2a + b = -18 \end{cases} \Rightarrow \begin{cases} b = -48 - 4a \\ b = -48 + 4 \cdot 15 \end{cases}$$

$$\begin{cases} a = -15 \\ b = 12 \end{cases}$$

$$2a = -30$$

$$b = 12$$

$$a = -15$$

~396

$$1) \begin{cases} 2^{2x} - (2a+1)2^x + a^2 + a = 0 \\ t^2 - (2a+1)t + a^2 + a = 0 \\ D = (2a+1)^2 - 4(a^2+a) = 1 \\ t_1 = \frac{2a+1-1}{2} = a \\ t_2 = \frac{2a+1+1}{2} = a+1 \end{cases} \quad \begin{cases} 2^x = t \\ 2^x = a \\ x = \log_2 a \\ 2^x = a+1 \\ x = \log_2(a+1) \end{cases}$$

$\mathcal{M}: \log_2 a; \log_2(a+1)$

$$2) \begin{cases} \log_a x + \log_a(x+1) = 1 \\ \log_a(x^2+x) = \log_a a \\ x^2+x-a=0 \\ D = 1+4a \\ x = \frac{-1 \pm \sqrt{1+4a}}{2} \end{cases}$$

$\mathcal{M}: \frac{-1 \pm \sqrt{1+4a}}{2}$

~397

$$1) \begin{cases} \log_2(4y+4a-3) = 1 + \log_2(a-x) \\ y = \sqrt{x} \end{cases}$$

$$\begin{aligned} \log_2(4y+4a-3) &= \log_2(2a-2y^2) \\ 4y+4a-3 &= 2a-2y^2 \\ 2y^2-4y+2a-3 &= 0 \\ D &= 16-8(2a-3) \geq 0 \\ 16-8a+24 &\geq 0 \\ a &\leq -5 \\ \mathcal{M}: &(-\infty; -5] \end{aligned}$$

$$2) \begin{cases} 1 + \log_2(b-2-y) = \log_2(b-x) \\ y + 2\sqrt{x} = 1 \Rightarrow y = 1-2\sqrt{x} \end{cases}$$

$$\begin{aligned} \log_2(2b-2-2(1-2\sqrt{x})) &= \log_2(b-x) \\ 2b-2-4+4\sqrt{x} &= b-x \\ x+4\sqrt{x}-(b+6) &= 0 \\ D &= 16+4(b+6) \geq 0 \\ 16+4b+24 &\geq 0 \\ b &\geq -10 \\ \mathcal{M}: &[-10; +\infty) \end{aligned}$$

~398

$$\begin{aligned} x^2 - 2(a+1)x + a^2 + 2a &< 0 \\ -2(a+1) &= -4 \\ -2a-2 &= -4 \\ -2a &= -2 \\ a &= 1 \end{aligned}$$

$\mathcal{M}: 1$

$$\begin{aligned} x^2 - 4x + 3 &< 0 \\ a^2 + 2a &= 3 \\ a^2 + 2a - 3 &= 0 \\ D &= 4+4\cdot 3 = 16 \\ a_1 &= \frac{-2+4}{2} = 1 \\ a_2 &= \frac{-2-4}{2} = -3 \end{aligned}$$

$a_2 = \frac{-2-4}{2} = -3$

~399

$$1) \frac{x-3}{\sqrt{x-a}} \leq 0$$

$$\begin{aligned} x-3 &= 0 & x-a > 0 \\ x &= 3 & x > a \end{aligned}$$

$\mathcal{M}: (a; 3]$

2)

# Өзіндік тапсырма!

$$1) \frac{2x-3}{x-2} = \frac{3x+1}{x+2} \quad \begin{matrix} x-2 \neq 0 \\ x \neq 2 \end{matrix} \quad \begin{matrix} x+2 \neq 0 \\ x \neq -2 \end{matrix}$$

$$2x^2 - 3x + 4x - 6 = 3x^2 + x - 6x - 2$$

$$x^2 - 6x + 4 = 0$$

$$D = 36 - 4 \cdot 4 = 20$$

$$x = \frac{6 \pm \sqrt{20}}{2} = 3 \pm \sqrt{5} \quad \text{D}$$

$$2) (\sqrt{x-1})^2 = (x-1)^2$$

$$x-1 = x^2 - 2x + 1$$

$$x^2 - 3x + 2 = 0$$

$$D = 9 - 4 \cdot 2 = 1$$

$$x_1 = \frac{3-1}{2} = 1 \quad x_2 = \frac{3+1}{2} = 2 \quad \text{D}$$

$$3) 3 \sin x + 4 \cos x = 5$$

$$3 \left( 2 \sin \frac{x}{2} \cdot \cos \frac{x}{2} \right) + 4 \left( \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} \right) - 5 \left( \sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} \right) = 0$$

$$6 \sin \frac{x}{2} \cos \frac{x}{2} - \cos^2 \frac{x}{2} - 9 \sin^2 \frac{x}{2} = 0 \quad / \cos^2 \frac{x}{2} \text{ қорықсыз}$$

$$6 \operatorname{tg} \frac{x}{2} - 1 - 9 \operatorname{tg}^2 \frac{x}{2} = 0 \Rightarrow 9 \operatorname{tg}^2 \frac{x}{2} - 6 \operatorname{tg} \frac{x}{2} + 1 = 0 \Rightarrow (3 \operatorname{tg} \frac{x}{2} - 1)^2 = 0$$

$$3 \operatorname{tg} \frac{x}{2} = 1 \Rightarrow \operatorname{tg} \frac{x}{2} = \frac{1}{3} \Rightarrow \frac{x}{2} = \operatorname{arctg} \frac{1}{3} + \pi n$$

$$x = 2 \operatorname{arctg} \frac{1}{3} + 2\pi n \quad n \in \mathbb{Z} \quad \text{D}$$

$$4) \begin{cases} x^2 + y^2 = 5 \\ 2x^2 - y^2 = 7 \end{cases} \quad \begin{cases} x = \sqrt{5-y^2} \\ 2(5-y^2) - y^2 = 7 \\ 10 - 3y^2 = 7 \\ y^2 = 1 \\ y = \pm 1 \end{cases} \quad \begin{cases} x = 2 \\ y = \pm 1 \end{cases} \quad \text{C}$$

$$5) \frac{\sqrt{21+x} + \sqrt{21-x}}{\sqrt{21+x} - \sqrt{21-x}} = \frac{21}{x} \quad x \neq 0$$

$$x\sqrt{21+x} + x\sqrt{21-x} = 21\sqrt{21+x} - 21\sqrt{21-x}$$

$$\sqrt{21+x}(x-21) + \sqrt{21-x}(x+21) = 0$$

$$\sqrt{441-x^2}(\sqrt{x-21} + \sqrt{x+21}) = 0$$

$$441 - x^2 = 0 \quad \sqrt{x-21} + \sqrt{x+21} \neq 0$$

$$x = \pm 21 \quad \text{B}$$

$$6) \sqrt{b} - \sqrt{3b-1} \geq 1$$

$$\begin{matrix} b \geq 0 \\ 3b-1 \geq 0 \\ b \geq \frac{1}{3} \end{matrix} \quad \begin{matrix} 0 & \frac{1}{3} & \rightarrow \end{matrix} \quad \text{MC: } \left[ \frac{1}{3}; +\infty \right) \quad \text{C}$$

$$7) \frac{3y+7}{2y-7} \geq 5$$

$$3y+7 = 10y-35 \quad 2y-7 \neq 0$$

$$7y = 42 \quad y \neq \frac{7}{2}$$

$$y \leq 6$$

$$\begin{matrix} - & + & - \\ 3.5 & 6 & \end{matrix} \quad \text{MC: } (3.5; 6] \quad \text{A}$$

$$8) \frac{2 \cdot 7}{14^{x+4}} > 2^{3x} \cdot 7$$

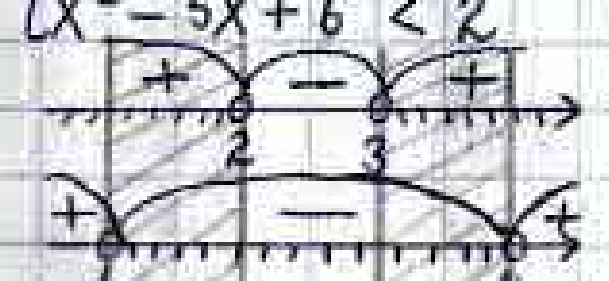
$$14^{x+4} > 14^{3x}$$

$$x+4 > 3x$$

$$-2x > -4$$

$$x < 2 \quad \text{B}$$

9)  $\log_2(x^2 - 5x + 6) < 1$   
 $\begin{cases} x^2 - 5x + 6 > 0 \\ x^2 - 5x + 6 < 2 \end{cases}$



$$x^2 - 5x + 6 = 0$$

$$D = 25 - 24 = 1$$

$$x_1 = \frac{5-1}{2} = 2$$

$$x_2 = \frac{5+1}{2} = 3$$

$$x^2 - 5x + 4 = 0$$

$$D = 25 - 16 = 9$$

$$x_1 = \frac{5-3}{2} = 1$$

$$x_2 = \frac{5+3}{2} = 4$$

MC:  $(1, 2) \cup (3, 4)$  (B)

10)  $\begin{cases} 96 \cdot 2^{y+1} > 12 \\ 16 \cdot 4^y < 1 \end{cases}$

$$\begin{cases} 2^{y+1} > \frac{1}{8} \\ 4^y < \frac{1}{16} \end{cases}$$

$$\begin{cases} 2^{y+1} > 2^{-3} \\ 4^y < 4^{-2} \end{cases}$$

$$\begin{cases} y > -4 \\ y < -2 \end{cases} \quad (-4, -2) \text{ (B)}$$

11)  $\begin{cases} 4 - \frac{y-1}{3} \leq y \\ \frac{7y-1}{8} < 6 \end{cases}$

$$4 - \frac{y-1}{3} \leq y$$

$$12 - y + 1 \leq 3y$$

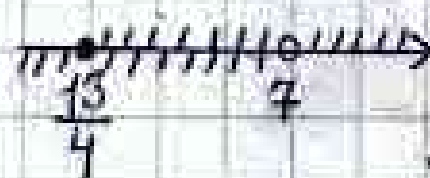
$$y \geq \frac{13}{4}$$

$$\frac{7y-1}{8} < 6$$

$$7y - 1 < 48$$

$$7y < 49$$

$$y < 7$$



MC:  $[\frac{13}{4}, 7)$  (C)

12)  $|x^2 - x + 2| = 4$   
 $x^2 - x + 2 = 4$   
 $x^2 - x - 2 = 0$   
 $D = 1 + 8 = 9$   
 $x_1 = -1 \quad x_2 = 2$

$$x^2 - x + 2 = -4$$

$$x^2 - x + 6 = 0$$

$$D = 1 + 24 = 25$$

$$x_1 = -2 \quad x_2 = 3$$

$$x_1 + x_2 + x_3 + x_4 =$$

$$-1 + 2 - 2 + 3 = 2 \text{ (B)}$$

13)  $\sqrt{4|x| - x^2} = 2$

$$4|x| - x^2 = 4$$

$$x^2 - 4|x| + 4 = 0$$

$$D = 16 - 16 = 0$$

$$x = \frac{4}{2} = 2$$

$$x^2 - 4|x| + 4 = 0$$

$$x^2 + 4|x| + 4 = 0$$

$$D = 16 - 16 = 0$$

$$x = \frac{-4}{2} = -2 \text{ (B)}$$

14)  $\begin{cases} y - |x + 2| = 1 \\ x^2 + (y - 5)^2 = 4 \end{cases}$

$$\begin{cases} y - (x + 2) = 1 \\ x^2 + (y - 5)^2 = 4 \end{cases}$$

НЕМЕЦЕ

$$\begin{cases} y + (x + 2) = 1 \\ x^2 + (y - 5)^2 = 4 \end{cases}$$

$$y - (x + 2) = 1$$

$$y = x + 3$$

$$x^2 + (x + 3 - 5)^2 = 4$$

$$x^2 + x^2 - 4x + 4 = 4$$

$$2x(x - 2) = 0$$

$$\begin{cases} x_1 = 0 \\ x_2 = 2 \end{cases} \quad \begin{cases} y_1 = 3 \\ y_2 = 5 \end{cases} \text{ (D)}$$

$$y + (x + 2) = 1$$

$$y = -x - 1$$

$$x^2 + (-x - 1 - 5)^2 = 4$$

$$2x^2 - 12x + 36 = 4$$

$$x^2 - 6x + 16 = 0$$

$$D = 36 - 4 \cdot 16 < 0$$

15)  $|3^x - 3| < 6$

$$\begin{cases} 3^x - 3 < 6 \\ 3^x - 3 > -6 \end{cases}$$

$$\begin{cases} 3^x < 3^2 \\ 3^x > -3 \end{cases}$$

$$x < 2 \text{ (C)}$$

16)  $|\log_2 x| \leq 1$

$$\begin{cases} \log_2 x \leq 1 \\ \log_2 x \geq -1 \end{cases}$$

$$\begin{cases} x \leq 2 \\ x \geq 0,5 \end{cases}$$



MC:  $[0,5; 2]$  (D)

17)  $|\cot x| < 0$

(A) шешімі жоқ, модульден теріс сан шықпайды

$$3) 2x^2 + 2ax + 5a - 6 = 0$$

$$D = 4a^2 - 4 \cdot 2(5a - 6) = 4a^2 - 40a + 48$$

$$x_1^2 + x_2^2 = 0 \quad \frac{(-2a + 2\sqrt{a^2 - 10a + 12})^2}{4} + \frac{(-2a - 2\sqrt{a^2 - 10a + 12})^2}{4} = 0$$

$$\frac{(-a + \sqrt{a^2 - 10a + 12})^2}{1} + \frac{(-a - \sqrt{a^2 - 10a + 12})^2}{1} = 0$$

$$a^2 - 2a\sqrt{a^2 - 10a + 12} + a^2 - 10a + 12 + a^2 + 2a\sqrt{a^2 - 10a + 12} + a^2 - 10a + 12 = 0$$

$$4a^2 - 20a + 24 = 0 \Rightarrow a^2 - 5a + 6 = 0$$

$$D = 25 - 24 = 1$$

$$a_1 = \frac{5-1}{2} = 2 \quad a_2 = \frac{5+1}{2} = 3 \quad \text{D}$$

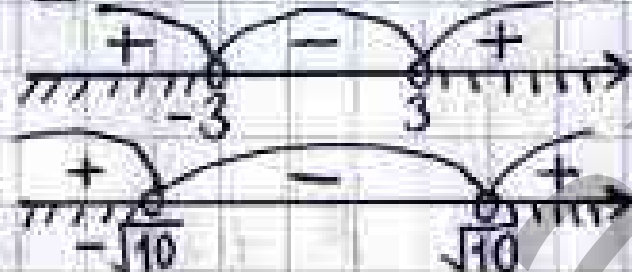
$$\text{B) } 1) 2x^2 = a + 4 \quad a + 4 \geq 0 \quad a \geq -4 \quad [-4; +\infty) \quad \text{C}$$

$$2) \log_2(x^2 - 9) > 0 \quad \log_2(x^2 - 9) > \log_2 1$$

$$\begin{cases} x^2 - 9 > 0 \\ x^2 - 9 > 1 \end{cases}$$

$$\begin{cases} x^2 - 9 = 0 \\ x = \pm 3 \end{cases}$$

$$\begin{cases} x^2 - 9 = 1 \\ x^2 = \pm \sqrt{10} \end{cases}$$



$$\text{MC: } (-\infty; -\sqrt{10}) \cup (\sqrt{10}; +\infty) \quad \text{D}$$

~400

$$P(A) = P(A_1) + P(A_2) + P(A_3) \quad P(A) = 0,45 + 0,30 + 0,15 = 0,9 \quad \text{MC: } 0,9$$

~401

$$P(A) = 1 - P(A_1) = 1 - 0,75 = 0,25 \quad \text{MC: } 0,25$$

~402

$$P(A) = 1 - (P(A_1) + P(A_2)) \quad P(A_3) = 1 - (0,6 + 0,1) = 0,3 \quad \text{MC: } 0,3$$

~403

1	5	2	4	3	3	4	2	5	1	6	1
1	6	2	5	3	4	4	3	5	2	6	2
		2	6	3	5	4	4	5	3	6	3
				3	6	4	5	5	4	6	4
						4	6	5	5	6	5
								5	6	6	6

Барлық мүсі  
қисқасы 36,  
бізге керек  
26, демек  
 $\frac{26}{36} = 0,72$

~404

$$P(AB) = P(A) \cdot P(B) = 0,9 \cdot 0,8 = 0,72$$

~405

$$P(AB) = P(A) \cdot P(B) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} \approx 0,0278$$

~406

$$1) P(AB) = P(A) \cdot P_A(B) \quad P(AB) = \frac{92}{100} \cdot \frac{72}{100} = 0,6626 \approx 0,6$$

2 - ақ  
3 - барлық ұбарыны 10  
5 - керек

MC: 0,6

2)  $a_3 = 7$ ,  $k_3 = 3$   $P(B) = \frac{6}{9}$   $P(C) = \frac{2}{9}$   
 $P(B) + P(C) = \frac{6}{9} + \frac{2}{9} = \frac{8}{9}$   $n.c.: \frac{8}{9}$

~408

$a_3 = 4$ ,  $k_3 = 7$   $P(A) = \frac{4}{11} \cdot \frac{7}{10} = \frac{28}{110} \approx 0,2545$   $n.c.: 0,2545$

~409

$P(A) = \frac{a}{a+b}$   $P(B) = \frac{b}{a+b}$   $P(A) \cdot P(B) = \frac{a}{a+b} \cdot \frac{b}{a+b} = \frac{ab}{(a+b)^2}$

~410

$a$   $i$   $T$   $K$   $\Pi$   $P_5 = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$   $n.c.: P(A) = \frac{1}{120}$

~411

$g$   $a$   $T$   $P$   $K$   $H$   $a$   $C$   $a$   $P_9 = 9! = 362880$   $P(A) = \frac{1}{362880}$

~412

X	1 мин	100 мин	10 мин
P	1/10000	100/10000	1000/10000

$\Rightarrow n.c.$

X	1 мин	100 мин	10 мин
P	0,0001	0,01	0,1

~413

1)  $X$  | 4 | 6 | 10  
 $P$  | 0,2 | 0,3 | 0,5  
 $M(X) = 4 \cdot 0,2 + 6 \cdot 0,3 + 10 \cdot 0,5 = 7,6$

2)  $X$  | 0,2 | 0,5 | 0,6  
 $P$  | 0,1 | 0,5 | 0,4  
 $M(X) = 0,2 \cdot 0,1 + 0,5 \cdot 0,5 + 0,6 \cdot 0,4 = 0,535$

~414

X	4-c	6-c	$X_3$
P	0	0,3	$P_3$

$P_3 = 1 - 0,3 = 0,7$ ;  $n.c.: M(X) = 8$  балла,  
 $(4-c) \cdot 0 + (6-c) \cdot 0,3 + 0,7 X_3 = 8$   
 $X_3 = \frac{6,2 + 0,3c}{0,7}$

~415

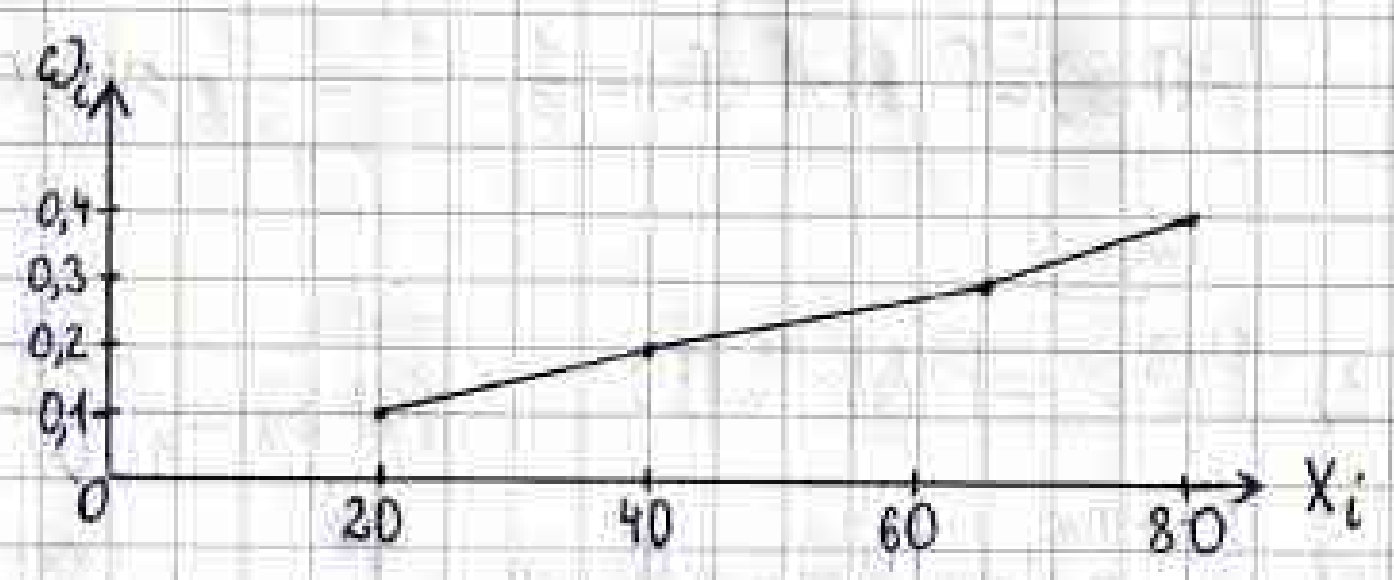
$X_i$	4	7	12	8
$n_i$	5	2	10	3
$\omega_i$	$\frac{5}{20}$	$\frac{2}{20}$	$\frac{3}{20}$	$\frac{10}{20}$

$n = 5 + 2 + 10 + 3 = 20$

$X_i$	4	7	8	12
$\omega_i$	0,25	0,1	0,15	0,5

~416

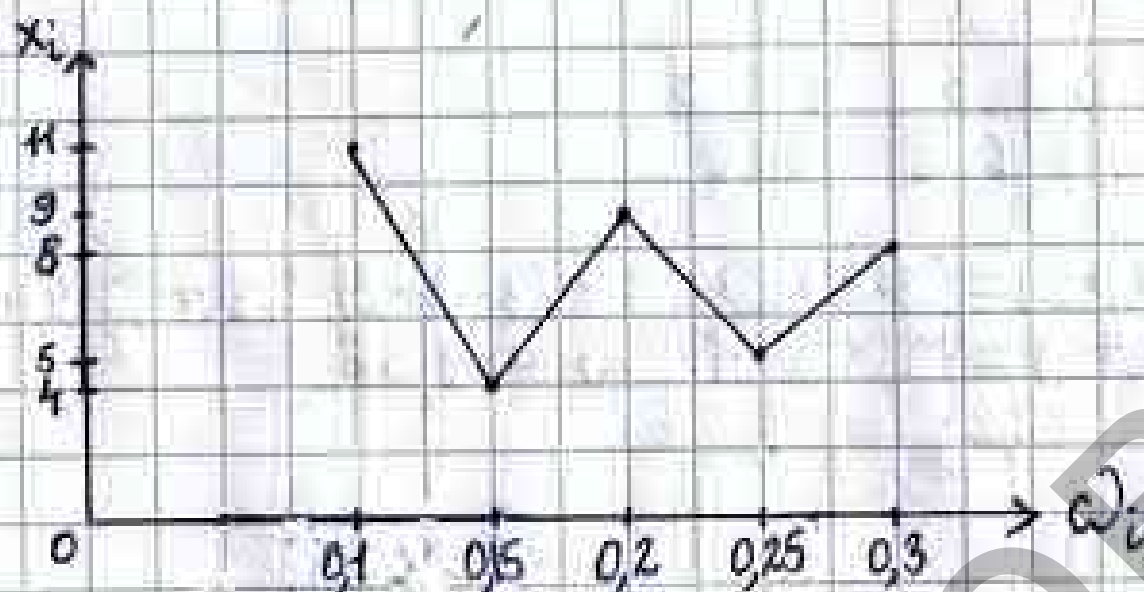
$X_i$	20	40	65	80
$\omega_i$	0,1	0,2	0,3	0,4





~417

$X_i$	4	5	8	9	11
$w_i$	0,15	0,25	0,3	0,2	0,1



~418

1) 

X	4,3	5,1	10,6
P	0,2	0,3	0,5

 $\sigma = ?$   
 $\Phi(X) = ?$   
 $\sigma = \sqrt{\Phi(X)}$      $\Phi(X) = M(X^2) - M^2(X)$   
 $M(X^2) = 4,3^2 \cdot 0,2 + 5,1^2 \cdot 0,3 + 10,6^2 \cdot 0,5 = 67,681$   
 $M^2(X) = (4,3 \cdot 0,2 + 5,1 \cdot 0,3 + 10,6 \cdot 0,5)^2 = 59,1361$   
 $\Phi(X) = 67,681 - 59,1361 = 8,5449$   
 $\sigma = \sqrt{8,5449} \approx 2,9$   
 MC: 8,5449; 2,9

2) 

X	131	140	160	180
P	0,05	0,1	0,25	0,6

 $\sigma = ?$   
 $\Phi(X) = ?$   
 $\sigma = \sqrt{\Phi(X)}$      $\Phi(X) = M(X^2) - M^2(X)$   
 $M(X^2) = 22258,05$   
 $M^2(X) = 16525,1025$   
 $\Phi(X) = 5732,9475$   
 $\sigma = \sqrt{5732,9475} \approx 75,7$   
 MC: 5732,9475; 75,7

~419

X	-1	0	1
P	$p_1$	$p_2$	$p_3$

$M(X) = 0,1$   
 $M(X^2) = 0,9$

$$\begin{cases} -p_1 + p_3 = 0,1 \\ p_1 + p_3 = 0,9 \end{cases}$$

$2p_3 = 1 \Rightarrow p_3 = 0,5$      $p_1 + 0,5 = 0,9 \Rightarrow p_1 = 0,4$      $p_2 = 1 - (0,5 + 0,4) = 0,1$

MC: 0,4; 0,1; 0,5

~420

3 - стандарт  
 7 - стандарт

$C_{10}^2 = \frac{10!}{2!8!} = \frac{9 \cdot 10}{1 \cdot 2} = 45$

1) 0 стандарт  
 $C_7^2 = \frac{7!}{2!5!} = 21$      $p_1 = \frac{C_7^2}{C_{10}^2} = \frac{21}{45} = \frac{7}{15}$

2) 1 стандарт  
 $C_3^1 = \frac{3!}{1!2!} = 3$      $C_7^1 = \frac{7!}{1!6!} = 7$   
 $p_2 = \frac{C_3^1 \cdot C_7^1}{C_{10}^2} = \frac{3 \cdot 7}{45} = \frac{7}{15}$

3) 2 стандарт  
 $C_3^2 = \frac{3!}{2!1!} = 3$      $p_3 = \frac{3}{45} = \frac{1}{15}$     генер

X	0	1	2
P	$\frac{7}{15}$	$\frac{7}{15}$	$\frac{1}{15}$

$M(X) = 0 \cdot \frac{7}{15} + 1 \cdot \frac{7}{15} + 2 \cdot \frac{7}{15} = 0,6$   
 MC: 0,6

~421

X	2	3	4	5
P	0,3	0,1	0,5	0,1

$M(2X) = 2 \cdot 2 \cdot 0,3 + 2 \cdot 3 \cdot 0,1 + 2 \cdot 4 \cdot 0,5 + 2 \cdot 5 \cdot 0,1 = 6,8$   
 $\Phi(2X) = 2^2 \cdot 4 \cdot 0,3 + 2^2 \cdot 9 \cdot 0,1 + 2^2 \cdot 16 \cdot 0,5 + 2^2 \cdot 25 \cdot 0,1 - (6,8)^2 = 4,16$   
 $\sigma(2X) = \sqrt{\Phi(2X)} = \sqrt{4,16} = 2,04$   
 MC: 6,8; 4,16;  $\approx 2,04$

~422

X	2	3	4	5
P	0,3	0,1	0,5	0,1

$M(3X+2) = (3 \cdot 2 + 2) \cdot 0,3 + (3 \cdot 3 + 2) \cdot 0,1 + (3 \cdot 4 + 2) \cdot 0,5 + (3 \cdot 5 + 2) \cdot 0,1 = 12,2$   
 $D(3X+2) = M(X^2) - M^2(X) = 9,36$   
 И: 12,2; 9,36

~423

Бер:  $n=5$   $p=0,2$   $q=1-p=1-0,2=0,8$   
 $\Phi(X) = ?$

$\Phi(X) = n \cdot p \cdot q$   $\Phi(X) = 5 \cdot 0,2 \cdot 0,8 = 0,8$   
 $n$  - таңырык саны  
 $p$  - тусу ыктымалдыгы  
 $q$  - түрлеу ыктымалдыгы  
 И: 0,8

~424

X	$X_1$	$X_2$	$M(X) = 2,6$	$X_1 < X_2$
P	0,2	0,8	$\sigma(X) = 0,8$	$p_2 = 1 - 0,2 = 0,8$

$M(X) = X_1 \cdot 0,2 + X_2 \cdot 0,8 = 2,6$   
 $\sigma(X) = \sqrt{D(X)} = \sqrt{M(X^2) - M^2(X)} = \sqrt{0,2X_1^2 + 0,8X_2^2 - 6,76} = 0,8$

$\begin{cases} X_1 \cdot 0,2 + X_2 \cdot 0,8 = 2,6 \\ 0,2X_1^2 + 0,8X_2^2 - 6,76 = 0,64 \end{cases}$ 
 $\begin{cases} 0,2X_1 + 0,8X_2 = 2,6 \\ 0,2X_1^2 + 0,8X_2^2 = 7,4 \end{cases}$ 
 $\begin{cases} X_1 = \frac{2,6 - 0,8X_2}{0,2} = 13 - 4X_2 \\ 0,2(13 - 4X_2)^2 + 0,8 = 7,4 \end{cases}$

$169 - 104X_2 + 16X_2^2 + 4X_2^2 = 37$   
 $5X_2^2 - 26X_2 + 132 = 0$   
 $D = 676 - 4 \cdot 5 \cdot 132 = 16$

$X_2 = \frac{26 - 4}{10} = 2,2 \Rightarrow X_1 = 13 - 4 \cdot 2,2 = 4,2$   $X_1 < X_2$  шартына сыйкес  
 келмейди

$X_2 = \frac{26 + 4}{10} = 3 \Rightarrow X_1 = 13 - 4 \cdot 3 = 1$

И: 

X	1	3
P	0,2	0,8

~425

X	1	$X_2$	$X_3$	$M(X) = 2,2$	$X_1 < X_2 < X_3$
P	0,3	0,2	0,5	$D(X) = 0,76$	$p_3 = 1 - (0,3 + 0,2) = 0,5$

$\begin{cases} 0,3 + 0,2X_2 + 0,5X_3 = 2,2 \\ 0,3 + 0,2X_2^2 + 0,5X_3^2 - (2,2)^2 = 0,76 \end{cases}$ 
 $\begin{cases} 0,2X_2 + 0,5X_3 = 1,9 \\ 0,2X_2^2 + 0,5X_3^2 = 5,3 \end{cases}$

$X_2 = \frac{1,9 - 0,5X_3}{0,2} = 9,5 - 2,5X_3$   
 $0,2(9,5 - 2,5X_3)^2 + 0,5X_3^2 = 5,3$   
 $90,25 - 47,5X_3 + 6,25X_3^2 + 2,5X_3^2 = 5,3$   
 $0,35X_3^2 - 1,9X_3 + 2,55 = 0$   
 $D = 3,61 - 4 \cdot 0,35 \cdot 2,55 = 0,04$   
 $X_3 = \frac{1,9 + 0,2}{0,7} = 3 \Rightarrow X_2 = 9,5 - 2,5 \cdot 3 = 2$

И: 

X	1	2	3
P	0,3	0,2	0,5

~426

1) 

X	1	2	3	4	5	6
P	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

 $M(X) = ?$   
 $\Theta(X) = ?$

$$M(X) = \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6} = \frac{21}{6}$$

$$\Theta(X) = M(X^2) - M^2(X) = \frac{1}{6} + \frac{4}{6} + \frac{9}{6} + \frac{16}{6} + \frac{25}{6} + \frac{36}{6} - \left(\frac{21}{6}\right)^2 = \frac{91}{6} - \frac{441}{36} = \frac{105}{36} = \frac{35}{12}$$

2) 

X	0	1	2	3
P	$P_1$	$P_2$	$P_3$	$P_4$

 $p = \frac{1}{2}$  естанба түсу ыктымалдыгы  
 $q = 1 - \frac{1}{2} = \frac{1}{2}$  естанба түспү ыктымалдыгы

$X_1 = 0$   $P(0) = q_1 \cdot q_2 \cdot q_3 = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

$X_2 = 1$   $P(1) = p_1 q_2 q_3 + q_1 p_2 q_3 + q_1 q_2 p_3 = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$

$X_3 = 2$   $P(2) = p_1 p_2 q_3 + p_1 q_2 p_3 + q_1 p_2 p_3 = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$

$X_4 = 3$   $P(3) = p_1 \cdot p_2 \cdot p_3 = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

демек 

X	0	1	2	3
P	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$M(X) = \frac{0}{8} + \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = 1,5$

$\Theta(X) = \frac{0}{8} + \frac{9}{8} + \frac{12}{8} + \frac{9}{8} = 3,75$

MC: 1,5; 3,75

3) 

X	1	2	3	4	5	6	7	8	9	10
n	1	2	2	3	2	4	2	4	3	4

n-бөлүштөр саны  
 $p = \frac{x}{n}$  демек  $\Rightarrow$

X	1	2	3	4	5	6	7	8	9	10
p	$\frac{1}{27}$	$\frac{2}{27}$	$\frac{2}{27}$	$\frac{3}{27}$	$\frac{2}{27}$	$\frac{4}{27}$	$\frac{2}{27}$	$\frac{4}{27}$	$\frac{3}{27}$	$\frac{4}{27}$

$$M(X) = \frac{1}{27} + \frac{4}{27} + \frac{6}{27} + \frac{12}{27} + \frac{10}{27} + \frac{24}{27} + \frac{14}{27} + \frac{32}{27} + \frac{27}{27} + \frac{40}{27} = \frac{170}{27}$$

$$\Theta(X) = M(X^2) - M^2(X) = \frac{422}{9} - \left(\frac{17}{27}\right)^2 = \frac{5822}{729}$$

Өзүңдү текшер!

1)  $\frac{12 - a_8}{8 - a_8} = \frac{12 + 4 - 2}{24 - 3}$  (A)

2) 

X	5	8	12	15	18
P	?	0,15	?	0,15	?

 $2x + 3x + 2x + 0,15 + 0,15 = 1$   
 $7x = 0,7$   
 $x = 0,1$

X	5	8	12	15	18
P	0,2	0,15	0,3	0,15	0,2

 (C)

3)

X	6	?	?	?	18
P	?	0,3	0,3	0,3	?

$$a_1 = 6 \quad a_5 = 18$$

$$a_5 = a_1 + 4d = 6 + 4d = 18 \Rightarrow d = 3 \text{ gemein } a_2 = 9, a_3 = 12, a_4 = 15$$

$$x + x + 0,3 + 0,3 + 0,3 = 1 \Rightarrow x = 0,05$$

X	6	9	12	15	18
P	0,05	0,3	0,3	0,3	0,05

Ⓟ

4)

X	2	4	5	7
P	0,1	0,2	0,4	0,2

$$M(X) = 2 \cdot 0,1 + 4 \cdot 0,2 + 5 \cdot 0,4 + 7 \cdot 0,2 = 4,4$$

$$D(X) = 4 \cdot 0,1 + 16 \cdot 0,2 + 25 \cdot 0,4 + 49 \cdot 0,2 - (4,4)^2 = 4,04$$

$$\sigma(X) = \sqrt{D(X)} = \sqrt{4,04} \approx 2$$

5)

$x_i$	3	5	8	10
$n_i$	1	2	3	4

$$n = 1 + 2 + 3 + 4 = 10$$

$$\omega_1 = \frac{1}{10} \quad \omega_2 = \frac{2}{10} \quad \omega_3 = \frac{3}{10} \quad \omega_4 = \frac{4}{10}$$

$x_i$	3	5	8	10
$n_i$	0,1	0,2	0,3	0,4

Ⓟ